

KINEMATIC PARAMETER ESTIMATION FOR TRANSPORT  
OF POLLUTANTS IN OVERLAND FLOW

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Abstract

A water quality time of travel model was developed to simulate the movement of nonpoint source pollutants in overland flow. The model was derived from basic kinematic wave theory which has been used by recent researchers to describe shallow flow in open channels and flow from surface runoff. The basic kinematic wave theory was adapted to describe the movement of water particles. Because particle motion is analogous to the movement of a conservative soluble pollutant, theory which defines this motion may be an elemental key to understanding the movement of pollutant particles in surface runoff.

Experiments to test the model were conducted on the half-acre Rainfall-Runoff Experimental Facility at Colorado State University. The Facility's size is such that it is an intermediate step between laboratory models and natural watersheds. The experimental watershed is impermeable and can be subjected to a variety of rainfall intensities and duration; also surface roughness and watershed configuration can be controlled. Six hundred experiments were performed to verify the model's predictive capabilities. A rectangular plane 30 meters long and 2 meters wide was selected as the test section in the water quality studies. Rhodamine WT was used to simulate a soluble, conservative pollutant. This fluorescent dye was injected at several points on the watershed surface so that travel times of the dye in overland flow for various conditions could be measured. The experiments were conducted using four surface roughnesses and four different rainfall intensities. The roughnesses ranged from a smooth plane surface to a surface of rills and eroded channels. Other variables investigated included rainfall duration and the quantity and location of the tracer within the watershed. A statistical regression analysis was performed on the data and indicated that the travel

times were mainly a function of rainfall intensity and location of tracer input. These two variables are the primary components in the theoretical model. A curve fitting technique was used on the observed data to obtain the best estimate in a least squares sense of the kinematic wave parameters. The theoretical model with these calculated parameters accurately predicted the observed tracer travel times.

Three separate techniques for quickly estimating the parameters of the kinematic wave equation were also developed. Each of these simple estimation techniques was tested using the observed data. The estimation method which assumed Chezy's relationship for turbulent flow proved quite precise and generated estimates of the kinematic parameters which could be used in the model to accurately predict travel times of the soluble tracer.

Results of the study serve as further verification of the validity of kinematic wave theory equations for overland flow and produce a model which could be utilized in the prediction of pollutant movement on watersheds and in the estimation of kinematic parameters. The research indicates that the theoretical kinematic equations with estimated parameters can predict the immediate impact of an instantaneous trace of pollutant carried in overland flow.

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## Introduction

Pollutants transported by storm water runoff from non-point sources are major contributors to the degradation of water quality in the nation's rivers. Studies have revealed that at certain times urban storm water runoff is a more serious threat to water quality than raw sewage discharges. During some storm events, runoff from street surfaces has been found to contribute significantly more BOD, (Biochemical Oxygen Demand) to area waters than the effluent from sewage treatment plants (1). Urban nonpoint pollution results primarily from debris and contaminants on the streets, contaminants from open land areas, publicly use chemicals, and dirt and contaminants washed from vehicles. Water soluble chemicals and pollutants have been found in urban storm water runoff at environmentally significant concentrations (2).

Much attention has been given in recent years to the development of models which can simulate storm water runoff from watershed surfaces. Theoretical equations have been developed to describe shallow flow in open channels and several adequately approximate overland flow. Although these models have been tested for their ability to describe unsteady, free-surface flow, none have been adequately tested as chemical

transport models. A number of models have been used to simulate the accumulation and removal of insoluble and partially soluble nonpoint source pollutants (3,4,5). None of these models, however, have components which can simulate the transport of soluble pollutants. The purpose of this research was to develop a transport model for conservative soluble pollutants on an impervious plane and to test it with data from carefully controlled experiments.

### Kinematic Wave Theory

One set of equations used to describe overland flow is based on kinematic wave theory. First introduced by Lighthill and Whitham in 1955 (6), the theory has been applied to surface runoff by Henderson and Wooding (7), Wooding (8), and Woolhiser (9).

Derivation of the kinematic equations is based upon the governing equations for unsteady free-surface flow introduced by DeSaint Venant (10). The equation for continuity on a plane surface is:

$$\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} = q \quad (1)$$

and the simplified momentum equation is:

$$S_o = S_f \quad (2)$$

where:  $h$  = depth of flow,  
 $q$  = lateral inflow rate,  
 $x$  = distance from upstream boundary,  
 $u$  = local average velocity,  
 $t$  = time,  
 $S_o$  = bed slope,  
 $S_f$  = friction slope.

Substitution of the Darcy-Weisbach equation or the Chezy equation into Eq. (2) will produce the following parametric relationship for depth of flow and velocity:

$$u = \alpha h^{m-1} \quad (3)$$

where:  $\alpha$  = roughness and slope parameter,  
 $m$  = flow regime parameter.

The parameter  $m$  has a value of approximately 1.5 for turbulent flow. Equation (3) has been used by researchers as the basic relationship in numerous watershed models (11,12). Models by Harley et al. (13) and Schaaque (14) have demonstrated

the applicability of the kinematic wave theory in prediction of runoff hydrographs.

### Formulation of Theoretical Model

The water quality model presented herein is derived to predict the convective movement of soluble, conservative pollutants in overland flow. Dispersion is ignored. The basic prediction model is derived by Brazil (15) from Eq. (3). The water particle velocities are represented by:

$$u = \left( \frac{dx}{dt} \right)_p \quad (4)$$

Consider spatially uniform rainfall at a constant rate  $i$  beginning at time 0, falling on an impervious surface. Prior to the time of equilibrium (within the shaded zone in Fig. 1 the theoretical depth of the water ( $h'$ ), is a function of the intensity of the rainfall ( $i$ ) and the time ( $t$ )

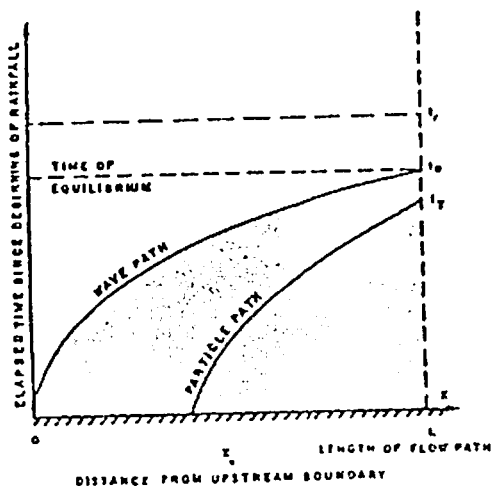


Fig. 1. Characteristic wave and particle paths in the  $x$ - $t$  plane

since the start of rainfall input. If the surface is initially dry, water depth at time,  $t$ , after rainfall begins is given by:

$$h' = i t \quad (5)$$

where:  $h'$  = theoretical depth,  
 $i$  = rainfall intensity.

The theoretical depth ( $h'$ ) will vary from the actual depth ( $h$ ) by a factor determined from the concentration of objects blocking flow paths on the flow plane. For most practical situations, however, the use of theoretical depths causes no problems in model applications. Substituting Eqs. (4) and (5) into Eq. (3) yields:

$$\left(\frac{dx}{dt}\right)_p = \alpha(it)^{m-1} \quad (6)$$

Integration and rearranging terms gives the time required for a particle starting at  $(x_0, t)$  to reach the lower boundary ( $x = L$ ).

$$t_T = \left(\frac{m(L-x_0)}{\alpha(i)^{m-1}}\right)^{1/m} \quad (7)$$

where:  $t_T$  = travel time (seconds),  
 $i$  = intensity of rainfall (meters/second),  
 $L$  = length of plane (meters),  
 $x_0$  = distance from top of plane (meters).

Eq. (7) is applicable only on the rising limb of the hydrograph. After equilibrium is reached on the watershed, the depth no longer increases with time and the flow is considered to be steady and nonuniform. Therefore, it is necessary to compare the observed and predicted  $t_T$  with time to equilibrium ( $t_e$ ) and with the time at which rainfall stops ( $t_r$ ) for partial equilibrium hydrographs. Time to equilibrium is dependent upon the characteristic wave velocities, which can be determined with an expression derived from the continuity equation and Eq. (6). Substituting Eq. (3) into Eq. (1) yields:

$$\frac{\partial h}{\partial t} + \alpha m h^{m-1} \frac{\partial h}{\partial x} = q \quad (8)$$

Solving Eq. (8) and the total differential of  $h(x, t)$  in matrix form yields a solution to the characteristic velocity in the  $x$ - $t$  plane:

$$\left(\frac{dx}{dt}\right)_w = \alpha m h^{m-1} \quad (9)$$

Eq. (9) represents the velocity of the characteristic wave down the plane. The notation  $(dx/dt)_w$  is used to denote the wave velocity as opposed to the particle velocity  $(dx/dt)_p$ . Note that the wave velocity is  $m$  times greater than the

particle velocity. The characteristic paths for both the wave and particle velocities are shown in Fig. 1.

By substituting Eq. (5) into Eq. (9) and integrating we obtain an expression for time to equilibrium,  $t_e$ .

$$t_e = \left( \frac{L}{\alpha(i)^{m-1}} \right)^{1/m} \quad (10)$$

It should be noted that for given values of  $i$  and  $L$  the time to equilibrium is determined by the kinematic parameters  $\alpha$  and  $m$ . A similar expression can be derived for the time at which recession begins for a partial equilibrium hydrograph.

### Verification of Theoretical Model

An experimental procedure was designed to collect data which could test the model's accuracy in predicting time of travel for soluble, conservative pollutants. Because water particle movement was investigated using a fluorescent dye, Rhodamine WT, its travel time across a watershed could be accurately recorded using a fluorometer.

Experimental facility. The Rainfall-Runoff Experimental Facility at Colorado State University was used as the model watershed. An impervious area of approximately 25,000 square feet, the facility simulates a watershed without infiltration, intermediate in size between laboratory models and natural watersheds. The general arrangement of the Rainfall-Runoff Facility is shown in Fig. 2. Characteristics such as surface roughness, imperviousness, and geometry can be changed to represent a wide variety of natural catchments. In addition, simulated rainfall can be generated at rates of 13, 28, 58, and 108 millimeters per hour. Holland (16) gives further details of the facility.

To simplify the conditions on the watershed, one plane section of the surface, two meters wide by thirty meters long, was partitioned from the rest of the facility for use in the experimental runs. Like the plane section of the facility, the experimental section was oriented so as to have a five percent slope. Discharge was measured through a 0.6 foot HS flume and the concentration of the fluorescent dye in the water was analyzed with a continuous flow Turner Model 111 fluorometer.

Experimental procedure. Equilibrium experimental runs were conducted with the dye injected at a point or a transverse line on the plane surface after the system had reached a steady state; for nonequilibrium runs the dye was injected

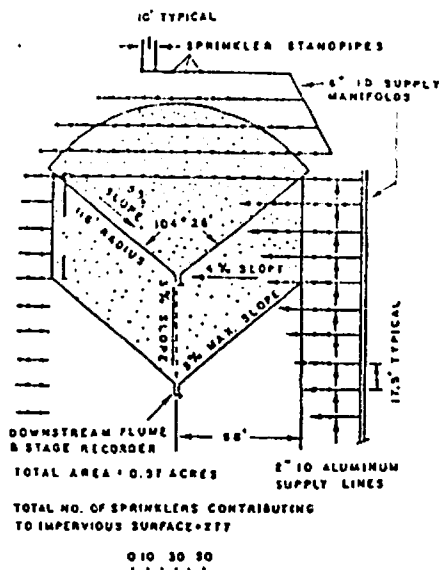


Fig. 2. Experimental Rainfall-Runoff Facility-general arrangement

simultaneous with the start of rain. Thus, for the nonequilibrium runs the dye represented an immediately soluble pollutant already on the surface at the beginning of a rainstorm. During each run, a stopwatch was used to determine the length of time the dye was actually traveling on the exposed plane. This time was equal to the time interval between tracer injection and peak concentration readout minus the time the tracer was pumped and transmitted to the fluorometer.

Variables investigated included surface roughness, rainfall intensity, rainfall duration, and the quantity and location of tracer input. Four surface roughnesses were included in the tests. These were:

- Set A - watershed covered with butyl rubber.
- Set B - 2 kg/m<sup>2</sup> of 1½-in. dia. gravel spread on the butyl surface.
- Set C - 10 kg/m<sup>2</sup> of 1½-in. dia. gravel spread on the butyl surface.
- Set D - artificial rilled surface (a smooth surface of a wet sand and cement mixture was eroded by simulated rainfall and allowed to harden for two days).

All four of the available rainfall intensities were utilized for each set of runs. Rainfall was applied for three durations: approximately one-half the time before the watershed runoff

reaches equilibrium, a time approximately equal to equilibrium time, and a time approximately 1.3 to 1.5 greater than the equilibrium time. Injections were made at two locations: distances of six meters and fifteen meters from the upper boundary. The four roughnesses, four intensities, three durations, and two locations made a total of ninety-six combinations of nonequilibrium travel time runs.

A few experiments were also conducted to collect data to examine the variables under conditions different from the normal travel time runs. One series included tracing the dye down the plane with the Set D roughness while the intensity of the rainfall was varied. These runs allowed the prediction model to be tested for rainfall patterns more characteristic of nature. Another set of runs was made with the dye injected as a line source across the flow plane to examine the effects of nonuniform cross sections. Approximately six hundred runs were made to test the model under various conditions.

## Results

Statistical analysis. A statistical regression analysis was performed on the travel time data to make sure it was predictable and could be correlated with the variables in the model. The variables examined included intensity, initial tracer location, rainfall duration, time of concentration, peak time, amount of dye injected, and dye concentration. The analysis indicated that the most important variables were rainfall intensity and the injection location and that the correlation coefficient was 0.90 or greater for combinations of these two variables alone. The theoretical model, Eq. (7) uses both intensity and injection location as input variables.

Theoretical analysis. The theoretical analysis was divided into three phases. The first phase of the study used equilibrium run data for finding the best estimate of the kinematic parameters  $\alpha$  and  $m$ . Next, the travel time data for nonequilibrium runs were compared with the predicted travel times using the best estimates of  $\alpha$  and  $m$  in Eq. (7) and finally, an analysis was conducted to see how well the model could predict travel times when rainfall intensity was varied during a single run.

For equilibrium runs the discharge at any unit width of cross section was only a function of rainfall intensity and distance from the top of the plane. The plane was divided into several reaches and during subsequent runs dye injections were made at a different section to determine the average velocity for each section. This information was used along with equilibrium discharge quantity for each section to calculate average depth of flow, the principle variable in Eq. (3). An expression for discharge per unit width,  $Q$ , can be derived



by substituting  $h'$  into Eq. (3) and by multiplying through by  $h'$ . The new equation is:

$$Q = \alpha(h')^m \quad (11)$$

The log form of this equation plots as a straight line on log-log paper with a dependent variable intercept of  $\log \alpha$  and a slope of  $m$ . Steady state data were taken for each surface roughness and showed that  $Q$  was highly correlated to  $h'$ . In every case the slope of the line,  $m$ , was approximately 1.5 indicating that the flow was turbulent. Equation (7) indicates that travel time is inversely proportional to  $\alpha$  and that in general a rougher surface will have a smaller value of  $\alpha$ . An analysis of the best estimates of  $\alpha$  and  $m$  from the steady-state runs showed that because of the intercorrelation of  $\alpha$  and  $m$ , and the fluctuation that can exist in the actual value of  $m$ , the roughest surface will not always have the smallest value of  $\alpha$ .

Best estimates of  $\alpha$  and  $m$  obtained from steady-state tracer studies were used in the model, Eq. (7), to predict travel times for the ninety-six combinations of input variables for unsteady flow. The model equations and observed data points were plotted on log-log paper with travel times as a function of rainfall intensity and location of injection. Figure 3 shows a graphical representation of the runs for Set C. A mean error analysis of the predicted travel times was determined using a sum of the errors square criterion for the observed data. The mean error for all four sets was 21 seconds using the best estimate of  $\alpha$  and  $m$  from the steady state runs. The observed data were generally within 10% of the predicted times and shorter travel times had lower deviations so that the percentage difference between the predicted and observed times was fairly consistent.

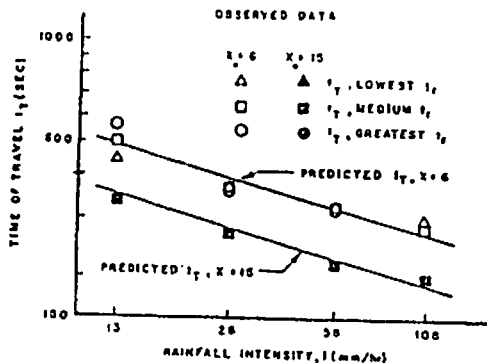


Fig. 3. Predicted and observed travel times versus rainfall intensities for Set C

For variable rainfall intensity the water depth within the shaded area on Fig. 1 is given by

$$R = i_1 t_1 + i_2 t_2 + \dots \quad (12)$$

where  $t_1$  is the duration of the rainfall of intensity  $i_1$  and  $t_2$  is the duration of the rainfall of intensity  $i_2$ , etc. Substitution of this expression into Eqs. (3) and (4) gives an expression which can be used to route the water particles down the plane. The mean error of the predicted travel times using the observed data from the variable intensity runs was 16 seconds. Equation (7) was also used to predict times with the average intensity calculated from total rainfall accumulation used as the input. The mean error for these travel times was 27 seconds, 11 seconds higher than the times using the routing technique.

Equation (7) was also used to predict time for the pollutants injected as a transverse line source. The mean error was 13 seconds, indicating that the model was even more accurate for the line source data examined than for the point source data.

Estimation techniques for kinematic parameters. Several techniques were examined to utilize tracer data to estimate the parameters in the kinematic equation. For injections of dye made at different distances,  $x_1$  and  $x_2$  from the top of the watershed but having equal rainfall intensities the travel times to the lower boundary will be  $t_1$  and  $t_2$  respectively. Substituting these values simultaneously into Eq. (7) yields

$$m = \frac{\ln(L-x_1) - \ln(L-x_2)}{(\ln t_1 - \ln t_2)} \quad (13)$$

$$\alpha = \frac{(L-x_1)^m}{i^{m-1} t_1^m} \quad (14)$$

The prediction model with estimated values of  $\alpha$  and  $m$  was tested against data from the ninety-six travel time runs. A mean error of 28 seconds was calculated showing that the pair of parameters could be accurately estimated with a small amount of data.

In an attempt to further simplify the estimating technique, the parameter  $m$  was held constant and Eq. (14) was solved to give  $\alpha$ . The technique was based on the Chezy turbulent flow relationship, giving a value of 1.5 for  $m$ . This method requires only one dye injection and observed travel time

to solve for  $\alpha$ . The model equation with  $m$  equal to 1.5 and  $\alpha$  estimated from the simplified technique gave predicted travel times with a mean error of 20 seconds. For the four surface roughnesses, Set A, B, C, and D, and assuming  $m=1.5$ ; the values of  $\alpha$  were 8.62, 4.71, 1.68 and 4.03 respectively. The method assuming a constant  $m$  value proved more accurate for the observed data than the technique which allowed both parameters to vary.

Two estimating methods were also tested with  $m$  assumed to be 1.67, the value obtained from Manning's equation. The model with  $m = 1.67$ , had a greater mean error indicating that Chezy relationship better represented the data. The one injection parameter estimation techniques were also used with variable intensity rainfall. Results were similar to the runs with constant intensity.

A nomograph was prepared to quickly estimate  $\alpha$  with data from one observed travel time run and is shown in Fig. 4. The graph displays a solution to Eq. (14) with input variables of travel time, length of flow plane, and rainfall intensity. Because the model is designed to predict only time for flows on the rising limb of the hydrograph, dye injection's must be made at a location within the watershed in order that the kinematic wave from the farthest boundary reaches the bottom

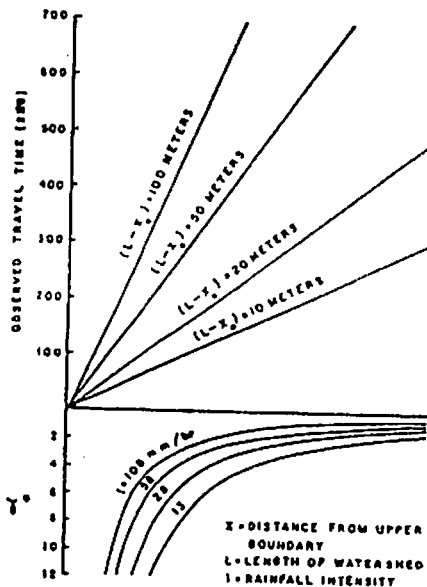


Fig. 4. Nomograph for predicting  $\alpha$ ,  $m = 1.5$

of the watershed after the dye concentration peak arrives. To insure that this condition is met, the dye must be injected at a distance greater than  $L(m-1/m)$  from the top of the plane (15). For  $m = 1.5$ , the injection must be made within the lower two-thirds of the watershed. The six meter injection point violates this condition but an analysis showed that the expected deviations are small (15).

Applications of the theoretical model. Numerous papers have been written on the value of kinematic wave theory in synthesizing hydrographs and on methods of calculating the kinematic parameters. The parameter estimation techniques described herein can be useful for selecting parameter values for the hydrograph equations as well as for the pollutant travel time model.

The correlation of pollutant strength and sediment combined with the simulation of sediment transport in a watershed has been recently proposed as a methodology for assessing the movement of insoluble and partially soluble nonpoint pollutants (5). Curtis has presented a model that uses the kinematic wave formulation and a set of relationships describing soil detachment and transport processes to simulate the discharge of sediment from an urban area (17). The experimental evidence presented herein suggests that the kinematic model is appropriate for pollutant transport as well as runoff hydrograph prediction.

One specific application of the model would be its use in the prediction of salt transport in urban runoff. Water soluble de-icing salts are applied to roads in the United States during the winter when snow and ice accumulate. The use of road salts has been increasing over the past three decades and has been linked to increases in chloride content of streams (18). The kinematic wave and chemical transport equations in finite difference form may be a useful tool in further understanding the processes involved in salt transport.

### Summary

A water quality time of travel model was developed based on the assumption that soluble pollutants in overland flow travel at velocities consistent with the relationships governed by the kinematic wave equations. Estimation techniques were presented for calculating the parameters in the model. The model equation was used with estimated parameters to predict pollutant travel times and was shown to be quite accurate when compared with the observed data.

## Conclusions

This study further verifies the validity of the kinematic wave equations for overland flow and also shows that the kinematic wave equations can be used to describe convective transport of pollutants. Data from the analyses indicate that the kinematic form for turbulent flow derived using Chezy's relationship is valid and that the presented model equations can be useful tools in the estimation of kinematic parameters as well as to predict movement of soluble pollutants on planes and, by inference, watersheds represented as networks of planes and channels.

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