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Precipitation Simulation Model for Mountainous Areas

Clayton L. Hanson[1] Member ASCE
David A. Woolhiser[1] Member ASCE

ABSTRACT

This study investigated the use of the Markov chain-mixed exponential (MCME) model as a means of obtaining synthetic daily precipitation amounts at a site, based on knowledge of the mean annual precipitation of the site. Data for this study were from 25 National Weather Service stations in Idaho. There was a relationship between annual precipitation and some of the parameters in the MCME model, and other parameters could be assumed constant.

INTRODUCTION

Daily precipitation is a primary climatic variable in several hydrologic and natural resource models. The Markov chain-mixed exponential (MCME) model was developed to simulate daily precipitation series in areas such as the western United States where there are few measuring stations. In the mountainous areas of the western United States, most precipitation stations are located at valley sites which do not necessarily represent the climatic conditions at higher elevations. The effects of elevation on precipitation occurrence and amount have been investigated by several scientists (Hanson et al., 1989). In this paper, the relationships between mean annual precipitation from 22 stations in Idaho and MCME model parameters discussed by Hanson and Woolhiser (1990) were used to evaluate how well the MCME model represented daily precipitation from three Idaho stations that were not in the original analysis.

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[1]Agricultural Engineer, USDA-Agricultural Research Service (USDA-ARS), 270 South Orchard, Boise, ID 83705, and Hydraulic Engineer, USDA-ARS, 2000 East Allen Road, Tucson, AZ 85719.

MODELING DAILY PRECIPITATION

The daily precipitation process was described by the MCME model (Woolhiser and Roldan, 1986). Precipitation occurrence was described by a first-order Markov chain specified by parameters $P_{00}(n)$, the probability of a dry day on day "n" given that day n-1 was dry, and $P_{10}(n)$, the probability of a dry day on day "n" given that day n-1 was wet.

The amount of precipitation on a "wet" day was simulated with the mixed exponential distribution:

$$f(x) = \frac{\alpha(n)}{\beta(n)} \exp\left[\frac{-x}{\beta(n)}\right] + \frac{1-\alpha(n)}{\delta(n)} \exp\left[\frac{-x}{\delta(n)}\right] \quad (1)$$

where the mean precipitation per "wet" day, $\mu(n)$, equals $\alpha(n)\beta(n) + [1-\alpha(n)]\delta(n)$. The parameter α is usually assumed constant throughout the year, but β and μ may vary seasonally. Note that if $\alpha(n) = \alpha$ = a constant and $\beta(n)$ and $\mu(n)$ are specified by Fourier series, $\delta(n)$ is determined by the above relationship.

The seasonal variations in the parameters P_{00}, P_{10}, β, and μ were described by the polar form of a finite Fourier series which was limited to six harmonics (Woolhiser and Pegram, 1979).

Data for this study were from 25 National Weather Service stations in Idaho (Table 1 in Hanson and Woolhiser, 1990). Based on data availability, these daily precipitation records were as close to 40-year periods, beginning March 1, 1940 as possible. Data from 22 stations were used for parameter estimation and data from three stations were used for testing the accuracy of the model. The three stations used for testing were selected a priori to represent different climatic regions of the state.

Based on each record, constant terms in the Fourier series, $\bar{P}_{00}$, $\bar{P}_{10}$, significant amplitudes and phase angles, as determined by the Akaike information criterion (Akaike, 1974), and the number of wet days were determined for the occurrence (Markov chain) portion of the MCME model. The constant terms $\bar{\alpha}$, $\bar{\beta}$, and the mean precipitation per wet day, $\bar{\mu}$, and significant amplitudes and phase angles were determined for the mixed exponential portion of MCME. MCME parameters were estimated by maximum likelihood techniques described by

Woolhiser and Roldan (1986) who also discussed data requirements for obtaining reliable parameter estimates. All statistical tests were at the 0.05 level of probability.

ANALYSIS

Analysis of the records by Hanson and Woolhiser (1990) showed that there were significant linear relationships between most of the parameters in MCME and mean annual precipitation. In this paper we used the individual relationships between the parameters P_{00}, P_{10}, α, β, and μ and mean annual precipitation to generate daily precipitation for the three Idaho sites not used in developing the relationships. This approach is used because mean annual precipitation values are available for many locations or estimates can be calculated from other sources (Hanson, 1984).

Occurrence of Wet Days

Because values of $\bar{P}_{00}$, $\bar{P}_{10}$, and $\bar{\alpha}$ vary between 0 and 1, the logit transformation (Hanson et al., 1989) of their values was used to develop relationships between each of them and mean annual precipitation. This transformation was used to prevent computing unrealistic values of the parameters at sites where the mean annual precipitation is considerably more or less than at the sites used to develop the relationships.

The following linear relationships between $\bar{P}_{00}$ and $\bar{P}_{10}$, and mean annual precipitation were used to calculate the values of $\bar{P}_{00}$ and $\bar{P}_{10}$:

$$\theta_1 = 2.046 - 0.0011X \qquad r = 0.815 \tag{2}$$

$$\theta_2 = 0.648 - 0.0013X \qquad r = 0.927 \tag{3}$$

where θ_1 and θ_2 are the logit transformations of $\bar{P}_{00}$ and $\bar{P}_{10}$, respectively, and X is mean annual precipitation in mm.

P_{00} varied seasonally with the first five harmonics being significant. The amplitudes of the first harmonics (C_{P001}) and second (C_{P002}) increased linearly with increasing mean annual precipitation. The data from the other three amplitudes and all of the phase angles associated with P_{00} had a considerable amount of scatter so they were assumed constant. The relationships between C_{P001} and C_{P002}, and mean annual precipitation were:

$$C_{P001} = 0.0344 + 0.0001X \qquad r = 0.631 \tag{4}$$

$$C_{P002} = 0.0117 + .00003X \qquad r = 0.625 \tag{5}$$

P_{10} varied seasonally with the first three harmonics being significant. None of the amplitudes or phase angles were related to mean annual precipitation and were assumed constant.

Amount of Precipitation on Wet Days

There was a significant linear decrease in $\bar{\alpha}$ and it could not be shown that α varied seasonally. Values of $\bar{\alpha}$ were computed for the test sites by the following equation:

$$\theta_3 = -0.379 - 0.0012X \qquad r = 0.411 \tag{6}$$

After values of $\bar{\alpha}$ were computed for each site, new values of $\bar{\beta}$, $\bar{\mu}$, and the Fourier coefficients for β and μ were obtained.

There was a significant linear increase in $\bar{\beta}$ with increasing mean annual precipitation which resulted in the following equation.

$$\bar{\beta} = 0.736 + 0.0021X \qquad r = 0.401 \tag{7}$$

Only the first harmonic of β was significant and neither the amplitude or phase angle were related to mean annual precipitation, so constant values were used for generating daily precipitation.

There was a significant positive linear increase in $\bar{\mu}$ with mean annual precipitation which is:

$$\bar{\mu} = 2.231 + 0.0046X \qquad r = .891 \tag{8}$$

The first three harmonics of μ were significant. However, only the first phase angle ($\phi_{\mu 1}$) was related to mean annual precipitation as shown by equation (9), and the other phase angles and amplitudes were set to constants for genenerating precipitation.

$$\phi_{\mu 1} = -0.411 - 0.0033X \qquad r = -0.586 \tag{9}$$

TEST OF ESTIMATION PROCEDURE AND DISCUSSION

Two sets of 50-year, daily precipitation records were generated for the three test stations, Coeur d'Alene, Idaho Falls, and Riggins (Table 1). The first record was generated using maximum likelihood (ML) parameters computed from each station's record and the second record using parameter sets estimated from regional mean values or equations (2) through (9).

Table 1. Sample monthly and annual summary of 50-year daily precipitation (mm) simulations.

	January		July		Annual	
Coeur d'Alene, ID	Ave.	Std.	Ave.	Std.	Ave.	Std.
Historical	91	45	19	19	660	122
Simulated (ML)*	85	29	27	21	671	85
Simulated (est.)**	76	29	22	23	638	98
Idaho Falls, ID	Ave.	Std.	Ave.	Std.	Ave.	Std.
Historical	20	14	11	9	240	63
Simulated (ML)	24	14	16	17	250	47
Simulated (est.)	29	14	17	16	263	53
Riggins, ID	Ave.	Std.	Ave.	Std.	Ave.	Std.
Historical	34	23	18	17	429	77
Simulated (ML)	36	19	23	19	428	69
Simulated (est.)	48	16	16	15	409	65

*Simulations using ML parameters.

**Simulations using regional mean parameters.

Historical and simulated precipitation for January, July, and mean annual and the standard deviations are shown in Table 1. Simulations of mean annual precipitation using ML parameter sets varied from the same precipitation at Riggins to 4% greater than the historical record at Idaho Falls. Simulated annual precipitation, using the estimated parameter set, varied from 5% less than the historical record at Riggins to 9% greater at Idaho Falls. There was no pattern of over- or under-estimating monthly precipitation from either parameter set and simulated monthly values followed the seasonal trends at the three test sites.

Standard deviations of the simulated annual precipitation means were considerably less for both parameter sets than that from the historical record. The annual standard deviations were greater for two of the three test stations for simulations which used the estimated parameter set than simulations using the ML parameters.

The range of simulated monthly high and low values was similar to that of the historical record for the three sites. The range of simulated annual precipitation was about the same for two of the three sites for both parameter sets, but both simulations underestimated the range at one site by about 30%. The average number of wet days was simulated within three days by the ML parameter set. The simulation based on the ML parameter set overestimated the number of wet days by 3 out of 86 days at the site with the least number of wet days which was the greatest difference between historical and simulated of the three sites. The simulation based on the estimated parameter set underestimated the average number of wet days by four days at two sites and nine days at the site with the least number of wet days.

The ML model preserved the important statistics within a year using both sets of parameters, but as shown in a previous study (Hanson et al., 1989) caution should be taken when using the model to study annual phenomena.

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