Multi-attribute Decision Making: Dominance with Respect to an Importance Order of the Attributes

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ABSTRACT

We examine a tool to assist a decision maker with the task of ranking a finite number of alternatives based on a finite number of attributes. The procedure indicates if domination with respect to an importance order (ordinal ranking) of the attributes can be easily detected if one assumes an additive value function. The concept of importance order (or partial information) domination has been developed in the literature but has been ignored in most multi-attribute applications. We show how closed form solutions of the linear programs needed to determine strong dominance are derived and present a graphical display of the results in the context of an example. The procedure outlined is practical to use and the results are easy to interpret. An example illustrates how one might apply the method to decision making in farm management with respect to water quality issues.

I. INTRODUCTION

Consider a multi-attribute decision-making problem in which a finite number of alternatives (choices) are evaluated based on a finite number of attributes (objectives or criteria). Often, a first step in the decision-making process is to eliminate from consideration those alternatives that are dominated according to Pareto (see [4, 12]). Once the set of nondominated

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alternatives is determined, a common practice is to continue the evaluation process by assuming an additive utility function and introducing weights on each attribute [4, 6, 9]. In this paper, we examine an improved version of the decision tool applied in [10] that aids the Decision Maker (DM) by finding a subset of alternatives that dominate with respect to an additive value function and a given importance order (partial information) of the decision criteria. The information made available as a result of this analysis can be used to rank the alternatives and may eliminate the necessity for determining a specific weight vector associated with each attribute for the purpose of ranking the alternatives. The decision aid also highlights the necessity to carefully justify a specific choice of attribute weights when importance order dominance is not indicated.

Many good methods for multi-attribute decision making exist (see [2] for numerous references). However, simple tools that can be quickly understood and applied are still needed. The method presented here is such a tool. The present analysis falls under the category of partial information in multi-attribute utility theory (see [5] for a discussion and numerous references). Our approach is conceptually simple and provides the DM with clear graphical evidence if one alternative is strongly dominant over another.

A generalized description of the problem can be stated as follows: Let v_{ij} be the result of evaluating the j^{th} alternative (Aj) with respect to the i^{th} attribute, $v_{ij} \in \Re$, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. The DM wishes to rank the alternatives assuming that the information in matrix $\mathbf{V} = [v_{ij}]$ provides a complete description of the alternatives.

The set of Non-Dominated Alternatives, **NDA**, can be defined as follows: $A_j \in \mathbf{NDA} \Leftrightarrow \nexists\, A_k$, such that $v_{ik} \geqslant v_{ij} \forall i$ and $v_{ij} > v_{ij}$ for some $i = 1, 2, \ldots, m$. We assume that higher values are more desirable. If there is more than one NDA, then a common practice is to assume an additive value function by assigning a weight, w_i, to attribute i and defining the total utility (or worth), U_j, of A_j by,

$$U_j = \sum_{i=1}^{m} w_i v_{ij}.$$

The alternatives can then be ranked based on their total utility. Numerous techniques for assessing weights have been proposed in the literature [6, 8]. Most of these techniques solicit weights directly or seek to discern them indirectly from the DM. The resultant ranking of the alternatives can be very sensitive to the weights. Of particular interest is the behavior of U if an importance order on the attributes is imposed.

Imposing an importance order implies that the attributes can be ordered so that the only allowable weight vectors are those for which

$$w_1 \geqslant w_2 \geqslant , \ldots , \geqslant w_m . \tag{1.1}$$

Therefore, assuming an additive utility function, a natural definition of dominance is the following.

DEFINITION. IMPORTANCE ORDER DOMINATION. We say that alternative k dominates alternative j with respect to an additive value function and a given importance order if and only if for *every* vector of weights, $\mathbf{w} = (w_i, \ldots, w_m)$, consistent with the importance order of the criteria we have

$$\sum_{i=1}^{m} w_i v_{ik} \geqslant \sum_{i=1}^{m} w_i v_{ij}, \tag{1.2}$$

with at least one vector $\mathbf{w}$ satisfying (1.2) as a strict inequality.

Determining whether an alternative dominates another with respect to an importance order of the attributes may be of particular interest in a group decision making setting, especially if the attributes can be ordered but an agreement on a particular weight vector is elusive. For the remainder of this paper, we assume that an importance order of the attributes has been determined by some means. We further assume that $v_{ij} \geqslant 0$ for all i and j, and that the higher the value of v_{ij}, the more desirable.

2. DETERMINING THE BEST AND WORST TOTAL UTILITY OF AN ALTERNATIVE

Once an importance order of the criteria is established, best and worst total utility for each alternative can be found without requiring the DM to set specific weights beforehand.

Let us assume that the indices of the attributes are such that if $i < j$, then attribute i is at least as important as attribute j (i.e., attribute 1 is at least as important as attribute 2 and so forth). This importance order suggests that we should require $w_1 \geqslant w_2 \geqslant , \ldots , \geqslant w_m$. Therefore, given the importance order, the highest total utility possible for alternative j can be determined by the following linear program.

Best Total Utility:

$$\text{Maximize}$$

$$\sum_{i=1}^{m} w_i v_{ij}$$

$$\text{subject to}$$

$$w_1 \geqslant w_2 \geqslant, \ldots, \geqslant w_m$$

$$\sum_{i=1}^{m} w_i = 1$$

$$w_m \geqslant 0.$$

The first constraint set is due to the importance order, (1.1); the second is a normalizing constraint, and the third restricts the weights to be nonnegative.

Similarly, the lowest total utility that alternative j can achieve under the same importance order is found by minimizing the objective function in the above linear program instead of maximizing it.

Worst Total Utility:

$$\text{Minimize}$$

$$\sum_{i=1}^{m} w_i v_{ij}$$

$$\text{subject to}$$

$$w_1 \geqslant w_2 \geqslant, \ldots, \geqslant w_m$$

$$\sum_{i=1}^{m} w_i = 1$$

$$w_m \geqslant 0.$$

For each alternative, the solutions to these two linear programs determine the maximum and minimum total utility possible for any combination of weights that do not violate the given importance order of the criteria. Having these two objective values immediately alerts the DM to the sensitivity of each alternative to a choice of weights.

The two linear programs must be solved for each alternative under consideration. This may, at first, seem to be a burdensome task, however, the linear programs are easily solved in closed form. For $k = 1, \ldots, m$, let s_{kj} be defined as follows:

$$s_{kj} = \frac{1}{k} \sum_{i=1}^{k} v_{ij}. \tag{2.1}$$

THEOREM 2.1. *Let BU_j and WU_j indicate the objective function values at the optimal solutions to the best and worst total utility linear programs, respectively. Then,*

$$BU_j = \max_k \{s_{kj}\},$$

and

$$WU_j = \min_k \{s_{kj}\}.$$

PROOF. Consider the best total utility linear program. Let $\alpha_1 \geqslant 0$, $i = 1, \ldots, m$, then the weights can be defined as

$$w_k = \sum_{i=k}^{m} \alpha_i,$$

and, consistent with the importance order, we have $w_1 \geqslant w_2 \geqslant, \ldots, \geqslant w_m$. Define $\beta_i = i\alpha_i$. Then,

$$\sum_{i=1}^{m} \beta_i = \sum_{i=1}^{m} i\alpha_i = \sum_{i=1}^{m} \left(\sum_{k=i}^{m} \alpha_k \right) = \sum_{i=1}^{m} w_i,$$

and, therefore,

$$\sum_{i=1}^{m} \beta_i = 1 \Leftrightarrow \sum_{i=1}^{m} w_i = 1.$$

Additionally, we have:

$$\begin{aligned}\sum_{i=1}^{m} w_i v_{ij} &= \sum_{i=1}^{m} \sum_{k=i}^{m} \alpha_k v_{ij} \\ &= \sum_{i=1}^{m} \sum_{k=i}^{m} \frac{1}{k} \beta_k v_{ij} \\ &= \sum_{k=1}^{m} \beta_k \left\{ \frac{1}{k} \sum_{i=1}^{k} v_{ij} \right\} \\ &= \sum_{k=1}^{m} \beta_k s_{kj}.\end{aligned}$$

The best total utility linear program is equivalent to the following linear program:

$$\text{Maximize} \quad \sum_{k=1}^{m} \beta_k s_{kj}$$

$$\text{subject to} \quad \sum_{k=1}^{m} \beta_i = 1$$

$$\beta_k \geqslant 0, \; k = 1, 2, \ldots, m.$$

Let $\hat{k}_j \in \{1, 2, \ldots, m\}$ be such that $s_{\hat{k}_j j} = \max_k \{s_{kj}\}$, then, clearly, an optimal solution to the program above is given by:

$$\beta_i = \begin{cases} 1, & \text{if } i = \hat{k}_j; \\ 0, & \text{otherwise.} \end{cases}$$

The objective function value for this program and, therefore, for the best total utility program is equal to $s_{\hat{k}_j j}$. The proof with regard to the worst total utility objective value is analogous if we replace Maximize by Minimize and max by min in the above argument. ∎

If the importance order is such that some of the criteria are considered to be of equal importance, then a slight modification of the closed form solution is necessary. In this case, there are strict equalities in the importance order constraint set, i.e., $w_j = w_{j+1}$ for j in a subset of the integers 1 through m that we deonte by J. Let $\hat{J} = \{1, 2, \ldots, m\} \setminus J$. Then, BU_j and WU_j are found as in Theorem 2.1 with the modification that the index k be restricted to the set $\hat{J}$.

The following theorem is a trivial result of the definition for importance order domination given in Section 1 and the best and worst total utility linear programs.

THEOREM 2.2. *Alternative k dominates alternative j with respect to a given importance order if* $WU_k \geqslant BU_j$ *and* $BU_k \neq WU_j$.

If $WU_k \geqslant BU_j$, the second condition only excludes the dominance of alternative k over alternative j iff $BU_k = WU_k = BU_j = WU_j$. This is the case in which all of the attribute values for alternatives k and j are equal to the same constant.

An alterative that dominates another by Theorem 2.2 is said to strongly dominate the other alternative since the definition of importance order domination is satisfied even if different weight vectors consistent with the importance order are used on either side of (1.2).

3. RANKING THE ALTERNATIVES

Theorems 2.1 and 2.2 suggest a method that can be used to determine whether an alternative can be recommeded based only on the values in **V** and an importance order of the attributes, or if additional input from the DM(s) is needed. However, if finding best and worst total utilities and then applying Theorem 2.2 does not yield a complete ranking of the alternatives and the partial ranking based is not satisfactory, then we suggest two courses of action for the DM:

(A) Average the best and worst total utilities for each alternative and rank the alternatives in descending order of the averages.

(B) Provide additional information on the weights.

It will be evident after the discussion of choice **(B)** that choice **(A)** will always rank an alternative at least as high as one that it dominates. Therefore, choice **(A)** yields a reasonable ranking of the alternatives based on the limited information on the weights with very little effort. This rule is especially useful if additional information on the weights is difficult to assess. The above analysis followed by **(A)** comprise the procedure followed in [10] and [11].

The additional information on the weights that we consider if **(B)** is selected is to ask the DM to supply constants $c_i \geqslant 1$, $i = 2, \dots, m$, which imply the following relationships:

$$w_1 \geqslant c_2 w_2, w_2 \geqslant c_3 w_3, \dots, w_{m-1} \geqslant c_m w_m \geqslant 0. \tag{3.1}$$

We seek conditions under which A_k dominates A_j. Let $\mathbf{W}\mathbf{w} \geqslant 0$ be the matrix and vector notation for the system of inequalities given above, and let $\mathbf{v_p}$ indicate the p^{th} column of the value matrix $\mathbf{V}$. Then,

$$\mathbf{w}^T(\mathbf{v_k} - \mathbf{v_j}) \geqslant 0, \forall \mathbf{w} \text{ such that } \mathbf{W}\mathbf{w} \geqslant 0$$

implies that the system

$$\mathbf{w}^T(\mathbf{v_k} - \mathbf{v_j}) < 0, \mathbf{W}\mathbf{w} \geqslant 0$$

has no solution. By Farkas' Lemma [1], this is true iff there exists $\lambda \in \Re^m$ such that the following system has a solution:

$$(\mathbf{v_k} - \mathbf{v_j}) = \lambda \mathbf{W}, \lambda \geqslant \mathbf{0}.$$

Therefore, we have

$$\begin{aligned} v_{1k} - v_{1j} &= \lambda_1 \\ v_{2k} - v_{2j} &= -c_2 \lambda_1 + \lambda_2 \\ \vdots \quad\quad &\vdots \quad\quad \vdots \\ v_{mk} - v_{mj} &= -c_m \lambda_{m-1} + \lambda_m. \end{aligned}$$

If we define $c_1 = 1$, this system has nonnegative solution for λ iff

$$\sum_{i=1}^{p} \frac{(v_{ik} - v_{ij})}{c_i} \prod_{r=i}^{p} c_r \geqslant 0, \; p = 1, \dots, m. \tag{3.2}$$

To satisfy the definition of importance order domination, one of the inequalities in (3.2) must be satisfied as a strict inequality for alternative k to dominate alternative j.

Conditions (3.2) are similar to results obtained in [7]. For $c_1 = c_2 = , \dots, = c_m = 1$, the vectors $\mathbf{s}_j^T = \{s_{1j}, s_{2j}, \dots, s_{mj}\}$, $j = 1, \dots, n$, defined in (2.1) should be compared to determine dominance. The conditions for this case are equivalent to the results obtained in [3] and [7]. Therefore, without any additional information on the weights beyond (1.1), alternative k dominates alternative j iff

$$s_{ik} \geqslant s_{ij}, \forall i = 1, 2, \dots, m, \text{ and } s_{ik} > s_{ij} \text{ for at least one } i. \tag{3.3}$$

From (3.3) it is clear that rule (**A**) will never rank an alternative below one it dominates. Before stating this formally in a theorem, let us define the best and worst total utilities as a function of $\mathbf{c} = \{c_1, c_2, \dots, c_m\}^T$. For $k = 1, \dots, m$ and $j = 1, \dots, n$, $c_1 = 1$, let

$$s_{kj}(c) = \frac{1}{\sum_{l=1}^{k} \frac{\prod_{p=l}^{k} c_p}{c_l}} \sum_{i=1}^{k} \frac{v_{ij}}{c_i} \prod_{r=i}^{k} c_r$$

Then, it can be shown that for all $j = 1, \dots, n$ we have

$$BU_j(\mathbf{c}) = \max_k \{s_{kj}(\mathbf{c})\}, \tag{3.4a}$$

$$WU_j(\mathbf{c}) = \min_k \{s_{kj}(\mathbf{c})\}. \tag{3.4b}$$

Where $BU_j(\mathbf{c})$ and $WU_j(\mathbf{c})$ indicate, respectively, the best and worst total utilities of alternative j given the restriction implied by (3.1). The definitions and discussion above yield the following theorem and related corollary.

THEOREM 3.1. *If A_k dominates A_j, then*

C1. $BU_k(\mathbf{c}) \geqslant BU_j(\mathbf{c})$
C2. $WU_k(\mathbf{c}) \geqslant WU_j(\mathbf{c})$.

COROLLARY 3.1. *If alternative k dominates alternative j with respect to* *the importance order given by* (3.1), *then*

$$\frac{BU_k(\mathbf{c}) + WU_k(\mathbf{c})}{2} \geqslant \frac{BU_j(\mathbf{c}) + WU_j(\mathbf{c})}{2}. \tag{3.5}$$

By Corollary 3.1, ranking the alternatives according to **(A)** will always rank an alternative at least as high as an alternative that it dominates with respect to the importance order and does not require the extra work to determine dominance explicitly by (3.2). If one wishes to insure that an alternative that is ranked higher by **(A)** dominates those ranked below it when Theorem 2.2 is not satisfied, it may be necessary to check if conditions (3.2) are satisfied for the given **c**. It may not be necessary, however, to check (3.2) for all alternative pairs. Note that Theorem 2.2 is sufficient but not necessary, and Theorem 3.1 is necessary but not sufficient. Therefore, to determine importance order dominance of A_k over another alternative, (3.2) need only be checked for those A_j that do not satisfy the conditions of Theorem 2.2 (otherwise we already know A_k dominates A_j) but do satisfy Theorem 3.1 (otherwise A_k cannot dominate A_j).

With the addition of the information provided above, the following steps summarize the approach:

Step 1. Determine the alternatives and attributes.

Step 2. Evaluate, by some means, each alternative with respect to each attribute.

Step 3. Discern an importance order (ordinal ranking) of the attributes.

Step 4. Determine best and worst total utilities by Theorem 2.1.

Step 5. Discern whether any alternative satisfies the importance order dominance of Theorem 2.2 and rank the alternatives where possible.

Step 6. If a unique best alternative, or complete ranking (whichever is desired) is available as a result of *Step 5*, recommend it, otherwise proceed to *Step 7a* if additional dominance information is desired, or to *Step 7b* for a complete ranking based on the current information.

Step 7a. Determine those pairs of alternatives that do not satisfy Theorem 2.2 but do satisfy Theorem 3.1. For these, determine if conditions (3.2) are satisfied and rank the alternatives where possible. If there are still unranked alternatives, proceed to *Step 7b*. or modify **c** and repeat from *Step 4* (using (3.4) in Theorem 2.1).

Step 7b. Rank the alternatives in descending order of the average of $BU_j(\mathbf{c})$ and $WU_j(\mathbf{c})$ ($\mathbf{c} = \{1, \ldots, 1\}^T$ by default).

A few comments about the procedure. If one does not want to specify the importance order beyond (1.1) and there are no two alternatives for which both the best and worst utilities are equal, performing *Steps 1* through *4* followed by *Step 7b* will order the alternatives preserving any dominance relations.

Except for the case mentioned above, the ranking (possibly partial) obtained by checking condition (3.2) in *Step 7a* will not conflict with the ranking obtained in *Step 7b*. Unless it is desired to know explicitly if the alternative with a strictly higher average in a pair of alternatives satisfying the conditions in *Step 7a* actually dominates the other, checking (3.2) is unnecessary. If equal ranking of two alternatives by *Step 7b* is not acceptable, then checking (3.2), if the two alternatives have equal values for best and worst total utilities, may resolve the issue. Otherwise, modifying $\mathbf{c}$ may break the tie.

To illustrate how the above procedure can be applied and presented graphically, we consider the following example from [11].

4. EXAMPLE: MANAGEMENT PRACTICES ON FARMLAND

In this example, the DM is concerned with the evaluation of different farming practices on crop land. The criteria with which the DM is concerned are the predicted values from a simulation model of variables that have an impact on surface and ground water quality as well as income. These include the amount of soil eroding from the field, the amount of pesticides and nutrients leaching below the root zone or in runoff, and net farm income.

The management alternatives considered are as follows. Alternative #1, the conventional or baseline practice, is continuous corn. Alternative #2 is continuous corn with a small grain winter cover crop. Alternative #3 is fair pasture. Information on the field, farm operations, chemical applications, and the simulation models used to determine the attribute values can be found in [10] and [11]. Alternatives #1 and #2 include annual applications of Nitrogen (N), Phosphorus (P), and pesticides Atrazine, Carbofuran, and Sevin. Alternative #3 required only a single application each spring of N.

The noncommensurable data obtained by simulating the three practices were converted to values in the interval [0, 1]. The conversion method discussed in [10] and [11] assigns 0.5 to each attribute value for Alternative #1 (the conventional practice). The criteria and value matrix appear in Table 1.

Since all of the alternatives under consideration are members of the set of Pareto nondominated alternatives, an importance order of the attributes is needed to continue the analysis. For this example, it was determined to be

1. N in percolation,
2. Sevin surface loss (in runoff and sediment),

TABLE 1
VALUE MATRIX

	ALT. #1	ALT. #2	ALT. #3
Sediment Yield	0.5	0.84	1.0
N (surface)	0.5	0.79	0.99
N (percolation)	0.5	0.86	0.02
P (surface)	0.5	0.78	0.99
Atrazine (surface)	0.5	0.84	1.0
Atrazine (percolation)	0.5	0.40	1.0
Sevin (surface)	0.5	0.83	1.0
Carbofuran (surface)	0.5	0.79	1.0
Net Income	0.5	0.71	0.46

3. Sediment yield,
4. Net income,
5. Atrazine in percolation,
6. Carbofuran surface loss,
7. P surface loss,
8. N surface loss, and
9. Atrazine surface loss.

Based on this ordering, the solutions to the best and worst linear programs were determined for each alternative by Theorem 2.1. The results are indicated in Figure 1. (Note: best and worst total utility for Alternative #1 are both 0.5.) Notice that Alternative #2 (corn with winter cover crop) clearly dominates, by Theorem 2.2, Alternative #1 (indicated by the broken line in Figure 1). The results for Alternative #3 indicate an extreme sensitivity to weight vectors consistent with the importance order. Clearly, Alternative #1 does not dominate Alternative #3 or vice versa. Additionally, we know by Theorem 3.1 that Alternative #3 does not dominate Alternative #2 given the importance order implied by (1.1). This graph indicates that regardless of the choice of weight, vector Alternative #2 is preferred to Alternative #1. Averaging the best and worst total utilities, according to *Step 7b* with $\mathbf{c} = (1, 1, 1, 1, 1, 1, 1, 1, 1)^T$ by default, yields the following ranking:

Alternative #2
Alternative #1 (conventional)
Alternative #3.

Suppose that the DM chooses to follow *Step 7a* in order to determine if

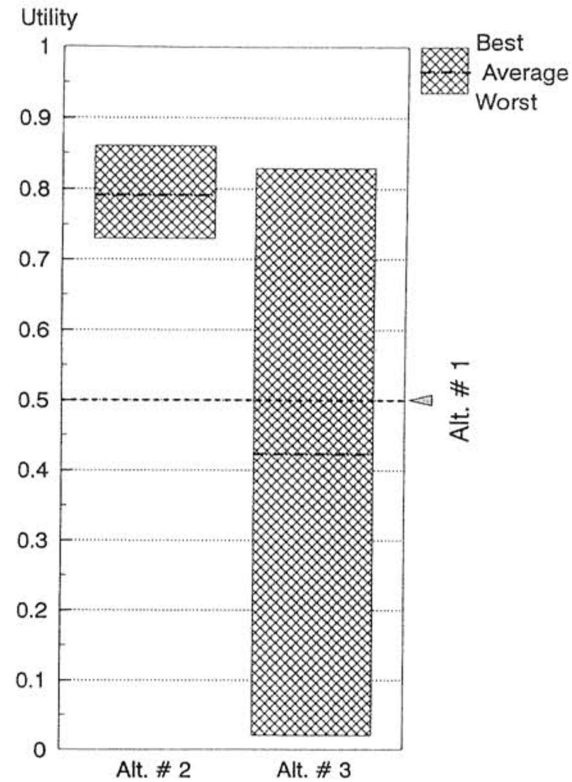

FIG. 1. Best and worst total utility.

Alternative #2 actually dominates Alternative #3. The conditions in (3.2) are not satisfied. The DM decides to provide additional information on the weights since domination is desired. In particular, suppose that information concerning existing nitrate levels in the groundwater indicate that this attribute should be weighted at least twice as much as the others, that is $w_1 \geqslant 2.0w_2$ (i.e., $c_2 = 2.0$). The best and worst total utilities with this additional information were computed according to (3.4) and are indicated in Figure 2. The shaded areas in Figure 2 indicate those values of the total utility that are no longer possible given the new information on the weights. Alternative #2 now dominates Alternative #3 according to Theorem 2.2, but this additional information does not affect the ranking of the alternatives based on *Step 7b*. Alternative #2 is the recommended choice among the three alternatives based on the importance order provided.

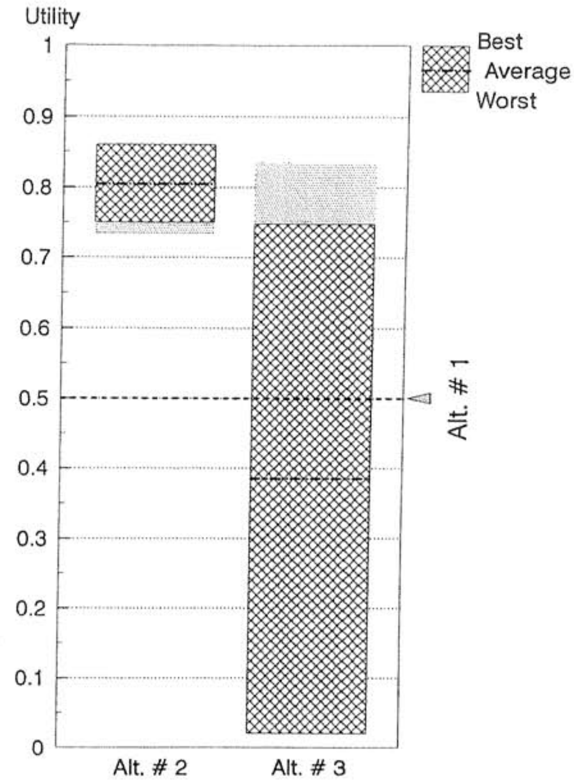

FIG. 2. Results for $c = (1, 2, 1, 1, 1, 1, 1, 1, 1)$.

5. CONCLUSIONS

In this paper, a theoretical basis for obtaining the best and worst total utilities used in [10] and [11] is provided. In addition, closed form solutions of the LP's for the best and worst total utilities were derived. Implementing the procedure or adding the steps to existing multi-attribute decision methods is conceptually and practically a simple task. We emphasize that this decision tool can be used to enhance, not replace, existing methods. Methods to determine the values in **V** and the importance order were not discussed.

An advantage of our approach is that if one alternative dominates all others in the sense of Theorem 2.2, then a recommendation can be made without ever requiring that a weight vector be specified. The example in the previous section illustrates that even the basic procedure, *Step 1–4* and *Step 7b*, without a strongly dominant alternative, will always rank an alternative at least as high as an alternative it dominates.

The graphical display of the range of possible values of the total utility of an alternative given the importance order alerts the DM to the sensitivity of that alternative to the weights. As we pointed out earlier, this tool may be particularly useful for group decisions. Importance order dominance of one alternative over another bolsters confidence in the ranking process, especially when there are disagreements about specific weights or a reluctance to base a decision on a single weight vector.

The closed form solutions also make it practical to examine the effects of uncertainty in the value matrix, **V** on the decision process. If the distributions of the random elements in **V** are known, estimates of the expected values of the best and worst total utilities can be computed, for example, by Monte Carlo sampling.

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