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The Impact of Recession Infiltration on Runoff Volume Computed by the Kinematic Wave Model

J. J. Stone, E. D. Shirley, L. J. Lane
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Abstract. *The effect of recession infiltration on runoff volume is quantified using the kinematic wave model for the case of lateral inflow made up of constant rainfall excess during the period of rainfall and constant infiltration after rainfall ends. A general solution is obtained using the following non-dimensional quantities;* $Q_* = Q/R_e$ *(runoff volume divided by rainfall excess volume),* $t_* = t_e/D$ *(time to kinematic equilibrium divided by the duration of rainfall excess), and* $f_* = f/r_e$ *(infiltration rate divided by rainfall excess rate). Using these quantities, the relationship for the reduction of runoff volume is* $Q_* = 1 - m/(m+1)\ t_*\ [f_*/(f_*+1)]^{1/m}$ *when* $t_* < [(f_*+1)/f_*]^{1/m}$ *and* $Q_* = 1/(m+1)\ t_*^{-m}\ (f_*+1)/f_*$ *when* $t_* \geq [(f_*+1)/f_*]^{1/m}$ *where m is the kinematic wave depth-discharge exponent. The first equation corresponds to the case when flow ceases after the characteristic from distance and time zero,* $C_{(0,0)}$*, reaches the end of the plane. The second equation corresponds to the case when the flow ceases and* $C_{(0,0)}$ *does not reach the end of the plane. These equations approximate the reduction of runoff volume for the more general case of time varying rainfall excess under constant and variable rainfall as would be the case when the rainfall excess is generated using the Green-Ampt infiltration equation.*
Keywords. Runoff, Infiltration, Recession hydrograph.

The lateral inflow rate, v (m/s), is the rate at which water that is available for runoff accumulates on a flow surface. For a constant rainfall and infiltration rate, it is defined as:

$$
\begin{array}{llll}
v = i - f = r_e & 0 < t < D & h > 0 & \\
v = -f & D < t < t_f & h > 0 & (1) \\
v = 0 & t > t_f & h = 0 &
\end{array}
$$

where
- i = rainfall rate (m/s)
- f = infiltration rate (m/s)
- r_e = rainfall excess rate (m/s)
- t = time
- D = duration of rainfall excess (s)
- h = flow depth (m)
- t_f = time (s) when runoff depth becomes zero at the end of the plane

A common assumption in many rainfall runoff simulation models is that the lateral inflow rate is only made up of the first component in equation 1, rainfall excess, and that the second component, infiltration during the recession phase of the hydrograph, is negligible. In effect, runoff volume, Q (m), is assumed to be equal to the rainfall excess volume, R_e (m). This assumption is popular because many design problems involve predicting the runoff hydrograph for extreme or large events in which most of the runoff occurs as infiltration approaches steady state. Consequently, the rainfall excess volume is large compared to the recession infiltration volume so that the overestimation of runoff volume is small. However, for small events or situations with long hydrograph recession durations, the recession infiltration can be a substantial portion of the total rainfall excess volume.

The most general solution for recession infiltration was presented by Smith and Woolhiser (1971) who used an infiltration equation coupled with a finite difference solution of the kinematic cascade model. The first analytical solution for the case of constant rainfall excess and infiltration was developed by Wooding (1965b) who showed in graphical form the effect of recession infiltration on the watershed runoff volume using an integer depth-discharge exponent for the kinematic wave model but did not explicitly develop a relationship between the rainfall excess volume and the routed runoff volume. Dunne and Dietrich (1980) computed the recession hydrograph assuming constant infiltration and considering an initial condition at the end of rainfall. Cundy and Tento (1985) developed an analytical solution for a coupled kinematic wave model and Philip's equation for the case of constant rainfall and variable infiltration. However, the amount of error in neglecting recession infiltration has not been well quantified in the literature.

The purpose of this article to examine the impact of neglecting recession infiltration for the simple case of constant rainfall and constant infiltration when coupled with the kinematic wave model for overland flow on a single plane. In order to generalize the analysis, non-dimensional quantities are defined and used to develop a relationship between rainfall excess volume and routed runoff volume as a function of the kinematic time to

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equilibrium and the final infiltration rate. The relationship is presented in equation and graphical form. The results for a range of slope lengths, gradients, and roughnesses obtained from the equations are compared with a general numerical solution of a coupled infiltration kinematic wave model.

Kinematic Wave Model for Overland Flow

The kinematic wave model for overland flow on a single plane consists of the continuity equation:

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = i - f = v \tag{2}$$

a depth-discharge relationship:

$$q = \alpha h^m \tag{3}$$

and initial and boundary conditions:

$$h(0, x) = h(t, 0) = 0 \tag{4}$$

where
- h = flow depth (m)
- t = time (s)
- q = discharge per unit width (m^3/ms)
- x = distance (m)
- α = depth-discharge coefficient
- m = depth-discharge exponent

If the Chezy relationship is used, $m = 3/2$ and $\alpha = C\,S_0^{1/2}$ where C = Chezy coefficient ($m^{1/2}$/s) and S_0 = slope of the plane (m/m). If the Manning relationship is used, $m = 5/3$ and $\alpha = S_0^{1/2}/n$ where n = Manning coefficient (s/$m^{1/3}$). Equations 2 and 3 can be combined (Wooding, 1965a; or Eagleson, 1970):

$$\frac{\partial h}{\partial t} + \alpha\, m\, h^{m-1}\frac{\partial h}{\partial x} = v \tag{5}$$

Equation 5 is solved by considering a curve, C_z, called a characteristic defined as starting at a boundary point $z = (t_0, x_0) = (t_0, 0)$ or an initial point $z = (t_0, x_0) = (0, x_0)$ (fig. 1) and parameterized by time t as $C_z(t) = [t, x_z(t)]$. The distance, $x_z(t)$, at time t on the characteristic C_z is given by:

$$\frac{dx_z(t)}{dt} = \alpha\, m\, h_z(t)^{m-1} \tag{6}$$

where $h_z(t) = h[t, x_z(t)]$. The depth at that distance on the characteristic C_z is defined by:

$$\frac{dh_z(t)}{dt} = v(t) \tag{7}$$

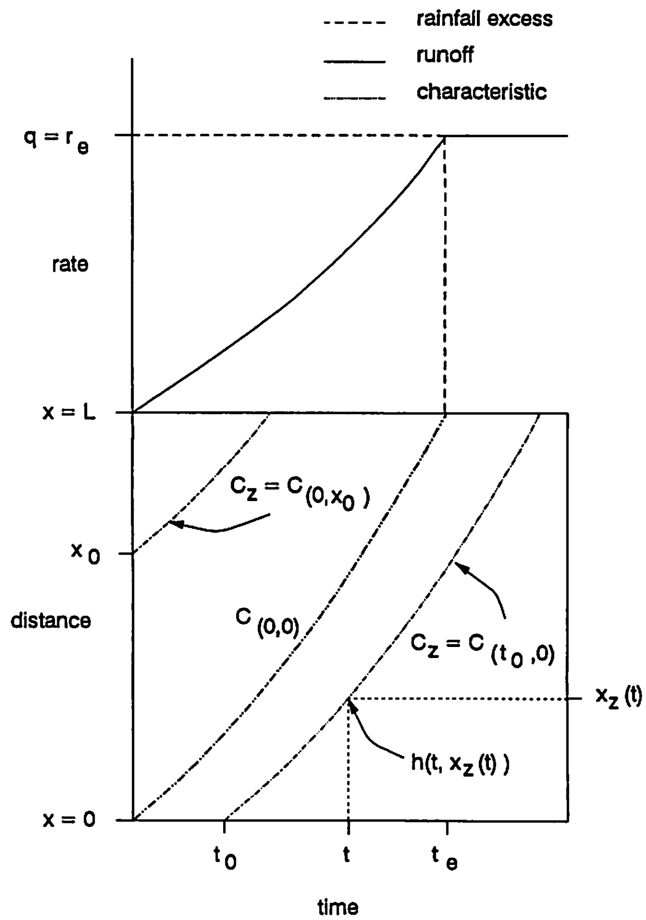

Figure 1–Definition sketch of the characteristic plane, $C_{(0,0)}$ characteristic, and hydrograph for constant rainfall excess.

For the case of constant rainfall excess, the time when the outflow rate equals the inflow rate or the time to kinematic equilibrium, t_e, is computed as:

$$t_e = \left(\frac{L}{\alpha\, r_e^{m-1}}\right)^{1/m} \tag{8}$$

where L represents the length (m) of the plane. This time corresponds to the time when the characteristic, $C_{(0,0)}$, which originates at the top of the plane at time zero, reaches the end of the plane (fig. 1).

A definition of lateral inflow which does not allow for infiltration during the recession of the hydrograph results in two physically unrealistic properties of the runoff hydrograph; partial equilibrium and a hydrograph of infinite duration. Referring to figure 2: (1) Partial equilibrium occurs when the duration of rainfall excess is less than the time to kinematic equilibrium. The result is that the flow depth at the end of the plane is constant (2) during the recession until the $C_{(0,0)}$ characteristic reaches the end of the plane (3). At that time, the flow depth begins to decrease (4), not because of infiltration, but because water is flowing off the plane. As the flow depth on the plane during the recession becomes small, the rate at which water flows off the plane becomes small. As a consequence, the flow depth approaches zero as time

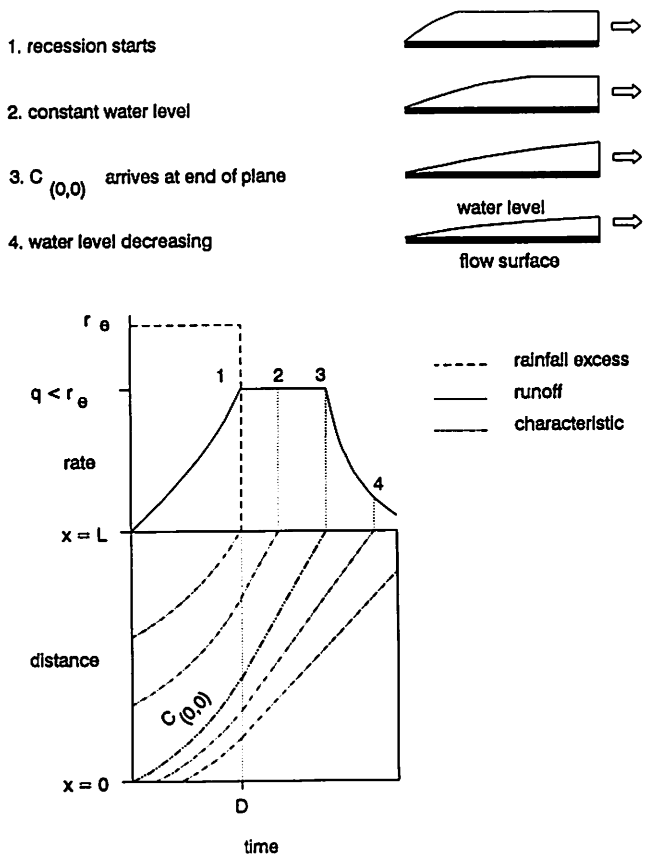

Figure 2–Definition sketch of the characteristic plane, hydrograph, and water surface profiles for the case of partial equilibrium under constant rainfall excess of finite duration.

approaches infinity and the hydrograph has an infinite duration.

The Relationship Between Rainfall Excess Volume and Runoff Volume

In order to develop the relationship between rainfall excess and runoff volume, the definition of lateral inflow from equation 1 is used. The total runoff volume when recession infiltration is computed is obtained by integrating equation 2 from 0 to ∞ with respect to time to obtain:

$$\int_0^\infty \frac{\partial h(t,x)}{\partial t}\,dt = 0 \tag{9}$$

$$\int_0^\infty \frac{\partial q(t,x)}{\partial x}\,dt = \frac{\partial}{\partial x}\int_0^\infty q(t,x)\,dt = \frac{dQ(x)}{dx} \tag{10}$$

$$\int_0^\infty v(t)\,dt = r_e D - f\left[t_f(x) - D\right] \tag{11}$$

$$\frac{dQ(x)}{dx} = r_e D - f\left[t_f(x) - D\right] \tag{12}$$

where $Q(x)$ is the total runoff volume at x and $t_f(x)$ is time runoff ends at x. The total runoff volume, Q, at the end of the plane is obtained by integrating equation 12 with respect to the length of the plane:

$$\int_0^L dQ(x) = \int_0^L \left\{r_e D - f\left[t_f(x) - D\right]\right\} dx$$

$$Q = r_e D\,L - f\int_0^L t_f(x)\,dx + f\,D\,L \tag{13}$$

It can be shown (Appendix I) that the form of $t_f(x)$, and thus, the solution of the integral in equation 13, depends on:

$$L < \alpha\, r_e^{m-1} D^m + \frac{\alpha}{f}\left(r_e D\right)^m \tag{14a}$$

$$L \geq \alpha\, r_e^{m-1} D^m + \frac{\alpha}{f}\left(r_e D\right)^m \tag{14b}$$

Equation 14a corresponds to the case when the flow ceases at L after the characteristic, $C_{(0,0)}$, from time and distance zero reaches the end of the plane and equation 14b corresponds to the case when the flow ceases and $C_{(0,0)}$ does not reach the end of the plane (fig. 3). For the first case, the integral of $t_f(x)$ in equation 13 is:

$$\int_0^L t_f(w)\,dw = \int_0^L \left[D + \frac{r_e}{f}\left(\frac{w}{\alpha\, r_e^{m-1}}\;\frac{f}{f + r_e}\right)^{1/m}\right] dw$$

$$= D\,L + \frac{m}{m+1}\,\frac{r_e L}{f}\left(\frac{L}{\alpha\, r_e^{m-1}}\;\frac{f}{f + r_e}\right)^{1/m} \tag{15a}$$

where w is the dummy variable of integration. Substituting into equation 13:

$$Q = r_e L\left[D - \frac{m}{m+1}\left(\frac{L}{\alpha\, r_e^{m-1}}\;\frac{f}{f + r_e}\right)^{1/m}\right] \tag{16a}$$

Equation 16a can be simplified by substituting the definition of t_e given by equation 8 and the definitions $Q_* = Q/R_e$ where $R_e = r_e\,D$ (m), $t_* = t_e/D$, and $f_* = f/r_e$, to get:

$$Q_* = 1 - \frac{m}{m+1}\left(\frac{f_*}{f_* + 1}\right)^{1/m} t_* \tag{17a}$$

For the second case, the integral of $t_f(x)$ is:

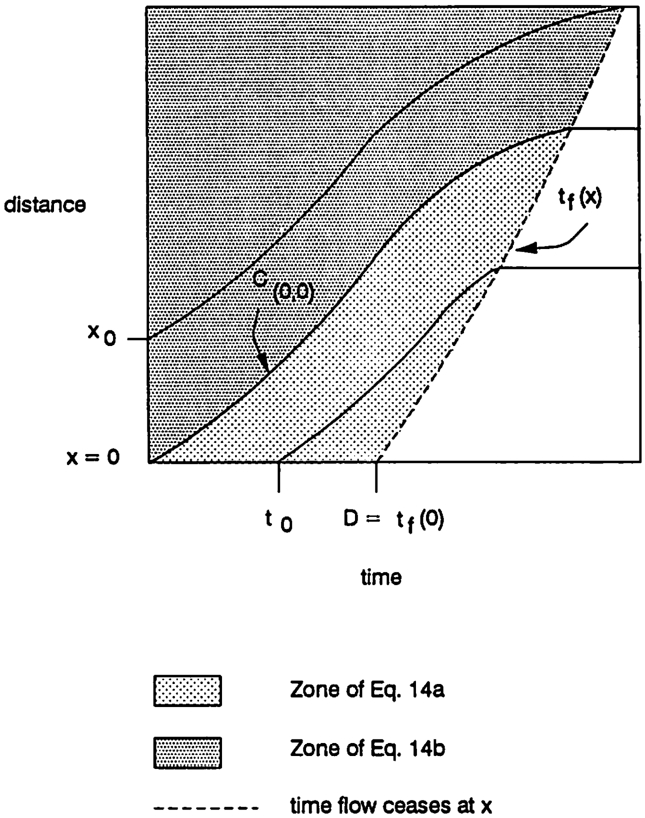

Figure 3–Definition sketch of the characteristic plane for cases when the flow ceases on the flow surface after $C_{(0,0)}$ reaches the end of the flow surface and flow ceases and $C_{(0,0)}$ does not reach the end of the flow surface.

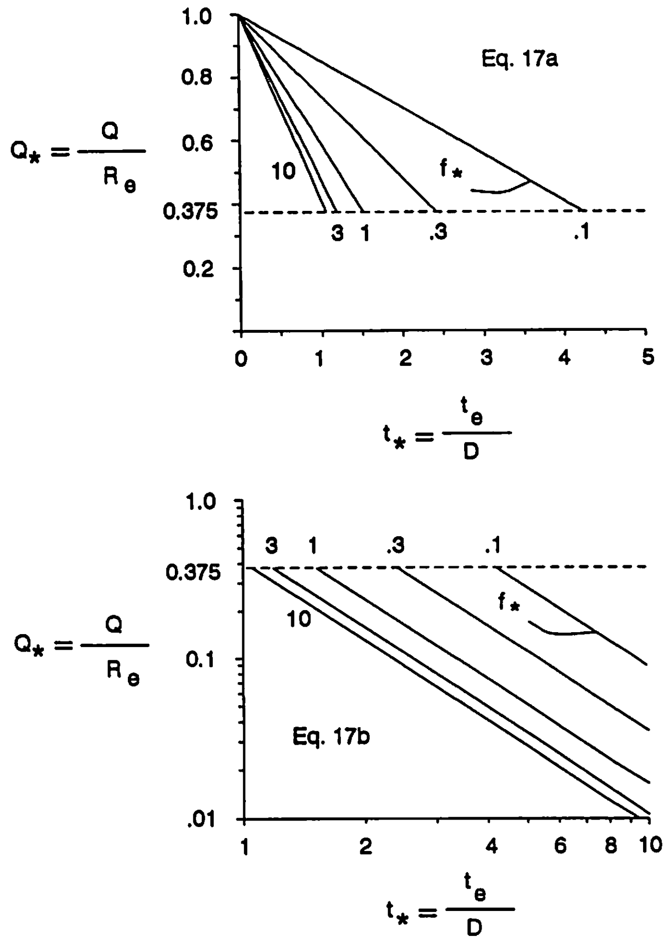

Figure 4–Plot of t_* vs. Q_* for equations 17a and 17b for the Manning discharge relationship.

$$\int_0^L t_f(w)\,dw$$

$$= \int_0^\beta \left[D + \frac{r_e}{f}\left(\frac{w}{\alpha\, r_e^{m-1}}\,\frac{f}{f + r_e}\right)^{1/m}\right] dw + \int_\beta^L \left[D\left(\frac{f + r_e}{f}\right)\right] dw$$

$$= D\,L + \frac{r_e D\,L}{f}\left[1 - \frac{1}{m+1}\left(D^m\,\frac{\alpha\, r_e^{m-1}}{L}\,\frac{f + r_e}{f}\right)\right] \qquad (15b)$$

where for convenience, β is defined as:

$$\beta = \alpha\, r_e^{m-1}\, D^m + \frac{\alpha}{f}\left(r_e\, D\right)^m$$

Then equation 13 becomes:

$$Q = r_e\, D\, L\, \frac{1}{m+1}\,\frac{\alpha\, r_e^{m-1}\, D^m}{L}\,\frac{f + r_e}{f} \qquad (16b)$$

and using the non-dimensional quantities defined above, equation 16b becomes:

$$Q_* = \frac{1}{m+1}\,\frac{f_* + 1}{f_*}\, t_*^{-m} \qquad (17b)$$

The indicator whether to use equation 17a or 17b is derived by considering the division between equation 14a and 14b which is for the characteristic $C_{(0,0)}$:

$$L = \alpha\, r_e^{m-1}\, D^m + \frac{\alpha}{f}\left(r_e\, D\right)^m \qquad (18)$$

Using the definition of t_* and f_*, equation 17 becomes:

$$t_* = \left(\frac{f_* + 1}{f_*}\right)^{1/m} \qquad (19)$$

By the inequalities in equations 14a and 14b, we get:

$$t_* < \left(\frac{f_* + 1}{f_*}\right)^{1/m} \quad \text{use eq. 17a} \qquad (20a)$$

$$t_* \geq \left(\frac{f_* + 1}{f_*}\right)^{1/m} \quad \text{use eq. 17b} \qquad (20b)$$

Table 1. Plane characteristics for verification of equations 17a and 17b for unsteady rainfall excess and infiltration

Case (1)	Length (m) (2)	Slope (%) (3)
1	10	1
2	50	1
3	100	1
4	10	9
5	50	9
6	100	9

Table 2. Rainfall distributions for verification of equations 17a and 17b for unsteady rainfall excess and infiltration

Distribution (1)	Time (min) (2)	Rate (mm/h) (3)
Constant	0	50
	30	0
Variable	0	30
	10	40
	20	50
	30	60
	40	30
	50	10
	60	0

DISCUSSION

CONSTANT RAINFALL EXCESS

In figure 4, equations 17a and 17b are plotted for selected values of f_* using the Manning discharge relationship. Note that equation 17a is plotted using an arithmetic scale and equation 17b is plotted using a log-log scale. The division between equation 17a and 17b is at the value of $Q_* = 0.375$. For the Chezy relationship the division would be at $Q_* = 0.4$. Referring to the plot of equation 17a (fig. 4a), even when kinematic time to equilibrium is reached and the outflow rate equals the inflow rate (i.e., $t_* \leq 1$), the runoff volume is less than the rainfall excess volume because of the infiltration during the recession. As would be expected, the greater the infiltration rate or the larger the value of f_*, the greater the reduction of the routed runoff volume. For example, if the rainfall excess duration is just equal to the time of kinematic equilibrium ($t_* = 1$) and the infiltration rate is 10 times the rainfall excess rate ($f_* = 10$), the reduction is about 62% while if the infiltration rate is only 0.1 of the excess rate ($f_* = 0.1$), the reduction is about 10%. As can be seen in the plot of equation 17b, the reduction in volume can be substantial if the flow ceases before the $C_{(0,0)}$ characteristic reaches the end of the plane. This reduction suggests that for a rough surface as would be the case after a tillage operation, runoff volume computed by an approach which neglects recession infiltration can be severely overestimated. For both cases, apart from increasing the infiltration rate with respect to the rainfall excess rate, factors which will reduce rainfall excess volume are increasing the roughness, decreasing the slope gradient, or increasing the slope length.

UNSTEADY RAINFALL EXCESS AND INFILTRATION

Equations 17a and 17b are only exactly true when lateral inflow is defined by equation 1. Methods which compute recession infiltration for the more general case of unsteady rainfall and variable rainfall excess generally involve a numerical solution of the kinematic wave model. As mentioned in the introduction, the KINEROS model (Woolhiser et al., 1990) uses a finite difference scheme to solve a coupled infiltration-kinematic wave model for overland and channel flow. Implementing a finite difference solution in a management model, particularly when used in a continuous simulation mode, can be impractical. Therefore, it is useful to examine how applicable equations 17a and 17b are to the more general case of unsteady rainfall excess and infiltration in relation to the more general solution contained in KINEROS.

To do the comparison, rainfall excess is computed using the Green-Ampt Mein-Larson (GAML) infiltration equation (Chu, 1978) for two rainfall intensity patterns, constant and variable. The amount of reduction of rainfall excess volume is then computed using equations 17a and 17b for a smooth and rough flow surface and several slope lengths and gradients. Finally, the results are compared with the KINEROS model which was modified to include Chu's solution of the GAML equation.

To apply equations 17a and 17b for unsteady rainfall and infiltration, the average rainfall excess during the duration of rainfall excess and the final infiltration rate at the end of rainfall excess is used. The plane characteristics and the two rainfall distributions, constant and variable, used in the analysis are listed in tables 1 and 2, respectively. Infiltration and kinematic parameters include initial soil moisture (0.20), effective saturated conductivity (6.5 mm/h), matric potential (110 mm), and porosity (0.43), and two Manning's n values, n = 0.35 corresponding to a no-till surface with 3 T/acre residue and n = 0.045 corresponding to a bare or fallow surface (Engman, 1989).

The infiltration and rainfall excess variables needed for equations 17a and 17b were computed using a GAML infiltration model (Stone et al., 1992). For the constant rainfall case, these values are:

$$D = 23.21 \text{ min} = 1393 \text{ s} \qquad R_e = 8.19 \text{ mm} = 0.00819 \text{ m}$$

$$f = 21.29 \text{ mm/h} = 0.00000591 \text{ m/s} \qquad \bar{r}_e = 21.17 \text{ mm/h} = 0.00000588 \text{ m/s}$$

and for the variable rainfall case:

$$D = 34.49 \text{ min} = 2189 \text{ s} \qquad R_e = 13.20 \text{ mm} = 0.0132 \text{ m}$$

$$f = 17.94 \text{ mm/h} = 0.00000498 \text{ m/s} \qquad \bar{r}_e = 21.70 \text{ mm/h} = 0.00000603 \text{ m/s}$$

The non-dimensional infiltration for the constant rainfall case is:

$$f_* = \frac{f}{\bar{r}_e} = \frac{21.29}{21.17} = 1.01$$

and for the variable rainfall case is:

$$f_* = \frac{17.94}{21.70} = 0.83$$

As an example, Case 1C in table 3 (0.01 slope, 10.0 m slope length, n = 0.35, constant rainfall) is used for the remainder of calculations. The depth-discharge coefficient, α, is:

$$\alpha = \frac{S^{1/2}}{n} = \frac{0.01^{1/2}}{0.35} = 0.29$$

the time to kinematic equilibrium, t_e:

$$t_e = \left(\frac{L}{\alpha\, \bar{r}_e^{\,m-1}}\right)^{1/m} = \left[\frac{10}{(0.29)(0.00000588)^{2/3}}\right]^{3/5} = 1035 \text{ s}$$

and the dimensionless time, t_*:

$$t_* = \frac{t_e}{D} = \frac{1035}{1393} = 0.75$$

To test to use equation 17a or 17b, compute the quantity:

$$\left(\frac{f_* + 1}{f_*}\right)^{1/m} = \left(\frac{1.01 + 1}{1.01}\right)^{3/5} = 1.51$$

Because $t_* < 1.51$, equation 17a is used to compute Q_*:

$$Q_* = 1 - \frac{m}{m+1}\left(\frac{f_*}{f_* + 1}\right)^{1/m} t_*$$

$$= 1 - (0.625)\left(\frac{1.01}{1.01 + 1}\right)^{3/5}(0.75) = 0.69$$

The runoff volume is found by using the definition of Q_* as:

$$Q = Q_* R_e = (0.69)(8.19) = 5.66 \text{ mm}$$

In the same manner as above, the values listed in tables 3 and 4 were computed.

Referring to tables 3 and 4, the values in columns 4 and 5 are the total amount of runoff which actually runs off the end of the plane. For example, in table 3 for Case 1C, the total runoff volume computed by KINEROS is 5.57 mm and by equation 17a is 5.66 mm which corresponds to a reduction of 32% and 31%, respectively, of the GAML computed rainfall excess volume (8.19 mm). Note that the relative difference between the runoff volume computed by equations 17a and 17b and those computed by KINEROS is very small for the case of constant rainfall, the difference between the two increases for the variable rainfall case, and the difference increases for the rougher surface (n = 0.35). The reason for this is that for the variable rainfall case, equation 8 is not a good estimate of the time $C_{(0,0)}$ reaches the end of the plane and the average rainfall excess rate is a poor approximation to compute $h_z(D)$ which is used to get $t_f(D)$. It is evident that within the range of slope gradients, lengths, and roughness values used, equations 17a and 17b can be applied for the case of constant rainfall intensity with little difference in computed runoff volume from the more complete solution of KINEROS. Although it is difficult to generalize accuracy of the equations for variable rainfall, it is reasonable to state that the smoother the overland flow surface, the more the runoff volume computed by equations 17a and 17b and KINEROS will agree. Equations 17a and 17b offer a simple and quick alternative to models such as KINEROS which in general use a numerical solution to the kinematic wave model.

The reduction of routed rainfall excess can be substantial if the flow surface has a high degree of roughness, even if the length of the flow plane is short. For example, for a 10 m plane and a slope of 0.01 (Case 1C), and Manning's n = 0.35, the reduction computed by equation 17a is 31% (table 3). In contrast, for the same plane length and slope but a Manning's n = 0.045, the reduction is only 9% (table 4). This illustrates, in a practical sense, when disregarding recession infiltration might cause substantial error in prediction or evaluation. For example, many management practices being recommended by U.S. action agencies such as the Soil Conservation Service to reduce or control erosion involve leaving residue on the soil surface after harvest. If a simulation model such as WEPP (Lane and Nearing, 1989)

Table 3. Results of verification of equations 17a and 17b for n = 0.35 and unsteady rainfall excess and infiltration (for the constant rainfall case, f_* = 1.01 and rainfall excess volume = 8.19 mm, for the variable rainfall case, f_* = 0.83 and rainfall excess volume = 13.20 mm)

			Runoff Volume (mm)		
Case (1)	t* (2)	Eq. No. (3)	KINEROS (4)	Eq. (17a) or (17b) (5)	Relative Difference (%)* (6)
1C†	0.75	17a	5.57	5.66	−1.6
2C	1.96	17b	1.95	1.98	−1.5
3C	2.98	17b	0.98	0.99	−1.0
4C	0.39	17a	6.86	6.87	−0.3
5C	1.02	17a	4.65	4.73	−1.7
6C	1.55	17b	2.94	2.95	−0.3
1V‡	0.47	17a	11.79	10.78	8.6
2V	1.24	17a	8.23	6.85	16.8
3V	1.88	17b	4.94	3.83	22.5
4V	0.25	17a	12.59	11.94	5.2
5V	0.64	17a	11.09	9.90	10.7
6V	0.98	17a	9.55	8.19	14.2

* Relative difference = (Column 4 – Column 5) / Column 4 × 100.
† 1 = case 1 from table 1, C = constant rainfall from table 2.
‡ 1 = case 1 from table 1, V = variable rainfall from table 2.

Table 4. Results of verification of equations 17a and 17b for n = 0.045 and unsteady rainfall excess and infiltration (for the constant rainfall case, f_* = 1.01 and rainfall excess volume = 8.19 mm, for the variable rainfall case, f_* = 0.83 and rainfall excess volume = 13.20 mm)

			Runoff Volume (mm)		
Case (1)	t* (2)	Eq. No. (3)	KINEROS (4)	Eq. (17a) or (17b) (5)	Relative Difference (%)* (6)
1C†	0.22	17a	7.47	7.45	0.3
2C	0.58	17a	6.19	6.24	−0.8
3C	0.87	17a	5.16	5.24	−1.6
4C	0.11	17a	7.87	7.81	0.8
5C	0.20	17a	7.19	7.18	0.1
6C	0.45	17a	6.63	6.67	−0.6
1V‡	0.14	17a	12.93	12.49	3.4
2V	0.36	17a	12.20	11.34	7.0
3V	0.45	17a	11.48	10.38	9.6
4V	0.07	17a	13.15	12.83	2.4
5V	0.19	17a	12.77	12.24	4.2
6V	0.28	17a	12.47	11.74	5.9

* Relative difference = (Column 4 – Column 5) / Column 4 × 100.
† 1 = case 1 from table 1, C = constant rainfall from table 2.
‡ 1 = case 1 from table 1, V = variable rainfall from table 2.

is used to evaluate the efficacy of a management practice, components of the model which are affected by the water balance can be substantially impacted by the recession infiltration.

Tolerable Error in Runoff Volume Estimation

Equation 17a can be used to compute a threshold in terms of t_* above which recession infiltration should be considered by rewriting equation 17a in the form of an inequality:

$$t_* \le (1 - Q_*)\,\frac{m+1}{m}\left(\frac{f_* + 1}{f_*}\right)^{1/m} \qquad (21)$$

For example, to compute a tolerable level of 10% error in runoff volume for the Manning discharge relationship, substitute $Q_* = 0.9$ into equation 21 to obtain:

$$t_* \le 0.16\left(\frac{f_* + 1}{f_*}\right)^{3/5}$$

Then characteristic conditions (slope length and gradient, roughness, infiltration, and rainfall) can be used to determine if neglecting the recession infiltration will cause more than a 10% error in runoff volume calculation. Using the cases from tables 3 and 4, t_* has to satisfy:

$$t_* \le 0.16\left(\frac{1.01 + 1}{1.01}\right)^{3/5} = 0.24$$

for the constant rainfall case, and:

$$t_* \le 0.16\left(\frac{0.83 + 1}{0.83}\right)^{3/5} = 0.26$$

for the variable rainfall case in order for the error in runoff volume estimation to be less than 10%. Referring to column 2 in table 3, only Case 4V satisfies the criteria while in table 4, Cases 1C, 4C, 5C, 1V, 4V, and 5V satisfy the criteria. The cases listed in tables 3 and 4 represent a large range of slope lengths, gradients, and roughness values, methods which neglect recession infiltration can frequently overestimate the amount of runoff leaving a flow surface.

Summary

The amount of reduction in rainfall excess volume that will occur during the recession of the hydrograph was quantified using non-dimensional quantities and solving the kinematic wave model for overland flow for the case of constant rainfall excess and infiltration rates. Two equations were developed, the use of which depend on if the characteristic from time and distance zero, $C_{(0,0)}$, reaches the end of the flow surface before the time the flow ceases or if the flow ceases and $C_{(0,0)}$ never reaches the end of the plane. The equations show that for rough surfaces as would occur after a tillage operation, the overestimation of runoff volume can be considerable if the infiltration during the recession of the hydrograph is neglected. The KINEROS model which computes recession infiltration was used to compare the results of the equations for the more general cases of time varying rainfall excess and infiltration under constant and variable rainfall intensity distributions. For the range of slope gradients, lengths, and roughness values tested, the equations and KINEROS computed similar reductions of rainfall excess for constant rainfall for both rough and smooth overland flow surfaces. For variable rainfall and rough surfaces, the difference between the equations and KINEROS increased because of the approximation of using the average rainfall excess rate in an unsteady state process. However, the equations offer a simple and quick alternative to the more computer intensive numerical solutions generally used in models like KINEROS. Finally, the equations were used to derive a check which determines under which conditions neglecting recession infiltration will exceed a pre-specified error criteria in runoff volume estimation. It was shown for the range of slope gradients, lengths, and roughness values tested that methods of runoff routing which neglect recession infiltration can frequently overestimate the amount of runoff leaving a flow surface.

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Appendix I*

The depth, $h_z(D) = h[D, x_z(D)]$, on the characteristic at time D is computed as:

$$h_z(D) = r_e(D - t_0) \quad (A.1)$$

For $t \le D$, by equations 2 and 3, the flow depth, $h_z(t)$, on a characteristic is:

$$h_z(t) = \int_{t_0}^{t} r_e(w)\,dw = r_e(t - t_0) \quad (A.2)$$

and for $t > D$ is:

$$h_z(t) = h_z(D) + \int_{D}^{t} -f\,dw = h_z(D) - f(t - D) \quad (A.3)$$

where $z = (t_0, x_0)$ and t_0 and x_0 are the starting points on the boundary of the t-x plane of the characteristic. For $t \le D$, by equations A.1, A.2, and 6, the distance, $x_z(t)$, on a characteristic is:

$$x_z(t) = x_0 + \alpha\, m \int_{t_0}^{t} [r_e\, t(w) - t_0]^{m-1}\, dw$$

$$= x_0 + \alpha\, r_e (t - t_0)^m \quad (A.4)$$

and for $t > D$:

$$x_z(t) = x_z(D) + \alpha\, m \int_{D}^{t} \{h_z(D) - f[t(w) - D]\}^{m-1}\, dw$$

$$= x_z(D) + \frac{\alpha}{f}\{h_z(D)^m - [h_z(D) - f(t - D)]^m\} \quad (A.5)$$

where $x_z(D)$, the distance on the characteristic with depth $h_z(D)$ at time D, is computed as:

$$x_z(D) = x_0 + \alpha\, r_e^{m-1} (D - t_0)^m \quad (A.6)$$

Substituting equation A.1 into equation A.3 when $h_z(t) = 0$ and solving for t, the time, $t_f(z)$, that flow ceases is:

$$t_f(z) = D + \frac{r_e(D - t_0)}{f} \quad (A.7)$$

Referring to figure 3, $t_f(z)$ will have a different form if flow ceases after $C_{(0,0)}$ reaches the end of the plane or if flow ceases and $C_{(0,0)}$ never reaches the end of the plane.

Case 1

The first case is when flow ceases at L after $C_{(0,0)}$ reaches L which means that $t_0 > 0$ and $x_0 = 0$. The time t_0 in equation A.7 is now the starting time of the characteristic which reaches L just when the depth becomes zero. Noting that when $t = t_f$, $h_z(D) = f(t_f - D)$ and substituting equation A.2 into equation A.3, equation A.5 becomes:

$$L = \alpha\, r_e^{m-1} (D - t_0)^m + \frac{\alpha}{f}[r_e(D - t_0)]^m \quad (A.8)$$

Because $D > t_0$, equation A.8 becomes equation 14a:

$$L < \alpha\, r_e^{m-1} D^m + \frac{\alpha}{f}(r_e D)^m \quad (14a)$$

Solving equation A.8 for t_0:

$$t_0 = D - \left(\frac{L}{\alpha\, r_e^{m-1}}\right)^{1/m} \left(\frac{f}{f + r^e}\right)^{1/m} \quad (A.9)$$

Substituting equation A.9 into equation A.7, the time flow ceases at L is:

$$t_z = D + \frac{r_e}{f}\left(\frac{L}{\alpha\, r_e^{m-1}}\right)^{1/m} \left(\frac{f}{f + r^e}\right)^{1/m} \quad (A.10)$$

Case 2

In the second case, flow ceases at L and $C_{(0,0)}$ never reaches L when $t_0 = 0$ and $x_0 > 0$. Making the same substitutions as with equation A.8:

$$L = x_0 + \alpha\, r_e^{m-1} D^m + \frac{\alpha}{f}(r_e D)^m \quad (A.11)$$

and because $x_0 > 0$, equation A.11 becomes equation 14b:

$$L \ge \alpha\, r_e^{m-1} D^m + \frac{\alpha}{f}(r_e D)^m \quad (14b)$$

Because $t_0 = 0$, equation A.3 is:

$$h_z(D) = r_e D \quad (A.12)$$

and t_f in equation A.7 becomes:

$$t_f = D\left(\frac{f + r_e}{f}\right) \quad (A.13)$$

* The original equation numbers are retained if the equation has already been introduced in the main body of the text.

Symbols

Symbol	Units	Description
C	$m^{1/2}/s$	Chezy coefficient
C_z	m/s	characteristic which originates at z
$C_{(0,0)}$	m/s	characteristic which originates at the top of the flow surface at time zero
D	s	duration of rainfall excess
f	m/s	infiltration rate
f_*	ND	dimensionless infiltration rate = f/r_e
h	m	flow depth on the plane
h_z	m	flow depth on a characteristic, C_z
i	m/s	rainfall rate
I	m	rainfall amount
L	m	length of the plane
m	ND	depth-discharge exponent
n	$s/m^{1/3}$	Manning's n coefficient
q	m^3/ms	discharge per unit width
Q	m^3/m^2s	runoff volume per unit area
Q_*	ND	dimensionless runoff volume = Q/R_e
r_e	m/s	rainfall excess rate
R_e	m	rainfall excess volume

Symbol	Units	Description
S_0	m/m	slope of the plane
t	s	time
t_e	s	time to kinematic equilibrium
t_f	s	time that flow ceases
t_L	s	time that a characteristic ($x = 0$, $t > 0$) reaches the bottom of the plane
t_0	s	origin point of a characteristic on the t-axis when $x_0 = 0$
t_*	ND	dimensionless time = t_e/D
v	m/s	lateral inflow rate
w	ND	dummy variable of integration
x	m	distance
x_0	m	origin of a characteristic on the x-axis when $t_0 = 0$
z	m, s	origin point of a characteristic at t_0 or x_0
α	$m^{1/2}/s$ - Chezy $s/m^{1/3}$ - Manning	depth-discharge coefficient