Reprinted from the *Soil Science Society of America Journal*
Volume 61, no. 3, May–June 1997
677 South Segoe Rd., Madison, WI 53711 USA

A SINGLE, CONTINUOUS FUNCTION FOR SLOPE STEEPNESS INFLUENCE ON SOIL LOSS

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Abstract

Recently proposed relationships for the effect of slope steepness on soil loss by water are linear functions of the sine of the slope angle. The Revised Universal Soil Loss Equation (RUSLE) uses two such functions: one for slopes <9% and another for slopes >9%. Recent research indicates that yet a different linear function is necessary for slopes greater than approximately 22%. The objective of this study was to develop a single slope steepness function that is representative of the data for all slopes. The resultant equation takes the form of a logistic function. It closely follows the RUSLE relationships for the slope steepness factor for slopes up to 22%, and also fits existing data for slopes greater than those from which the RUSLE relationships were derived.

Early relationships that describe the effect of slope steepness on soil loss caused by rainfall and runoff on hillslopes were of various forms (Zingg, 1940; Smith and Whitt, 1947; Musgrave, 1947; Smith and Wischmeier, 1957; Wischmeier and Smith, 1978), but recent relationships have described the effect as a linear function of the sine of the slope angle (e.g., McIsaac et al., 1987; McCool et al., 1987). The RUSLE uses the relationships proposed by McCool et al. (1987), which are:

$$S = 10.8 \sin\theta + 0.03 \text{ when } \sin\theta < 0.0896 \quad [1]$$

and

$$S = 16.8 \sin\theta - 0.5 \text{ when } \sin\theta \leq 0.0896 \quad [2]$$

These relationships follow the "unit plot" convention used in USLE technology wherein the slope factor, S, is the soil loss at slope with angle θ, normalized to the soil loss at 9% ($\sin\theta = 0.0896$) slope (Wischmeier and Smith, 1978). Equation [1] is based on erosion plot studies on slopes up to 18% from LaCross, WI, and Eq. [2] is based on data from a study conducted by Murphee and Mutchler (1981) on slopes from 0.1 to 3%. The equations were compared with 50 sets of natural runoff plot data from slopes up to 22% (McCool et al., 1987).

Liu et al. (1994) presented data from three sites located in the Yellow River loess plateau of China for slopes up to 55%. They presented a relationship of the form

$$S = 21.91 \sin\theta - 0.96 \quad [3]$$

While both Eq. [2] and [3] are normalized to 9% slope, the RUSLE relationship (Eq. [2]) underpredicts the slope factor for the data presented by Liu et al. (1994) for slopes greater than approximately 22%.

Equations [1], [2], and [3] represent linear forms of the slope steepness function for soil loss as slope progressively increases. That is, Eq. [1] was derived from low slopes, Eq. [2] was derived from moderate slopes,

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Published in Soil Sci. Soc. Am. J. 61:917–919 (1997).

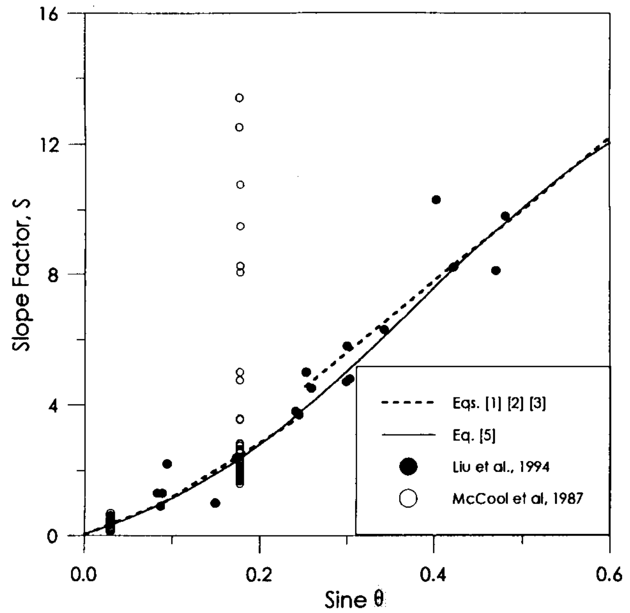

Fig. 1. Slope steepness factor, *S*, vs. the sine of the slope angle, θ, for values of sinθ up to 0.6.

and Eq. [3] was derived using data from slopes up to 55%. Note also that the first coefficient in each of the three equations (the slope of the line between *S* and sinθ) increases from Eq. [1] to [3], while the intercept decreases. At first glance, one might attempt to derive a general equation using the form

$$S = m \sin\theta + b \qquad [4]$$

where *m* and *b* change as a function of sinθ. As an example, if *m* and *b* are considered to be linear functions of sinθ, the resultant equation for *S* is a quadratic function of sinθ, not unlike the form used in the USLE. The problem with this approach, which is also exhibited with

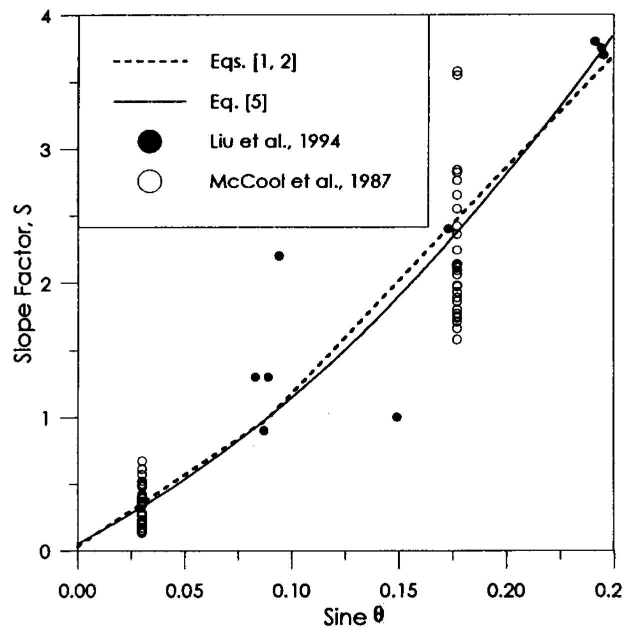

Fig. 2. Slope steepness factor, *S*, vs. the sine of the slope angle, θ, for values of sinθ up to 0.25.

the USLE slope relationship, is that the resultant quadratic form of the equation generates predictions of *S* that are too great at high slopes.

The purpose of this study was to develop a slope steepness function that: (i) is consistent with the RUSLE relationships (Eq. [1] and [2]), which are based on extensive data for slopes up to 22%, (ii) is consistent with the data from the Yellow River loess plateau from very steep slopes, and (iii) is a single, continuous function of sinθ.

Methods

The approach taken was to fit a curve as closely as possible to the curves given by Eq. [1], [2], and [3]. The RUSLE relationships (Eq. [1] and [2]) for slope steepness are based on a very wide array of data sets (McCool et al., 1987), and Eq. [3] is based on possibly the only significant amount of reliable data for slopes steeper than 22%. Since most applications of prediction models are intended for slopes under 22%, and since a majority of the data is from such slopes, we considered it necessary to fit the RUSLE curves as exactly as possible, while giving results for steeper slopes that are statistically representative of the steeper slopes. Thus, the method followed was to derive a single curve that fits closely with Eq. [1] for $\sin\theta < 0.0896$, with Eq. [2] for $0.896 < \sin\theta \leq 0.25$, and Eq. [3] for $0.25 < \sin\theta \leq 0.6$.

Several types of curves might have been used to give an equivalent level of fit to Eq. [1], [2], and [3] for their representative ranges. However, visual observations indicated that this curve should be S-shaped in order to give an adequate fit. The natural choice, therefore, was the use of a logistic function. This function was also chosen based on physical considerations. Recent experimental data from Nearing et al. (1997) indicate a fit between rill erosion rates and a logistic function of flow energy.

Evaluation of the proposed equation was made using graphical comparisons of Eq. [5] to Eq. [1], [2], and [3], and with comparisons to previously reported data. Data for slope factor *S* derived from erosion plots and presented in Table 3 of McCool et al. (1987) for 3 and 18% slope were used to evaluate Eq. [5] relative to Eq. [1] and [2], respectively. Erosion plot data from China and reported by Liu et al. (1994) were used to evaluate Eq. [5] relative to Eq. [2] and [3]. The error term used for statistical comparisons was the absolute value of the differences between the slope factors, *S*, from erosion plot data and the equation predictions, ΔS. Paired sample, two-tailed *t*-test comparisons of means ($\alpha = 0.02$) of ΔS were made between Eq. [5] and Eq. [1] using the 3% slope data of McCool et al. (1987), between Eq. [5] and Eq. [2] using the 18% slope data from McCool et al. (1987), and between Eq. [5] and Eq. [2] and [3] using the data from China of Liu et al. (1994).

Results

The resultant logistic equation for the effect of slope steepness on soil loss on hillslopes by rainfall and runoff is given by

$$S = -1.5 + 17/[1 + \exp(2.3 - 6.1 \sin\theta)] \qquad [5]$$

The curves for Eq. [1], [2], [3], and [5], as well as data from Liu et al. (1994) for steep slopes and from McCool et al. (1987) are presented in Fig. 1 for slopes in the range $0 \leq \sin\theta \leq 0.6$ and in Fig. 2 for a closer view of

Table 1. Averages and standard deviations (SD) of the absolute values of the difference between the erosion plot data and the equation predictions, ΔS.

	Data from 3% slope		Data from 18% slope		Data from Liu et al., 1994		
	Eq. [1]	Eq. [5]	Eq. [2]	Eq. [5]	Eq. [2]	Eq. [3]	Eq. [5]
Avg. ΔS	0.095	0.102	1.68	1.66	0.95	0.65	0.59
SD ΔS	0.084	0.079	2.78	2.80	1.04	0.59	0.67
n	41	41	41	41	19	19	19

the range $0 \leq \sin\theta \leq 0.25$. Average absolute errors, ΔS, for comparisons to data are reported in Table 1.

Discussion

Differences between Eq. [5] and Eq. [1] and [2] for slopes of $\sin\theta \leq 0.25$ are minor (Fig. 2). The maximum difference between values of Eq. [5] and Eq. [1] and [2] for $\sin\theta \leq 0.25$ was 0.16, which is only 10% of the average error for the RUSLE equation compared with the data from which it was developed (Table 1). The paired sample, two-tailed t-test comparisons of means showed that the calculated error term, ΔS, was not statistically different ($\alpha = 0.02$) between Eq. [1] and [5] for the 3% data or between Eq. [2] and [5] for the 18% data. We conclude from these comparisons that Eq. [5] adequately represents Eq. [1] and [2] and the data from which they were derived for slopes up to 25%.

The greatest difference between S values from the logistic curve and from Eq. [3] was at $\sin\theta = 0.25$ where the difference was 0.66. The paired sample, two-tailed t-test comparisons of means ($\alpha = 0.02$) indicated that Eq. [5] gave a better fit to the data from China as reported by Liu et al. (1994) than did Eq. [2]. There was no statistical difference in performance between Eq. [3] and [5] relative to the China data.

Conclusions

A slope steepness function for soil loss caused by rainfall and runoff was derived. The proposed equation has the advantages that: (i) it is a single, continuous function for the USLE or RUSLE slope factor that is equivalent in its accuracy in matching existing soil loss data to the linear functions currently used for RUSLE at slopes <25%, and (ii) it fits the existing data for slopes of >25% better than the current RUSLE slope steepness function, and as well as the equation proposed by Liu et al. (1994).

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