NORMAL-DEPTH CALCULATIONS IN COMPLEX CHANNEL SECTIONS

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ABSTRACT: The general problem of solving for normal flow depth in open-channel flow has a complication in that some types of channel cross sections do not always have a unique solution. This paper analyzes an alternative iterative procedure for quickly and accurately solving the implicit problem of determining the normal flow depth in complex channel sections. Conditions that guarantee a unique solution and guarantee that the iterative procedure will converge to the solution are developed. A computer program for quickly and accurately finding the unique solution, using the Chezy or Manning flow resistance equations, is available. Test runs for a rectangular, a triangular, a trapezoidal, and two complex channel cross sections are used to evaluate the effectiveness of the procedure. The test results show that the iterative procedure presented here meets the requirements of guaranteed convergence, computational efficiency (speed and accuracy), and the ability to handle both trapezoidal and complex channel cross sections.

INTRODUCTION

The Chezy and Manning equations are widely used for determining the relations between the mean velocity of a turbulent steady uniform flow, the hydraulic roughness, and the slope of the channel bottom. There are no computational difficulties in solving these equations when the channel slope or channel discharge is the unknown. However, when the channel cross section is the unknown, the solution generally cannot be found explicitly, and for some types of channel cross section the problem does not always have a unique solution (Henderson 1966). For example, sufficiently high flow in a circular conduit will not have a unique solution (Barr and Das 1986). Chow (1959) provided a graphic procedure for the direct solution of the normal depth in rectangular and trapezoidal channels and in circular conduits running partially full. Graphic solutions were also presented by Jeppson (1965) for particular channel geometric shapes. Barr and Das (1986) presented a numerical solution for rectangular channels and both numerical and graphic procedures for trapezoidal channels and circular conduits running partially full using the Manning equation.

Although the concepts behind these methods are still valid, there is a need for replacing these particular approaches by computational algorithms to be implemented in modern high-speed computers. The Newton-Raphson method has been the usual numerical technique for solving the implicit problem of determining normal flow depth in a computer (McLatchy 1989). However, the method is sensitive (timewise) to starting position and, for some types of channel cross section, there is no guarantee that the method will converge to a unique solution (Press et al. 1986).

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The purpose of this paper is to accomplish the following: (1) Develop an alternative iterative procedure for quickly and accurately finding normal flow depth in complex channel cross sections; (2) find conditions guaranteeing both a solution that is unique and an iterative procedure that will converge to the solution; (3) present computational forms optimized for the special trapezoidal and more general complex channel cross sections; and (4) perform test runs to verify convergence and computational efficiency (speed and accuracy of the program). The findings from this paper will be helpful in contributing directly to the development of computer simulation models by providing an efficient algorithm with guaranteed convergence for computing normal flow depth when necessary.

FLOW RESISTANCE EQUATIONS

In this paper, A and P denote flow cross-sectional area (L^2) and wetted perimeter (L), respectively, as a function of flow depth. (Note: L represents length and T time for all variables.) The hydraulic radius, R (in L), is defined by:

$$R = \frac{A}{P} \dotfill (1)$$

The Chezy equation for a turbulent uniform flow (Chow 1959) may be written as:

$$V = C(S_0 R)^{1/2} \dotfill (2a)$$

and the Manning equation may be written as:

$$V = \left(\frac{S_0^{1/2}}{n}\right) R^{2/3} \dotfill (2b)$$

where V = the mean flow velocity (LT^{-1}); S_0 = the slope of the channel bottom in the direction of flow; C = the Chezy factor of flow resistance $(L^{1/2}T^{-1})$; and n = the Manning coefficient of hydraulic roughness $(L^{-1/3}T)$. Velocity and area are related to flow discharge, Q (in $L^3 T^{-1}$), by:

$$Q = AV \dotfill (3)$$

Combining Eqs. 1–3 gives:

$$Q = KA^\alpha P^{-\beta} \dotfill (4)$$

where for the Chezy equation:

$$K = CS_0^{1/2}, \quad \alpha = \frac{3}{2}, \quad \beta = \frac{1}{2} \dotfill (5a)$$

and for the Manning equation:

$$K = \frac{S_0^{1/2}}{n}, \quad \alpha = \frac{5}{3}, \quad \beta = \frac{2}{3} \dotfill (5b)$$

In the following section, an iterative procedure is presented to solve the

uniform flow equations quickly and accurately for the normal depth, y, when Q, A, and P satisfy certain conditions.

NUMERICAL ANALYSIS

Consider a channel cross section in which the flow rate can be expressed by Eq. 4 and satisfying the following:

1. K, α, and β are positive.
2. Q, A, and P are nonnegative, continuous, and strictly increasing.
3. $Q(0) = A(0) = 0$, and $Q(\infty) = A(\infty) = \infty$.

For a given $Q_0 \geq 0$, one wishes to solve $Q(y) = Q_0$ for $y \geq 0$. Since, by the last two conditions, Q is continuous and strictly increasing from 0 to ∞, a unique solution to the equation exists. Since Q can be mathematically inverted only in special cases, one seeks a numerical solution.

By the second condition, A is strictly increasing and continuous, and thus invertible, and its function inverse, A^{-1}, is continuous and increasing. It can be assumed that there is either a mathematical formula for the inverse (as in the case of a trapezoidal channel) or a numerical procedure for the inverse (as when A is calculated by interpolation from a table of depth and area values). It can also be assumed that A and P may be computed using mathematical formulas or numerical procedures. For a given flow rate $Q_0 > 0$, $f(y)$ is defined as:

$$f(y) = A^{-1}\left[\left(\frac{Q_0}{K}\right)^{1/\alpha} P(y)^{\beta/\alpha}\right] \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (6)$$

so that if $y = f(y)$, then $Q_0 = Q(y)$. Given any initial $y_1 > 0$, y_n is iteratively defined by:

$$y_{n+1} = f(y_n) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (7)$$

Let y_0 denote the true solution of $Q_0 = Q(y_0)$; then $y_0 = f(y_0)$.

1. If $y \leq y_0$, then $y \leq f(y) \leq y_0$.
2. If $y_0 \leq y$, then $y_0 \leq f(y) \leq y$.

are true according to the following: Assuming that $y \leq y_0$, from condition 2, one gets $Q(y) \leq Q(y_0) = Q_0$. Since A and P are increasing, so are A^{-1} and f. Rearranging the inequality $Q(y) \leq Q_0$ gives $A(y) \leq (Q_0/K)^{1/\alpha}P(y)^{\beta/\alpha}$; applying A^{-1} to both sides of the inequality gives $y \leq f(y)$. Applying f to both sides of the inequality $y \leq y_0$ gives $f(y) \leq f(y_0) = y_0$. This establishes the first premise; the second is similarly established.

It now can be shown that the sequence y_n converges to y_0. If the initial value, y_1, is less than or equal to y_0, then premise 1 says that y_n is an increasing sequence bounded above by y_0. Such a sequence must converge to some value y_*. Eq. 7 and the continuity of f give $y_* = f(y_*)$, and hence $Q_0 = Q(y_*)$. Since the latter equation has y_0 as its unique solution, $y_* = y_0$. If the initial value y_1 is greater or equal to y_0, then one gets y_n to converge downward to y_0, using a similar argument with premise 2.

In the following, let P_n be $P(y_n)$ and Q_n be $Q(y_n)$. Numerically, the pre-

ceding iteration has to be done until y_n is close to y_0 and Q_n is close to Q_0. Since y_0 is not available, one settles for having y_{n+1} close to y_n and Q_n close to Q_0. Relative errors are defined by:

$$\text{re } Q(n) = \text{abs} \left(\frac{Q_n - Q}{Q} \right) \dots \dots \dots \dots \dots \dots (8)$$

and

$$\text{re } y(n) = \text{abs} \left(\frac{y_n - y_{n+1}}{y_{n+1}} \right) \dots \dots \dots \dots \dots \dots (9)$$

It can be shown, under appropriate conditions, that:

$$\text{re } Q(n) = \text{abs} \left[\left(\frac{P_n}{P_{n+1}} \right)^\beta - 1 \right] \leq \text{abs} \left(\frac{P_n}{P_{n+1}} - 1 \right) \leq \text{re } y(n) \dots \dots \dots (10)$$

The first equality is true in general. The first inequality requires $\beta < 1$, and the last inequality is special for a trapezoidal channel. Thus for a trapezoidal channel one needs only to check for re $y(n)$ small.

Efficient computational procedures for f are presented in the following sections, depending on whether the shape of the channel cross section is triangular, rectangular, trapezoidal, or complex (defined by points connected by straight lines). Conditions 1–3 will be proved for the complex cross-section shape. Since the other cross-section shapes are special cases of the complex cross section, conditions 1–3 are also true for the other cross-section shapes.

GEOMETRIC PROPERTIES OF CHANNEL SECTION

Consider a fixed location in a trapezoidal channel (Fig. 1). For any given depth of flow, y, the following geometric relations can be defined:

$$A(y) = (b + c_1 y)y \dots \dots \dots \dots \dots \dots (11)$$

$$P(y) = b + c_2 y \dots \dots \dots \dots \dots \dots (12)$$

$$R(y) = \frac{(b + c_1 y)y}{b + c_2 y} \dots \dots \dots \dots \dots \dots (13)$$
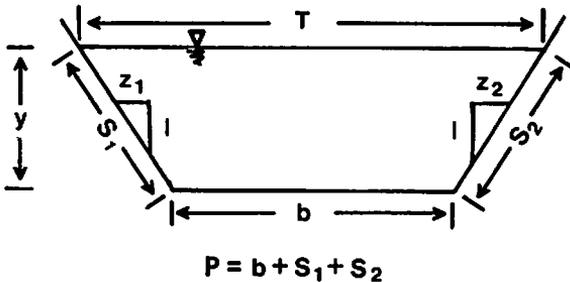

$$P = b + S_1 + S_2$$

FIG. 1. Geometric Elements of Channel Cross Section

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where b = the width of the channel bottom (L), and c_1 and c_2 = constants defined in terms of the side slopes z_1 and z_2 (Fig. 1) as:

$$c_1 = \frac{z_1 + z_2}{2} \quad \text{...} \quad (14)$$

$$c_2 = (1 + z_1^2)^{1/2} + (1 + z_2^2)^{1/2} \quad \text{................................} \quad (15)$$

Triangular Channel Section

For a triangular-shaped channel section, $b = 0$ and $c_1 > 0$, and the flow depth can be determined in terms of Q directly as follows. For the Chezy equation:

$$y = \left[\left(\frac{Q}{c_1 C} \right)^2 \left(\frac{c_2}{c_1 S_0} \right) \right]^{1/5} \quad \text{......................................} \quad (16a)$$

and for the Manning equation:

$$y = \left[\left(\frac{nQ}{c_2 S_0^{1/2}} \right)^{3/5} \left(\frac{c_2}{c_1} \right) \right]^{5/8} \quad \text{....................................} \quad (16b)$$

Rectangular Channel Section

For a rectangular channel section, $b > 0$, $c_1 = 0$, and $c_2 = 2$. The numerical procedure must be used to compute the flow depth. Using the Chezy equation, Eq. 6 for f becomes:

$$f(y) = a_2(a_1 + y)^{1/3} \quad \text{...} \quad (17a)$$

$$a_1 = \frac{b}{2} \quad \text{..} \quad (17b)$$

$$a_2 = \left[\left(\frac{Q}{bC} \right)^2 \left(\frac{2}{bS_0} \right) \right]^{1/3} \quad \text{......................................} \quad (17c)$$

and, using the Manning equation:

$$f(y) = a_2(a_1 + y)^{2/5} \quad \text{...} \quad (18a)$$

$$a_1 = \frac{b}{2} \quad \text{..} \quad (18b)$$

$$a_2 = \left(\frac{nQ}{2S_0^2} \right)^{3/5} \left(\frac{2}{b} \right) \quad \text{...} \quad (18c)$$

General Trapezoidal Channel Section

For a general trapezoidal channel section, $b > 0$ and $c_1 > 0$. The numerical procedure must be used to compute the flow depth. Using the Chezy equation, Eq. 6 for f becomes:

$$f(y) = \frac{w}{a_1 + (a_2 + w)^{1/2}} \quad \text{.......................................} \quad (19a)$$

$$w = [(a_3 + y)a_4]^{1/3} \quad \text{.......................................} \quad (19b)$$

$$a_1 = \frac{b}{2c_1} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (19c)$$

$$a_2 = a_1^2 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (19d)$$

$$a_3 = \frac{b}{c_2} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (19e)$$

$$a_4 = \left(\frac{c_2}{c_1 S_0}\right)\left(\frac{Q}{c_1 C}\right)^2 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (19f)$$

and using the Manning equation:

$$f(y) = \frac{w}{a_1 + (a_2 + w)^{1/2}} \cdot \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (20a)$$

$$w = [(a_3 + y)a_4]^{2/5} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (20b)$$

$$a_1 = \frac{b}{2c_1} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (20c)$$

$$a_2 = a_1^2 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (20d)$$

$$a_3 = \frac{b}{c_2} \cdot \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (20e)$$

$$a_4 = \left(\frac{c_2}{c_1}\right)\left(\frac{nQ}{c_1 S_0^{1/2}}\right)^{3/2} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (20f)$$

Complex Channel Section

Natural channel sections are in general very irregular, usually varying from an approximate parabola to an approximate trapezoidal. A complex channel cross section may be defined in terms of N specified flow depths (y_i) and the distances to the left (wL_i) and right (wR_i) sides of the channel at those depths, as shown in Fig. 2. The following conditions are assumed to hold:

1. The flow depth values, y_i, are strictly increasing and $y_1 = 0$ (point at the bottom of channel cross section).

2. The section widths, $w_i = wL_i + wR_i$, are nondecreasing and $w_1 \geq 0$, and $w_2 > 0$.

3. The sides of the channel are vertical above the last specified flow depth. (This is done to prevent channel overflow. If extension at an angle is desired, set the last y_i value extremely high and set the wL_i and wR_i values to get the desired angles.)

The following definitions are used in the next sections:

$$\Delta wL_i = wL_{i+1} - wL_i \quad \text{for } 1 \leq i < N \dots\dots\dots\dots\dots\dots\dots\dots \quad (21a)$$

$$\Delta wR_i = wR_{i+1} - wR_i \quad \text{for } 1 \leq i < N \dots\dots\dots\dots\dots\dots\dots\dots \quad (21b)$$

$$\Delta w_i = w_{i+1} - w_i \quad \text{for } 1 \leq i < N \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (21c)$$

$$\Delta y_i = y_{i+1} - y_i \quad \text{for } 1 \leq i < N \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (21d)$$
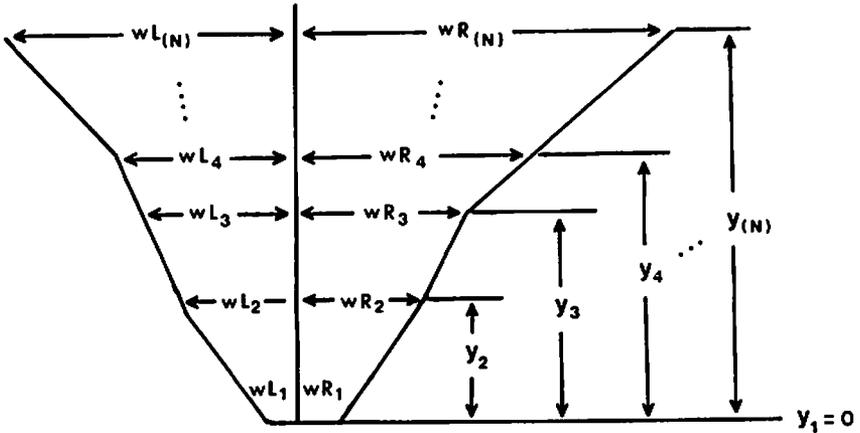
FIG. 2. Complex Channel Cross Section Defined in Terms of Horizontal Sub-sections at Specified Flow Depths (y_i) and Distances to Left (wL_i) and Right (wR_i) Sides

For flow depth y with $0 \le y \le y_N$, let i be the index such that $y_i \le y \le y_{i+1}$ For $y > y_N$, let $i = N$. Cross-sectional area, $A(y)$, and wetted perimeter, $P(y)$, are defined as:

$$A(y) = A_i + (y - y_i)a1_i + (y - y_i)^2a2_i \dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(22)}$$

where

$$A_1 = 0\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(23a)}$$

$$A_i = A_{i-1} + \Delta y_{i-1}a1_{i-1} + (\Delta y_{i-1})^2a2_{i-1} \quad \text{for } 1 < i \le N \dots\dots\dots\dots \text{(23b)}$$

$$a1_i = w_i \quad \text{for } 1 \le i \le N \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(23c)}$$

$$a2_i = \frac{2\Delta w_i}{\Delta y_i} \quad \text{for } 1 \le i < N\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(23d)}$$

$$a2_N = 0\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(23e)}$$

and

$$P(y) = P_i + (y - y_i)p1_i \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(24)}$$

where

$$P_1 = w_1\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(25a)}$$

$$P_i = P_{i-1} + (\Delta y_{i-1})p1_{i-1} \quad \text{for } 1 < i \le N\dots\dots\dots\dots\dots\dots\dots \text{(25b)}$$

$$p1_i = \left[1 + \left(\frac{\Delta wL_i}{\Delta y_i}\right)^2\right]^{1/2} + \left[1 + \left(\frac{\Delta wR_i}{\Delta y_i}\right)^2\right]^{1/2} \quad \text{for } 1 \le i < N\dots\dots \text{(25c)}$$

$$p1_N = 2\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(25d)}$$

The second set of conditions are sufficient to show that the first set is true.
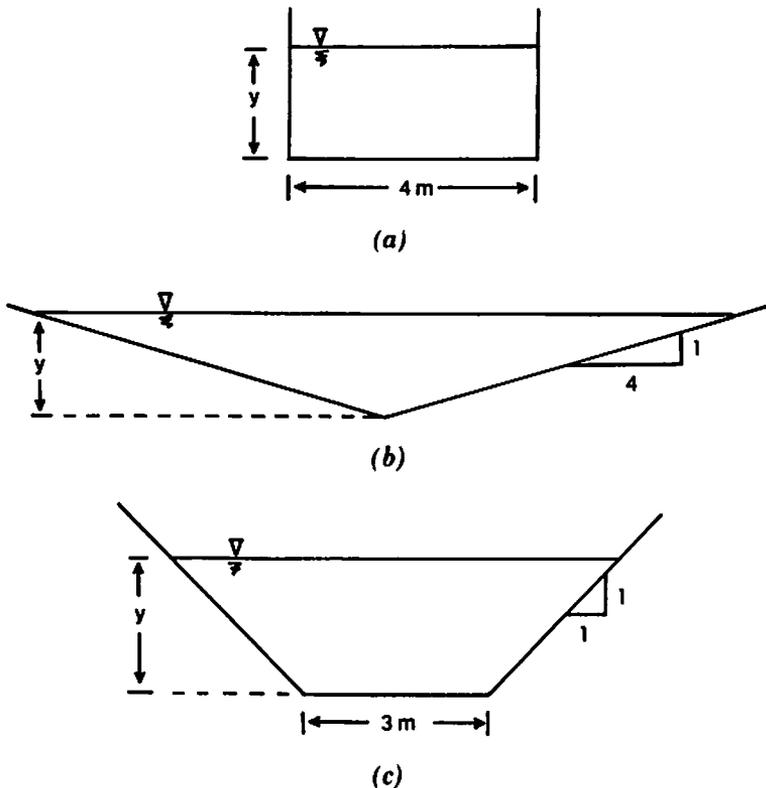
(a)

(b)

(c)

FIG. 3.   Channel Cross Sections: (a) Rectangular; (b) Triangular; and (c) Trapezoidal

The only significant problem is showing that $Q(y)$ given by Eq. 4 is continuous at 0 and is strictly increasing.

Since $Q = KA^{\alpha}P^{-\beta}$, with $K > 0$, $\alpha > 0$, and $\beta > 0$, and A is strictly increasing from 0, it suffices to show that $R = A/P$ is continuous at 0 and strictly increasing. The hydraulic radius $R(0)$ is defined to be zero. To show continuity of $R(y)$ at 0, one needs to show that $\lim_{y \to 0} R(y) = 0$. For $0 < y \le y_2$, it can be shown that:

$$R(y) = \frac{A(y)}{P(y)} = \frac{ya1_1 + y^2 a2_1}{w_1 + yp1_1} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (26)$$

because $i = 1$, $y_1 = 0$, and $a1_1 = 0$. In this form, it is easily seen that $R(y)$ tends to 0 as y tends to 0. $R(y)$ is strictly increasing when its derivative:

$$R'(y) = \frac{A'(y)P(y) - A(y)P'(y)}{P(y)^2} > 0 \qquad \text{for } y > 0 \dots\dots\dots\dots\dots\dots (27)$$

Since $P(y)^2 > 0$ for $y > 0$, to establish Eq. 27 one needs to show that $s(y) = A'(y)P(y) - A(y)P'(y) > 0$ for $y > 0$. Since the second derivative $A''(y)$
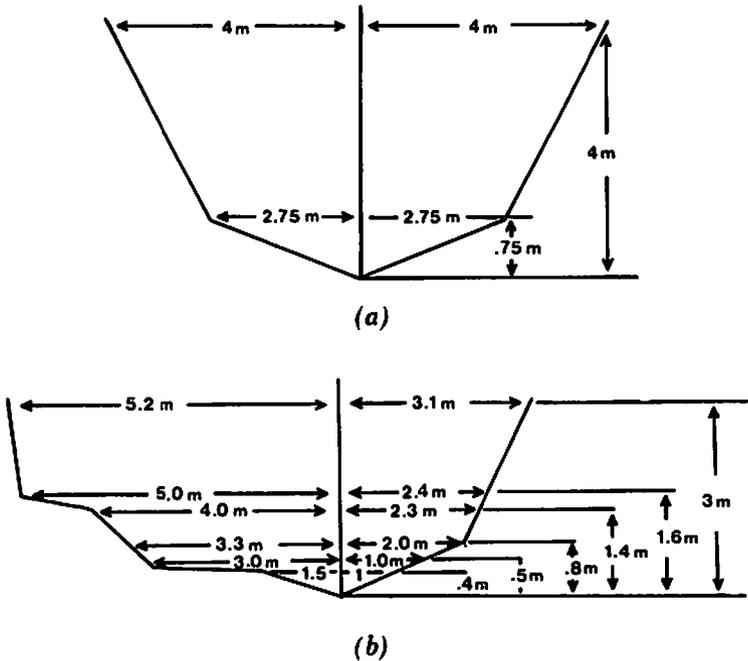
FIG. 4. Complex Channel Cross Sections: (a) Section 1; and (b) Section 2

≥ 0, and $P''(y) = 0$, one gets $s'(y) = A''(y)P(y) - A(y)P''(y) = A''(y)P(y)$ ≥ 0. Since $A'(0) = P(0) = w_1$ and $A(0) = 0$, one gets $s(0) = w_1^2$. These statements show that $s(y)$ is a nondecreasing function and $s(0) \geq 0$.

In the case that $a2_1 > 0$, since $P(y) > 0$ for $y > 0$, one gets $s'(y) > 0$ for $y_2 \geq y > 0$. Since $s(0) \geq 0$, $s(y)$ immediately becomes positive; since $s(y)$ is nondecreasing, it stays positive.

In the case that $a2_i = 0$, $w_1 = w_2 > 0$; hence $s(0) = w_1^2 > 0$. Thus function $s(y)$ starts out positive, and since $s(y)$ is a nondecreasing function, it stays positive.

The first conditions 1 and 2 are valid in a more general setting but not in complete generality. For example, in a circular conduit running partially full, when the flow depth is sufficiently close to the top of the circle, there are two possible solutions to normal depth. The width restrictions cover most practical open channel situations.

PERFORMANCE AND EVALUATION

Test runs of the numerical procedure were performed as follows. For a given channel cross section, bottom slope, flow resistance (Chezy's C or Manning's n), and flow depth, the outflow rate was computed directly from the equations. The given cross section, bottom slope, flow resistance, and computed outflow rate were then used in the numerical procedure to estimate the flow depth at a specified convergence tolerance and an initial flow depth

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TABLE 1. Range of Flow-Resistance Coefficients, Channel Slopes, and Flow Depths Used in Test Runs

Variable (1)	Lower value (2)	Upper value (3)
C	30.0	90.0
n	0.01	0.15
S_0	0.00001	0.10
y_n	0.01	3.0

Note: C = Chezy's C; n = Manning's n; S_0 = channel slope; and y_n = normal flow depth.

TABLE 2. Maximum Error as Function of Tolerance with an Initial Flow-Depth Guess of 2 m

Tolerance (1)	Maximum Absolute Error	
	Chezy (2)	Manning (3)
10^{-7}	4.77×10^{-8}	4.77×10^{-8}
10^{-6}	7.15×10^{-7}	7.15×10^{-7}
10^{-5}	4.04×10^{-6}	5.01×10^{-6}
10^{-4}	6.99×10^{-5}	6.68×10^{-5}
10^{-3}	3.62×10^{-4}	8.67×10^{-4}
10^{-2}	5.06×10^{-3}	4.61×10^{-3}

TABLE 3. Maximum Number of Iterations as Function of Initial Flow-Depth Guess with Convergence Tolerance of 10^{-4}

Initial flow-depth guess (m) (1)	Maximum Number of Iterations	
	Chezy (2)	Manning (3)
10^{-10}	9	10
10^{-4}	9	10
10^{-3}	8	9
1.0	7	8
2.0	7	8
3.0	7	8
6.0	7	8
20.0	8	9
50.0	8	9
10^2	8	9
10^3	9	10
10^4	9	10

guess. The absolute value of the difference between the given flow depth and the estimated flow depth is a measure of the numerical procedure error. The number of iterations required for convergence is a measure of the numerical procedure speed.

For each selected convergence tolerance, initial flow depth guess, and flow

TABLE 4. Number of Iterations and Absolute Errors Using Chezy's C with Tolerance = 10^{-4} and $y_0 = 2$ m

Cross section (1)	N_{min} (2)	E_{min} (3)	N_{max} (4)	E_{max} (5)
Triangle	1	0.0	1	2.38×10^{-7}
Rectangle	4	0.0	6	6.99×10^{-5}
Trapezoidal	4	0.0	6	2.62×10^{-5}
Complex 1	5	1.67×10^{-7}	7	1.45×10^{-5}
Complex 2	5	1.76×10^{-7}	7	1.14×10^{-5}

Note: y_0 = initial flow-depth guess; N_{min} = minimum iterations; E_{min} = minimum absolute error; N_{max} = maximum iterations; and E_{max} = maximum absolute error.

TABLE 5. Number of Iterations and Absolute Errors Using Manning's n with Tolerance = 10^{-4} and $y_0 = 2$ m

Cross section (1)	N_{min} (2)	E_{min} (3)	N_{max} (4)	E_{max} (5)
Triangle	1	0.0	1	1.19×10^{-7}
Rectangle	4	1.86×10^{-9}	7	4.98×10^{-5}
Trapezoidal	4	9.31×10^{-10}	6	6.68×10^{-5}
Complex 1	5	1.18×10^{-7}	7	4.34×10^{-5}
Complex 2	5	1.25×10^{-7}	7	4.08×10^{-5}

Note: y_0 = initial flow depth guess; N_{min} = minimum iterations; E_{min} = minimum absolute error; N_{max} = maximum iterations; and E_{max} = maximum absolute error.

resistance equation (Chezy or Manning), a set of 625 test runs were performed. These tests used five channel cross sections [Fig. 3(a–c) and 4(a–b)] and five equally spaced values for each of bottom slope, flow resistance, and flow depth. Table 1 shows the ranges used for these values. Each set of tests took less than 5 s of processing time on a VAX 750.

To determine accuracy as a function of tolerance, six sets of test runs were performed for each flow resistance equation with the initial flow-depth guess set to 2 m and convergence tolerance varying from 10^{-7} to 10^{-2}. Table 2 summarizes the results of these tests. Tolerances above 10^{-6} are not recommended because truncation errors may be such that the procedure never converges.

To determine speed as a function of initial flow-depth guess, 12 sets of test runs were performed for each flow-resistance equation with convergence tolerance set to 10^{-4} and initial flow-depth guess varying from 10^{-10} m to 10^4 m. Table 3 summarizes the results of these tests.

Tables 4 and 5 give a more detailed summary of the test set with a convergence tolerance of 10^{-4} and an initial flow-depth guess of 2 m.

SUMMARY AND CONCLUSIONS

An iterative procedure was presented for quickly and accurately solving the implicit problem of determining the normal depth in complex channel cross sections using the Chezy or Manning flow-resistance equations. Test

runs were performed to evaluate the iterative numerical technique, using a rectangular, a triangular, a general trapezoidal [Figs. 3(a–c)], and two complex channel cross sections [Figs. 4(a) and 4(b)] and using Chezy's C values from 30.0 to 90.0, Manning's n values from 0.01 to 0.15, channel bottom slopes from 10^{-5} to 10^{-1}, and normal flow depths, from 0.01 m to 3.0 m (Table 1). It was verified during the test runs that the algorithms always converged for a convergence tolerance of 10^{-6} or more and that absolute errors were not affected by initial flow depth guess. A maximum number of 12 iterations was observed for the complex cross section 2 [Fig. 4(b)] when using the Chezy equation and a tolerance of 10^{-7} and an initial flow-depth guess of 2 m. The same tolerance and initial flow-depth guess resulted in a maximum number of 13 iterations for the second complex cross section when using the Manning equation. Absolute errors decreased with decreasing tolerance. A tolerance of 10^{-3} and an initial flow-depth guess of 2 m resulted in a very satisfactory maximum absolute error of only 8.67×10^{-4} m. It also was verified during the test runs that the initial flow depth guess does not significantly affect the algorithm performance (Table 3).

Tests similar to those devised for the iterative procedure were also devised for the Newton-Raphson method, except that the two complex channel cross sections were not used. It was observed that the iterative procedure is computationally efficient comparable to the Newton-Raphson method with suitable starting position (number of iterations and computation times were generally a little larger for the Newton-Raphson method) and very insensitive (timewise) to starting position.

In conclusion, the test results have shown that the iterative procedure presented herein meets the requirements of guaranteed convergence, computational efficiency (speed and accuracy), and ability to handle both trapezoidal and complex channel cross sections. Because uniform flow is a condition of fundamental importance in channel-design problems and natural stream calculations, the techniques described here are useful because they offer a fast, accurate solution and can be implemented in computer simulation models.

A computer program has been written to implement the alternative iterative procedure on a computer. The source code was written in standard FORTRAN 77 for efficiency and portability, especially among personal computers. A copy of the program can be obtained from the writers.

APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A	=	flow cross-sectional area;
a_1, a_2, a_3, a_4	=	constants;
b	=	channel bottom width;
C	=	Chezy factor of flow resistance;
c_1, c_2	=	constants;
K	=	coefficient;
N	=	number of horizontal subsections describing complex cross section;
n	=	Manning coefficient of hydraulic roughness;
P	=	wetted perimeter;
Q	=	flow discharge;
R	=	hydraulic radius;
S_0	=	channel bottom slope;
V	=	mean flow velocity;
w_i	=	width of subsection i;
wL_i	=	left-hand-side distance from centerline of channel cross section corresponding to flow depth y_i;
wR_i	=	right-hand-side distance from centerline of channel cross section corresponding to flow depth y_i;
y	=	flow depth;
y_i	=	flow depth at subsection i;
z_1, z_2	=	side slopes;
α	=	coefficient; and
β	=	coefficient.

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Normal-Depth Calculations in Complex Channel Sections
Edward D. Shirley and Vicente L. Lopes, Members, ASCE
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These are corrected equations:

$$a_2 = \left(\frac{nQ}{2S_0^{1/2}}\right)^{3/5}\left(\frac{2}{b}\right) \qquad (18c)$$

$$a2_i = \frac{\Delta w_i}{2\Delta y_i} \qquad for\ 1 \leq i < N \qquad (23d)$$