

The Water Erosion Prediction Project:
Erosion Parameter Estimation

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The USDA Water Erosion Prediction Project (WEPP) includes a large field program for collection of experimental data to evaluate soil erodibility. Rainfall simulation techniques are used to measure runoff and soil loss on experimental plots. Given these data on rates and amounts of soil loss, techniques are needed to estimate model parameters from field data to allow subsequent application of the simulation model in erosion prediction. An optimization method for determining erodibility parameters from field data is outlined.

Introduction

The governing equations for interrill and rill erosion were derived by Foster and Meyer (1972) and are discussed in detail by Foster (1982). The specific forms of the equations used in WEPP are presented by Lane et al. (1988). Briefly, the sediment continuity equation for overland flow is:

$$dG/dx = D_r + D_i \quad (1)$$

where G (kg/m/s) is sediment load per unit of rill width, x is distance downslope (m), D_i (kg/m²/s) is interrill delivery rate which is considered uniform with x , and D_r (kg/m²/s) is detachment or deposition by flow in the rill or concentrated flow channel. Only detachment is considered here and the deposition equations will not be presented. However, sediment load is primarily controlled by transport capacity when deposition occurs so that

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erodibility parameters cannot be estimated from field data where deposition occurred.

For the case of detachment of bare soil, the steady state continuity equation is:

$$dG/dx = K_i I^2 + K_r (\tau - \tau_c)(1-G/T_c) \quad (2)$$

where K_i is interrill erodibility (kg s/m^4), I is rainfall intensity (m/s), K_r (s/m) is the rill erodibility parameter, τ (Pa) is shear stress in the rill acting on the soil, τ_c is a critical or threshold shear stress for detachment to occur, T_c (kg/s/m) is sediment transport capacity in the rill, and G is sediment load as described above.

This equation cannot be solved analytically. The simplest case for solving Eq. 2 is to assume that G is small compared to T_c , that τ and T_c are independent of x , and that K_i can be found independently from K_r and τ . In this case, the equation is readily solvable and erodibility parameter values can be obtained with linear regression techniques. However, estimated values of K_r will be distorted from their optimal value depending on the ratio of G to T_c and the rate of variation of τ and T_c with x .

The purpose of this paper is to outline and evaluate an optimization method for determining erodibility parameters from field data for use in the WEPP representative profile model. The model uses a steady state sediment continuity equation (Eq. 2) as a basis for calculating sediment loads, hence the parameters evaluated in this study are for the steady state equation. The proposed method may be used to optimize simultaneously for all three erodibility parameters (K_i , K_r , τ_c), or if the interrill parameter, K_i , is known, the two rill erodibility parameters alone may be determined. Thus, the method may be used to evaluate parameters from experiments designed specifically for determination of rill erosion parameters (i.e., Laflen et al. 1987) or from large plot data alone (i.e., Simanton et al. 1986), where all three erodibility parameters must be evaluated simultaneously. Response surfaces for hypothetical parameter values are discussed, the optimization scheme is described, and examples using field data are presented.

Model Response

A least squares objective function was used to describe the difference between predicted and measured field values of sediment load. Optimization involves minimizing this objective function for

a set of field data. The objective function may also be used to characterize the sensitivity of the model to variation of input parameters. Given a known optimum set of parameter values (K_i , K_r , r_c), a sum of squares difference between the calculated sediment loads at the optimum point and the calculated sediment loads at points away from the optimum can be found. A contour mapping of the values of the least squares objective function in the parameter space is a response surface. Figure 1 shows an example of this for a set of K_r and r_c values from the data set described below.

Seven sets of synthetic data were generated with the model to evaluate the potential of optimizing the erodibility parameters for the model. Table 1 lists the parameters for the seven sets. The data generated were for the case of a ten meter long plot with rainfall intensity varying from 30 to 180 mm/hr and runoff ranging from 15 to 90 l/min per meter width. Six synthetic data points were generated for each data set. Contours of the objective function, which was the least squares difference between sediment loads calculated at the known optimum point and those calculated for erodibility values away from the known optimum, were plotted. In plotting these surfaces one of the parameters was held constant while the other two were varied around the optimum values.

Table 1. Model input and optimization results for synthetic erosion data.

Case	actual parameter values			2 parameter optimization		3 parameter optimization		
	K_r	r_c	K_i	K_r	r_c	K_r	r_c	K_i
	s/m	Pa	kg $\frac{1}{m^4}$ * 10^{-6}	s/m	Pa	s/m	Pa	kg $\frac{1}{m^4}$
1	.025	3.0	2.5	.0250	3.004	.0248	2.988	2.504
2	.025	3.0	1.0	.0250	3.004	.0253	3.008	0.969
3	.025	3.0	5.0	.0250	3.004	.0257	3.047	5.009
4	.025	0.5	2.5	.0250	0.514	.0250	0.494	2.501
5	.005	3.0	2.5	.0049	2.963	.0054	3.169	2.488
6	.050	3.0	2.5	.0500	3.004	.0491	2.962	2.513
7	.025	6.0	2.5	.0252	6.008	.0240	5.967	2.529

The response surfaces of the objective function were relatively well behaved, although some elongation of the surface was evident. The elongation of the response surface indicated a dependence between the parameters. For each synthetic data set, the response function remained low in a valley of increasing K_r and r_c ; the

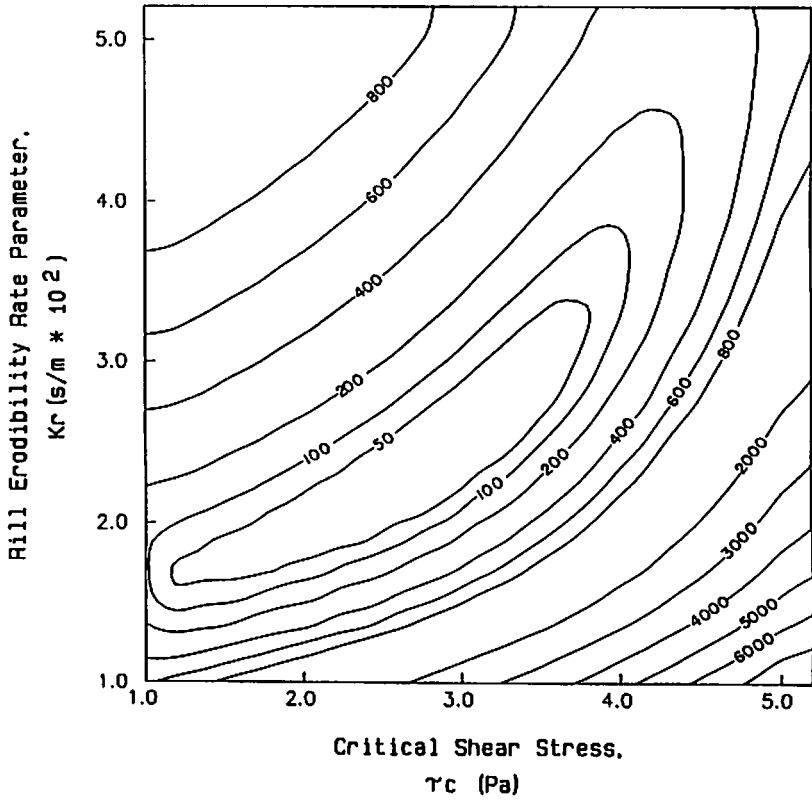


Figure 1. Example of a response surface for a set of K_r and τ_c values.

increase in K_r was offset by a decrease in r_c in terms of sediment load prediction. Similar dependence existed between K_r and K_i , but was not as evident between K_i and r_c for the synthetic data.

The shape of the model response surfaces and the dependency of the erodibility terms indicated the need for a relatively large range of output values and flow conditions to obtain the erodibility parameters. Three or even two erodibility parameters cannot be determined from an experimental condition with only one rainfall intensity, discharge rate, slope gradient, and slope length. The greater the number of flow conditions in an experiment, the greater will be the number of degrees of freedom for determining the parameter values.

Overall the response surfaces in the erodibility parameter space were considered to be relatively well behaved. Dependencies between parameters were not as great as those found for the model studied by Blau et al. (1988). The fact that the optimization procedure worked well in finding response surface minima, as will be shown below, further suggested that the model is a viable description of the steady state erosion process.

Optimization

An optimization algorithm was used to obtain erodibility parameter values from sediment load data for the steady state erosion model. The procedure is described below for optimizing two parameters, K_r and r_c .

Initial (best guess) values of K_r and r_c are input. The program then uses the erosion model to calculate sediment loads for an array of K_r and r_c values around the central values. The least squares objective function is calculated for each point on the array, the minimum of the function is found, and the central K_r and r_c values are reset to correspond to the minimum point of the array. Then, the program calculates a new array of the objective function around the new central values with a finer grid mesh, and finds the minimum.

The process is repeated for successively finer grids. If the minimum of the objective function is found on the boundary of one of the grids, the central values are readjusted to that point on the grid, but the grid is not made finer. This is necessary when searching for the minimum value when it is located in a "valley" on the response surface since the line of minimum objective function in the valley may pass between points of the grid set by the program. The accuracy of the method is dependent upon the number of times that the grid size is reduced and the mesh is made finer. Experience in using the procedure indicated that a good optimum set of parameter values could be obtained after five reductions in the grid size.

The process for optimizing for three parameters is the same as for optimizing two, except that a three dimensional grid is used instead of two.

The optimization technique was tested with twenty sets of field data. The model erodibility parameters were optimized for field data from five WEPP rangeland sites, the study of Stein et al. (1986), the study of Meyer et al. (1975), and four WEPP cropland sites. The results of the field data optimizations are presented in Table 2. Figure 2 is a plot of optimized vs. measured sediment load results using the parameters derived from the optimization procedure for selected data sets. There were no apparent problems using the optimization technique to obtain erodibility values from the data used in this study, and the model represented the data quite well.

Table 2. Results of parameter optimization for field data sets.

Data Set	Experimental Site Condition	K_r	r_c	K_i	Coefficient of Determination	Number of Observations
Meeker 1	Range	1.58	6.61	1.58	0.98	6
Meeker 2	Range	5.31	6.24	1.95	0.98	6
Cuba 1	Range	0.78	2.12	0.22	0.99	6
Cuba 2	Range	0.52	1.38	0.27	0.89	6
Woodward 1	Range	5.59	4.65	0.025	0.99	5
Woodward 2	Range	3.66	5.91	0.16	0.95	5
Chickasha 1	Range	0.12	1.69	0.30	0.99	5
Chickasha 2	Range	0.55	2.16	0.48	0.86	6
Cottonwood 1	Range	0.75	0.22	0.93	0.93	6
Cottonwood 2	Range	1.24	2.06	0.57	0.97	7
Stein et al.	Crop	4.86	1.11	0.72	0.96	4
Meyer et al.	Crop	1.36	4.50	1.58	0.73	9
Sharpsburg 1	Crop	2.63	3.07	NA	0.93	5
Sharpsburg 2	Crop	2.14	1.98	NA	0.93	5
Amarillo 1	Crop	29.47	2.53	NA	0.97	5
Amarillo 2	Crop	23.54	2.74	NA	0.93	5
Heiden 1	Crop	8.07	0.00	NA	0.98	5
Heiden 2	Crop	5.43	0.27	NA	0.81	5
Walla Walla 1	Crop	12.18	2.10	NA	0.98	5
Walla Walla 2	Crop	14.49	2.43	NA	0.98	5

Summary

A method was outlined for optimizing erodibility parameters with field erosion data for a steady state erosion model. The use of

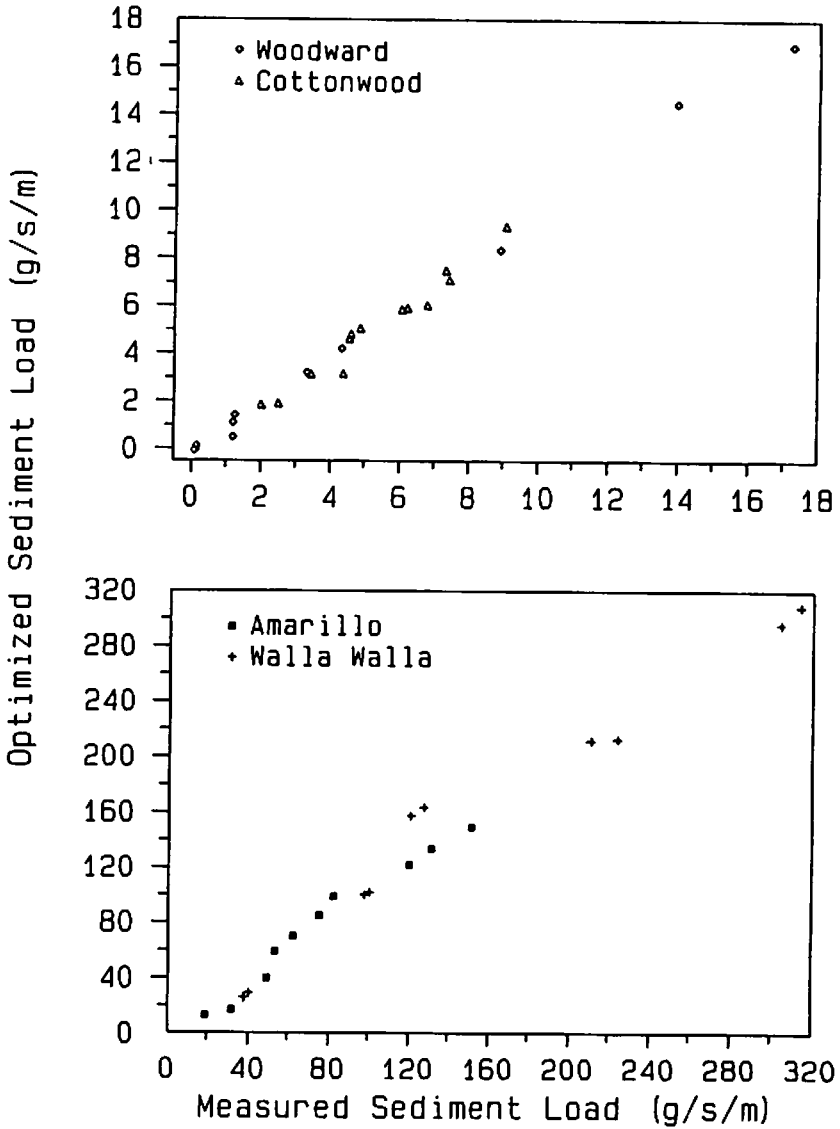


Figure 2. Measured and optimized sediment loads (g/s/m) for selected data sets. Values shown are for: a) 2 WEPP rangeland sites and b) 2 WEPP cropland sites.

optimization to obtain the erodibility parameters ensured that they were derived using the same assumptions and solution techniques used in the model so that the parameters are compatible with the model. The optimization technique provided a method for deriving three erodibility parameters simultaneously for experiments where preformed rills or channels were not appropriate.

The method presented here calculated erodibility parameter values which characterized the measured data quite well. This result has important implications. On many rangeland experiments, detachment by surface flow is not apparent in the form of incised rills, yet, the rill-interrill model appears to represent the data quite well.

References

- Blau, J.B., Woolhiser, D.A., and Lane, L.J., (1988). "Identification of erosion model parameters," Transactions, ASAE, 31, 839-845.
- Foster, G.R., (1982). "Modeling the erosion process," Ch. 8 In: Hydrologic Modeling of Small Watersheds, C. T. Haan, H. P. Johnson, and D. L. Brakensiek (editors), Monograph No. 5, ASAE, St. Joseph, MI.
- Foster, G.R., and Meyer, L.D., (1972). "A closed-form soil erosion equation for upland areas," In: Sedimentation: Symposium to Honor Professor H.A. Einstein, H.W. Shen (editor), Fort Collins, CO.
- Laflen, J.M., Thomas, A.W., and Welch, R., (1987). "Cropland experiments for the WEPP project," Paper No. 87-2544, 1987 Winter Meeting of ASAE, Chicago, IL.
- Lane, L.J., Shirley, E.D., and Singh, V.D., (1988). "Modeling erosion on hillslopes," In: Modelling Geomorphological Systems, M.G. Anderson (editor), John Wiley & Sons Ltd., Chichester, England.
- Meyer, L.D., Foster, G.R., and Nikolov, S., (1975). "Effect of flow rate and canopy on rill erosion," Transactions, ASAE, 18, 905-911.
- Simanton, J.R., Johnson, C.W., Nyhan, J.W., and Romney, E.M., (1986). "Rainfall simulation on rangeland erosion plots," In: Erosion on Rangelands: Emerging Technology and Data Base, L.J. Lane (editor) Proc. Rainfall Simulator Workshop, Tucson, AZ, Soc. for Range Mgt., Denver, CO.
- Stein, O.R., Neibling, W.H., Logan, Y.J., and Moldenhauer, W.G., (1986). "Runoff and soil loss as influenced by tillage and residue cover," Soil Sci. Soc. Am. J. (50), 1527-1531.