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URBAN STORM RUNOFF MODEL[a]

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INTRODUCTION

The problems of stormwater drainage have increased in proportion to the rate of growth of urban areas. Major hydrologic effects of urbanization include: increased peak rate and volume of runoff, and reduced response time, and reduced hydrograph-base length. The high cost of storm sewers, drainage channels, and flow detention structures requires adequate estimates of runoff for the best stormwater system design. One method of estimating runoff is to use computer models based upon the mathematical equations describing the physical process being studied.

The rainfall-runoff process is complex with many interrelated components. Attempting to precisely model all components of the process is complicated and expensive, so to simplify the modeling process, components are approximated. The computer model used in this study is based upon proven simplifications of infiltration and surface runoff. Rainfall input to the model is considered as several discrete pulses of rain and is assumed uniform over the watershed. A watershed is presented as a series of cascading planes and channels (Fig. 1). Each plane is assumed to have uniform surface and subsurface characteristics.

SURFACE FLOW

The kinematic equations for overland flow and flow in an open channel have been developed previously (4,6,12). The one-dimensional continuity equation with lateral inflow is

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$$\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} = q \quad (1)$$

in which h = depth of flow; q = lateral inflow (rainfall); t = time; u = local average velocity; and x = the distance from the upstream end. The momentum equation for one-dimensional flow can be written as

$$\frac{1}{g}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) + \frac{\partial h}{\partial x} = S - S_f - \frac{q}{g}\frac{u}{h} \quad (2)$$

in which g = the acceleration of gravity; S = the bed slope, and S_f = the friction slope. Woolhiser and Liggett (12) showed that for most hydrologically significant cases of overland flow Eq. 2 can be closely approximated by

$$S = S_f \quad (3)$$

This approximation is used in writing a parametric equation of the form

$$u = \alpha\, h^{N-1} \quad (4)$$

in which α and N are parameters related to surface roughness (flow resistance) and geometry. The hydraulic radius for a plane or wide channel is approximately

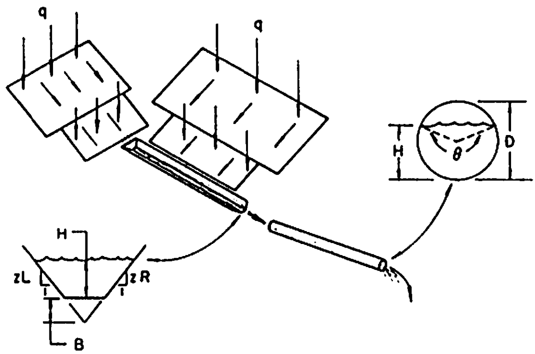

FIG. 1.—Watershed Represented as Kinematic Cascade

the depth of flow. For the Chezy equation $\alpha = C\sqrt{S}$, in which S is the slope of the plane or channel, and $N = 3/2$. For Mannings equation $\alpha = (1.49/n)\sqrt{S}$ and $N = 5/3$. Eq. 4 is substituted into Eq. 1 with the following result

$$\frac{\partial h}{\partial t} + \frac{\partial(\alpha h^N)}{\partial x} = q \quad (5)$$

Completing the differentiation with respect to x, one obtains

$$\frac{\partial h}{\partial t} + \alpha N h^{N-1}\frac{\partial h}{\partial x} = q \quad (6)$$

Eq. 6 is referred to as the kinematic flow equation and can be solved analytically for simplified initial and boundary conditions. A finite different method can be used to obtain a general solution to Eq. 6. Previous workers (4,6) found that a second-order Lax-Wendroff explicit finite difference method gave satisfactory results when modeling overland flows on an experimental watershed.

The same principles may be applied to channels with a general trapezoidal shape. The trapezoidal shape can be used to approximate geometry from nearly rectangular to very broad swale-like channels.

An important problem in urban hydrology is routing runoff through storm sewers. One routing method is simply to translate the hydrograph in time and assume that the peak does not attenuate. Another method is to apply the kinematic flow equations to storm sewers where surcharged conditions do not exist. Rovey (6) showed that kinematic routing in circular storm sewers based only upon the hydraulic characteristics of the system did as well as a lag and route method that was "fitted" to the data (3). The continuity equation for a closed conduit is

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad (7)$$

in which A = the cross-sectional area of flow; and Q = the discharge. The storm sewer receives no lateral inflow but rather receives flow at the upstream end from other storm sewers or an inlet hydrographs from a gutter or overland flow. The kinematic approximation of the momentum equation allows substituting the slope of the storm sewer for the friction slope. If the Darcy-Weisbach equation is used, the discharge equation is

$$Q = \alpha \frac{A^N}{P^{N-1}} \quad (8)$$

in which P = the wetted perimeter; $\alpha = \sqrt{(8g/f)S}$; f = the Darcy-Weisbach friction factor; and $N = 3/2$.

The explicit Lax-Wendroff finite difference scheme used to solve the overland flow and trapezoidal channel equations requires that numerical stability criteria be maintained. Initial testing of the Lax-Wendroff scheme for routing in circular conduits resulted in very small time increments being required to maintain stability. To eliminate excessive computer time, a four-point implicit finite difference scheme with a fixed time increment was used. Eq. 7 is rewritten as

$$\frac{\partial A}{\partial t} + \frac{dQ}{dA}\frac{\partial A}{\partial x} = 0 \quad (9)$$

The four-point implicit finite difference equation is

$$\frac{(A_k^{j+1} - A_k^j) + (A_{k-1}^{j+1} - A_{k-1}^j)}{2\Delta t} + \omega \left[\frac{dQ^{j+1}}{dA}\left(\frac{A_k^{j+1} - A_{k-1}^{j+1}}{\Delta x}\right)\right]$$

$$+ (1 - \omega)\left[\frac{dQ^j}{dA}\left(\frac{A_k^j - A_{k-1}^j}{\Delta x}\right)\right] = 0 \quad (10)$$

in which ω is a weighting factor between 0.5 and 1. The Newton method was used to solve this implicit nonlinear equation.

INFILTRATION

Smith (8) reported on extensive numerical solutions of Richard's equation of flow in an unsaturated porous medium for a range of soils from fine clay to a moderately uniform sand where initially the infiltration rate is limited by the rainfall rate, i. After the time of ponding, t_p, the infiltration curve has the form of a decay-type function and can be expressed as

$$f = f_\infty + A_1 (t - t_o)^{-\alpha_1} \quad (11)$$

in which f = the infiltration rate; f_∞ = the steady-state infiltration rate; t_o = the vertical asymptote of the infiltration function; and A_1 and α_1 are parameters related to the soil texture, initial water content, and rainfall rate. Fig. 2 shows a definition sketch of this infiltration relation. Smith found it advantageous

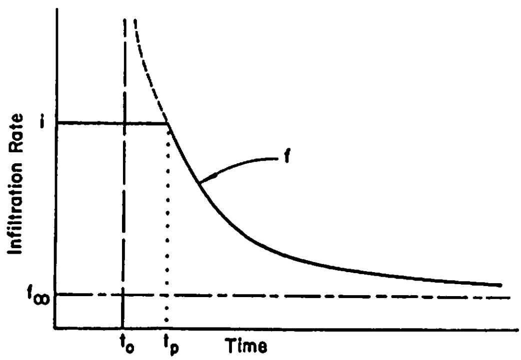

FIG. 2.—Infiltration Function

to use dimensionless variables resulting in the following normalized infiltration equation

$$f_* = 1 + (1 - \alpha_1)(t_* - t_{o*})^{-\alpha_1} \quad (12)$$

in which the dimensionless variables are defined as

$$f_* = \frac{f}{f_\infty}; \quad t_* = \frac{t}{T_o}; \quad t_{o*} = \frac{t_o}{T_o} \quad (13)$$

The normalizing time, T_o, is the time required for the volume, $f_\infty t$, to equal one-half of the accumulated infiltration.

From numerical experiments Smith found that T_o was a linear function of the initial relative soil water content, S_i. This relationship can be expressed as

$$T_o = C_1 (S_o - S_i) \quad (14)$$

in which S_o = the maximum relative saturation; and C_1 is a coefficient dependent on soil texture. A value of S_o = 0.95 is a good approximation for all soils, and the coefficient C_1 can be determined by experiment or estimated from results presented by Smith.

The steady-state infiltration rate, f_{∞}, can be obtained from field experiments or estimated from published data for various soil textures. For numerical experiments with six soils ranging from sand to clay, Smith found that α ranged from 0.4 to 0.7. When α_1, f_{∞}, and T_o have been estimated, the parameter A_1 in Eq. 11 can be obtained from the following relationship which results from the definition of T_o

$$A_1 = T_o^{\alpha}(1 - \alpha) f_{\infty} \quad (15)$$

The dimensionless time to ponding, t_{p*}, is not a function of the initial soil water content but depends on the dimensionless rainfall intensity, $i_* = i/f_{\infty}$

$$t_{p*} = \gamma (i_*)^{-\beta} \quad (16)$$

Smith tabulated values of γ for several soil textures and found that β could be estimated from γ by the regression equation

$$\beta = 1.712 + 0.279\gamma \quad (17)$$

The dimensionless time, t_{o*}, is obtained from the following relation which follows from Eq. 12 since $f_* = i_*$ at $t_* = t_{p*}$

$$t_{o*} = t_{p*} - \left(\frac{1 - \alpha}{i_* - 1}\right)^{1/\alpha} \quad (18)$$

Thus, all of the parameters in Eq. 11 can be estimated from field experiments, published data, or relationships between parameters developed by Smith.

Computer Model

A computer program is formulated to calculate the response of a watershed to a specified rainfall event. The model can be used to compute flows for the following elements: overland flow on a rectangular surface, open channel flow in a trapezoidal channel, and free surface flow in a circular conduit. Watershed geometry is represented by combinations of these geometrical segments. The computer model parameters are estimated from information obtained from topographic maps, aerial photographs, soil surveys, property development records, watershed reconnaissance, and other hydrologic data sources. Input data are utilized by the computer model to sequentially compute the outflow hydrograph from each segment. Outflow from one segment may become inflow to another segment. Fig. 3 is a flow chart of program KINGEN and provides a brief outline of the computational logic utilized by the model.

Parameter Estimation

Model parameters required are the geometric parameters, surface resistance to flow, and infiltration parameters if the surface is pervious. The geometric parameters are estimated from a topographic map of the watershed. Sloping planes are fitted to the watershed to conserve surface slope and the watershed area. The number of planes affects the computational costs; the greater the number of planes the higher the computing costs.

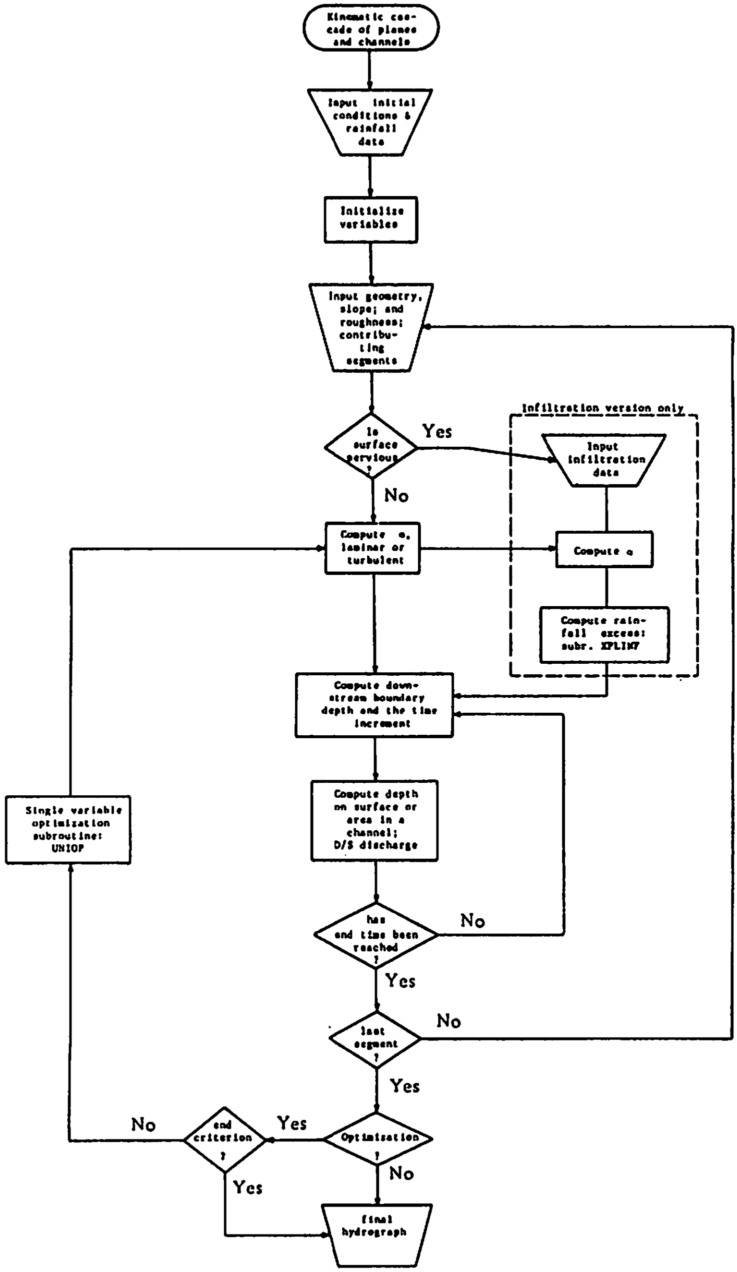

FIG. 3.—Flow Chart of Program KINGEN

Flow resistance estimation is often subjective. Friction coefficients for several surfaces have been reported in the literature (2,5). For hydrologic systems, the flow resistance is a parameter that cannot be measured directly. It is often "optimized" to obtain the best fit when comparing computed results with observed

TABLE 1.—Overland Flow Resistance Coefficients (10)

Surface (1)	Mannings *n* (2)	Chezy *C* (3)
Concrete or asphalt	0.010-0.013	73-38
Bare sand	0.010-0.016	65-33
Graveled surface	0.012-0.030	38-18
Bare clay-loam soil (eroded)	0.012-0.033	36-16
Sparse vegetation	0.053-0.130	11-5
Short grass prairie	0.10-0.20	6.5-3.6
Bluegrass sod	0.17-0.48	4.2-1.8

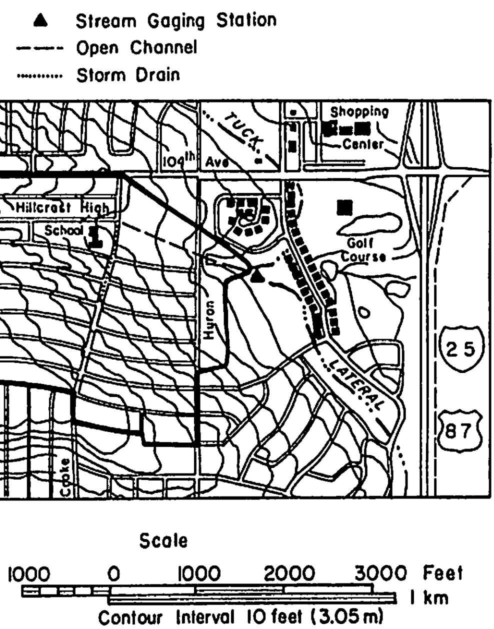

FIG. 4.—Hillcrest Drain Watershed near Northglenn, Colo.

data. Table 1 summarizes some appropriate flow resistance coefficients. Methods of estimating infiltration parameters were previously examined.

Urban Watershed

In 1968, the United States Geological Survey began a cooperative project with the Denver Regional Council of Governments and the Urban Drainage and Flood Control District to collect and analyze rainfall-runoff data from small, urban drainage basins in the Denver area (1). By 1972, there were 30 urban rainfall-runoff stations. This study is limited to a single watershed in the suburb of Northglenn; Fig. 4 is a topographic map of the watershed. Rainfall-runoff data were collected at the watershed by simultaneous operations of two digital records that record values for accumulated rainfall and water stage on paper tape at 5-min intervals. A single timer operates both recorders, eliminating any timing discrepancies between rainfall and runoff. The record of water stages is converted to flow rates by a stage-discharge curve.

The Denver area has a mean annual precipitation of about 33 cm (13 in.) unevenly distributed throughout the year. Winter precipitation is snow. Most stormwater runoff is between April and September when there is a series of

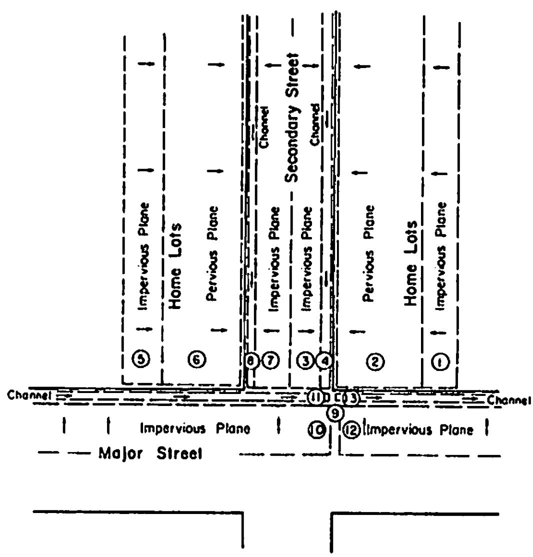

FIG. 5.—Geometric Segments for Typical Block in Urban Watershed

short-duration, high-intensity thunderstorms. Within the 670,000 m^2 (165 acres) Hillcrest Drain watershed are single-family residences, one school and adjacent grounds, plus the playground of a second school, and a small business area at the northern edge.

The soils are Fort Collins clay and clay loam with a moderate to heavy texture. These soils developed from alluvial material transported from the mountains to the west and deposited atop sandstone and shale formations. The soils are well developed and can infiltrate water moderately if protected by vegetation and not compacted. Presently, most of the pervious area is covered with bluegrass lawns. The land slopes generally to the northeast at about a 1%-2% grade. The detailed surface layout and storm sewer information were obtained from drawings and specifications filed with the city of Northglenn by the area developers.

The computer model described previously was used to simulate runoff hydrographs from the Hillcrest Drain watershed. The complex geometry of the pervious and impervious areas would require many planes and channels to represent in detail the physiographic features. The computer storage required to accommodate the large number of geometric segments is too costly for many applications. The watershed geometry may also be approximated by two or three segments, and the watershed parameters optimized by minimizing the difference between computed and measured results. However, physically interpreting parameters facilitates the estimation of model parameters from other urban watersheds.

TABLE 2.—Typical Computer Segments

Segment number (1)	Physical significance (2)	Lateral inflow (3)	Upstream Inflow	
			Plane (4)	Channel (5)
1	Roof	Rainfall	—	—
2	Lawn	Rainfall	1	—
3	Street	Rainfall	—	—
4	Gutter	Seg. 2, 3	—	—
5	Roof	Rainfall	—	—
6	Lawn	Rainfall	5	—
7	Street	Rainfall	—	—
8	Gutter	Seg. 6, 7	—	—
9	None (add channels)	—	—	4, 8
10	Street	Rainfall	—	—
11	Gutter	Seg. 10	—	—
12	Street	Rainfall	—	—

TABLE 3.—Infiltration Parameters

Parameter (1)	Numerical value (2)	Source (3)
α_1	0.53	Smith (8)
γ	0.45	Smith (8)
C_1	400	Smith (8)
f_∞	0.1	Rovey (6)
S_1	variable	—

This dilemma can be solved by a compromise of the two extremes. The watershed is represented by enough segments to maintain a resemblance to the physiologic features but this number is limited to keep the computer storage to an acceptable level. Over 150 cascading planes and channels were used to represent the watershed. Fig. 5 shows the computer segments, indicated by dashed lines, used to represent a typical block of the watershed. Each segment is numbered in the order that computations are performed. Table 2 lists the computer segments, their corresponding physical significance, and the inflow sources.

The soil characteristics previously described represent the conditions when the area was rural. The soil characteristics probably changed somewhat when the area was urbanized. The clayey subsurface material excavated for basements and foundations is often spread atop or mixed with the topsoil. The soil was probably compacted during the movement of construction equipment and the planting of lawns. These factors reduced the infiltration rate from its condition of cultivated agricultural land. Table 3 is a list of the infiltration parameters

TABLE 4.—Resistance Parameters for Hillcrest Watershed

Type of surface (1)	Chezy *C* friction coefficient (2)
Street	50
Roof	50
Lawn	4.2
Gutter	85
Storm sewer	Mannings n = 0.013

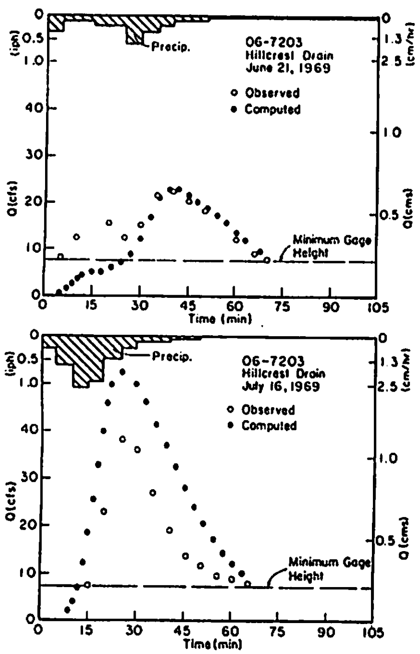

FIG. 6.—Hillcrest Drain, June 21 and July 16, 1969

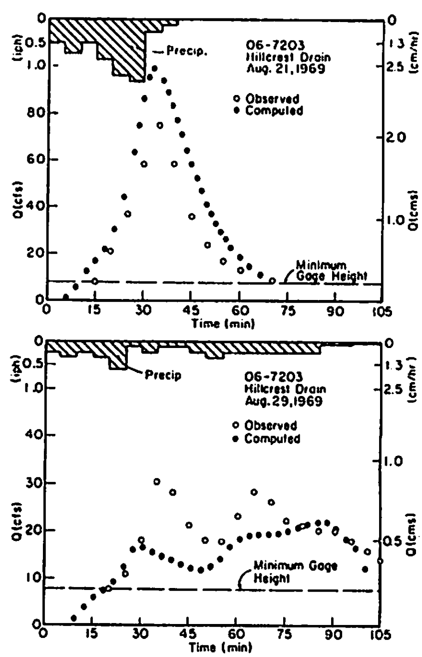

FIG. 7.—Hillcrest Drain, August 21 and 29, 1969

used in the computer model for the pervious segments of the Hillcrest Drain watershed. The initial moisture, S_i, was estimated as a function of the antecedent rainfall conditions.

The flow resistance parameters were estimated from the range of values published in the literature. Table 4 lists the values used for the urban watershed.

Results

Several observed rainfall hyetographs were utilized as input to the computer model. The model parameters were estimated as previously covered and their values have been previously listed. The results of some of the simulations of the runoff hydrographs are shown in Figs. 6, 7, and 8. These results are from simulations based upon prior estimates of model parameters and were not obtained by optimizations of events.

Simulation of the storm of June 21, 1969 (Fig. 6) resulted in an underestimation of the initial peak of discharge. The storms of July 16, 1969 (Fig. 6) and August 21, 1969 (Fig. 7) contained the highest rainfall intensities among the simulated events. The peak rate and volume of runoff were overestimated for both of these storms. However, the timing of the peaks is very good.

The storm of August 29, 1969 (Fig. 7) consisted of several moderate bursts of rainfall on a nearly uniform base rate of rainfall. The computed rising limb

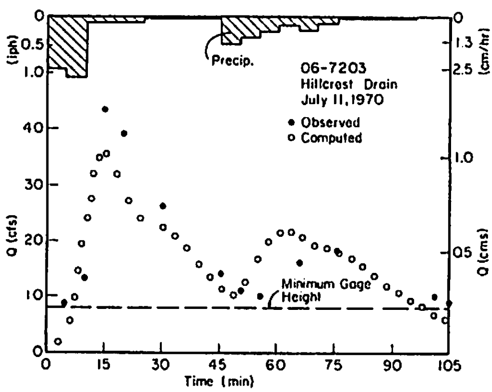

FIG. 8.—Hillcrest Drain, July 11, 1970

of the hydrograph nearly matches that of the observed; however, peak runoff rates are underpredicted.

The storm of July 11, 1970 (Fig. 8) consisted of two definite pulses of rainfall. Simulation of this event resulted in underprediction of the first and highest peak rate, but overpredicting the second peak. The timing of the first peak is good but the computed second peak occurs about 10 min before the observed peak.

From the results of the simulations, it is concluded that the inability to predict the rainfall excess (difference between actual rainfall and all losses) has significantly contributed to the differences between computed and observed results. A significant problem was estimating the antecedent moisture condition of the soil. Rainfall was reported for the gaging station only when runoff was significant. Between the time of the significant runoff events there was no information about minor rainfall amounts that did not cause appreciable runoff but which may have affected soil moisture. It is possible to have varying amounts of rainfall over the watershed even though the area is only 670,000 m^2 (165 acres). The single rain gage at the runoff site may not be sufficient to completely define the spatial distribution of localized rainfall over the watershed.

Since this model is based upon the watershed's physical characteristics, it can be used to predict changes in the hydraulic response due to changes in the land use. A practical application of this feature would be in determining changes in runoff when a watershed is transformed from an agricultural to urban environment. This particular use of the model has not yet been made although the model has been tested on agricultural watersheds (6,7,11). Further research is needed in the prediction of runoff changes due to the urbanization of watersheds.

Summary and Conclusions

A flow routing model based upon the kinematic equation was used to simulate overland flow and free surface flow in trapezoidal or circular-shaped channels. Kinematic routing does not have the capability to account for backwater effects; applications must be limited accordingly. Routing of flows was based upon the hydraulic characteristics of the surface or channel. Computation of infiltration losses on pervious surfaces was based upon a parametric decay-type function. The most sensitive of infiltration parameters had physical significance. The original study using these infiltration components listed appropriate parameters for a wide range of soil types. Runoff from a 67,000-m^2 (165-acre) urban watershed was simulated using the computer model. The complex geometry was represented by a series of cascading planes and channels. Storm sewers within the watershed were simulated as free-surface flowing circular conduits.

It is concluded that this physically based model was satisfactory for predictive purposes. The greatest challenge to using the computer model is to adequately estimate the rainfall excess available for surface runoff. An objective means of estimating the initial moisture content from antecedent precipitation and climatic data would be very useful. Further refinements of the routing components would be justified only with an improved method of estimating infiltration.

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KEY WORDS: Computers; Hydrographs; Infiltration; Kinematics; Overland flow; Runoff; Storm sewers; Urban development

ABSTRACT: A surface flow routing model based upon a kinematic cascade of planes and channels is combined with a parametric infiltration model to constitute a watershed model. The kinematic approximation is used to route flows through storm drains of trapezoidal or circular cross section. Model parameters are estimated from the hydraulic characteristics of the surface and soils. A Chezy friction relationship is used to model surface flow resistance. The predictive ability of the watershed model is tested by simulating runoff hydrographs from a 67-ha urban watershed near Denver, Colorado.

REFERENCE: Rovey, Edward W., and Woolhiser, David A., "Urban Storm Runoff Model," *Journal of the Hydraulics Division*, ASCE, Vol. 103, No. HY11, Proc. Paper 13369, November, 1977, pp. 1339-1351