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PARAMETER TRANSFERABILITY FOR A DAILY RAINFALL DISAGGREGATION MODEL

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ABSTRACT

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The model for disaggregating daily rainfall into sequences of individual showers presented by Hershenhorn and Woolhiser is modified and applied to data from two midwestern stations in the U.S.A. Statistical tests indicate that number of showers, the start times, shower depths, and shower durations of simulated sequences compare favorably with observed sequences, and that the disaggregation model parameters identified at one gage provide a satisfactory fit for three test stations up to 470 km away, provided the test stations are in the same climatological region.

INTRODUCTION

Methods that use readily-available daily rainfall data to stochastically simulate short-time period rainfall have recently been developed. The simulated short-time period rainfall may then be used as input for time-varying infiltration models allowing them to be used at locations where only daily rainfall records are available. At least two methods of disaggregating daily rainfall into shorter time periods exist in the literature — the discrete–discrete (D–D) and the discrete–continuous (D–C), (Woolhiser and Goodrich, 1988). The D–D method disaggregates the discrete process of daily rainfall into the discrete process of a 60 min (or less) rainfall. The methods devised by Betson et al. (1980), and Srikanthan and McMahon (1985) are good examples of the D–D method. The D–C method as developed by Hershenhorn (1984) and Hershenhorn and Woolhiser (1987), disaggregates the discrete daily rainfall process into a continuous process of wet periods (showers) and dry periods within a day. The showers can then be disaggregated into a continuous intensity process (Woolhiser and Osborn, 1985). The D–C method appears to require fewer parameters than the D–D method. Hershenhorn and Woolhiser (1987) successfully developed a method of disaggregating daily summer (July and August) rainfall, from various locations in southeastern Arizona, into a continuous process of wet and dry periods within a day. If this technique is to be used on a large scale (i.e. nationwide) the technique must be flexible enough to handle regional variations in the daily rainfall process. Also, parameter values obtained at one location should provide some information about the daily rainfall process at nearby locations, i.e. the parameters should be transferable. In this paper we report on an investigation of the adaptability of Hershenhorn's techniques to the midwestern United States, and the transferability of the model parameters within the Midwest.

DATA SET

The breakpoint precipitation data (rainfall depth is tabulated for each time the intensity changes within a shower) were collected on experimental watersheds operated by the Agricultural Research Service and were obtained on magnetic tape from the USDA Hydrology Laboratory. Two of the stations, Hastings, Nebraska and McCredie (now Kingdom City), Missouri, were used to determine the feasibility of using the daily disaggregation model in the midwestern United States for the period May–August. Three additional locations (Fennimore, WI; Monticello, IL; Treynor, IA), were used to determine if the parameters obtained at Hastings, or McCredie could be used to disaggregate daily rainfall at other locations. Some precipitation statistics for May–August for the stations investigated are listed in Table 1. The annual precipitation cycle for this geographic region has a summer maximum and a winter minimum. The maximum monthly precipitation usually occurs during June, and over half the total annual precipitation occurs during the months of May–September. Approximately 80% of the summer precipitation has been classified as frontal in nature (Rudd, 1961). In contrast, most of the rainfall during the July and August season in southeastern Arizona is a result of local, convective, air mass type thunderstorms (Trewartha, 1981). Figure 1 illustrates the precipitation types in North America with regionalization based on the

TABLE 1

Selected precipitation statistics

Location	Period of record	Mean annual precipitation (mm)	Mean monthly precipitation (mm)			
			May	June	July	August
Hastings	1938–1967	574.5	90.9	125.2	73.2	65.8
McCredie	1941–1974	909.6	111.5	116.3	90.2	74.7
Fennimore	1939–1968	814.1	94.0	125.0	106.0	95.3
Monticello	1941–1974	819.2	85.1	97.3	101.0	88.1
Treynor	1964–1977	811.3	122.0	118.0	90.9	94.2

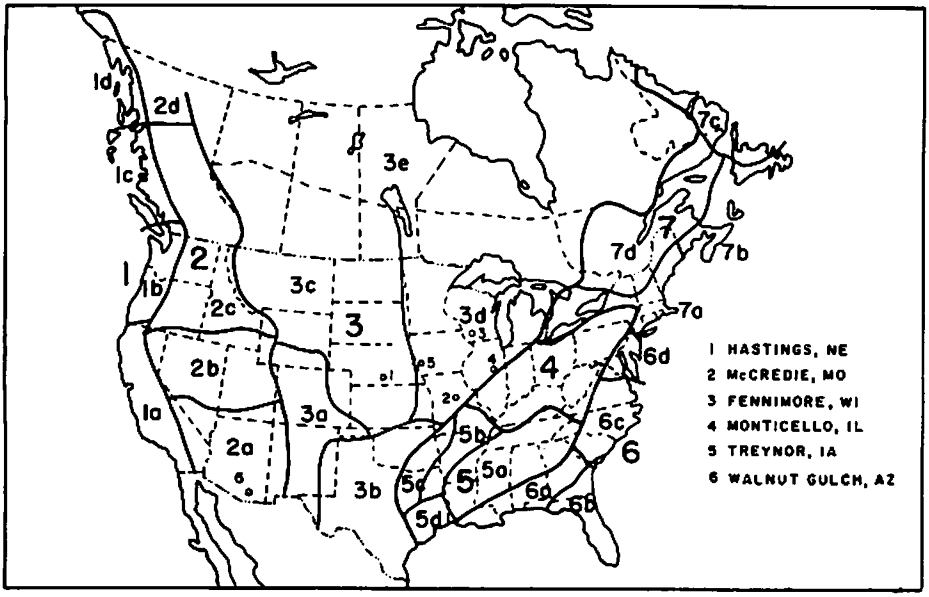

Fig. 1. Location of test and primary sites within precipitation regions of North America based on the annual march of precipitation (after Trewartha, 1981).

characteristics of the annual march of precipitation (Trewartha, 1981). All five stations fall within type 3; Hastings is in subtype 3c and the other stations are in subtype 3d. Precipitation at Hastings exhibits the typical monthly variation for subtype 3c. The crest of the precipitation profile is characteristically sharp, with June usually being the wettest month. It is common for the profile to halt its decline in August and September, so that a bench or a hint of a slight secondary September maximum may be evident. The monthly precipitation profile of McCredie is typical of a location in the subtype 3d. This subtype experiences more precipitation than 3c and occasionally has a double maximum in the summer. The first peak usually occurs in June (occasionally May or July) and the second peak occurs most commonly in September.

MATHEMATICAL MODEL

The mathematical model used in this study was described in detail by Hershenhorn (1984) and Hershenhorn and Woolhiser (1987), and will be briefly described here. Let $\xi(t)$ denote the continuous process of precipitation intensity at a point in space. $\xi(t)$ is non-negative and will take on positive values over random intervals of time. A possible realization of this function in an arbitrary period of time is illustrated in Fig. 2. Let a positive $\xi(t)$ pulse which is bounded on both sides by $\xi(t)$ of zero for ten or more minutes be defined as a shower. The starting time of the nth shower is T_n, the ending time is T'_n, and the duration is D_n. A complete shower is defined as one that occurs within one day, and a

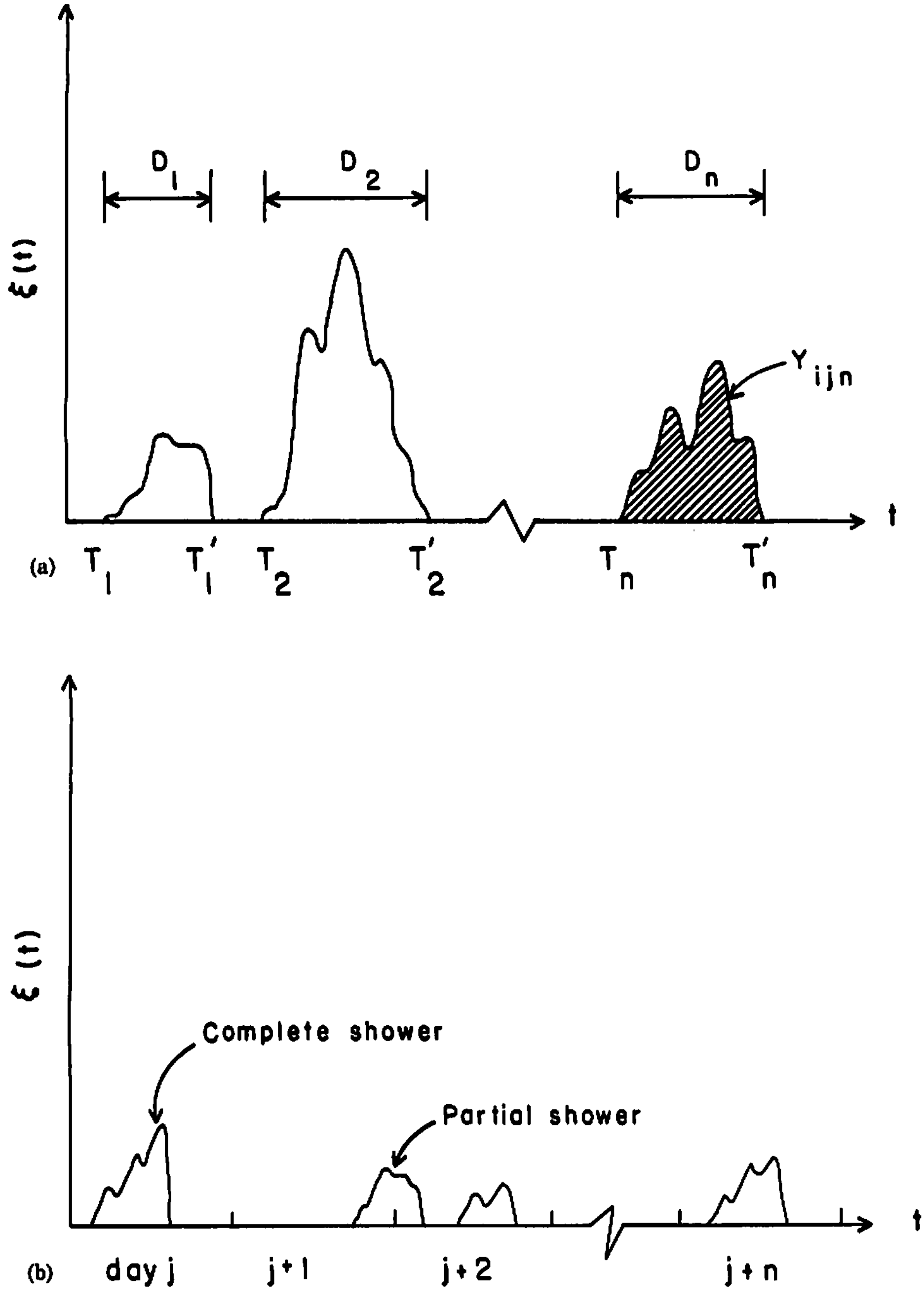

Fig. 2. (a) Possible sample function of the rainfall intensity process (from Hershenhorn, 1984). (b) Definition sketch of complete and partial showers.

partial shower is one which begins on one day and ends the following day. Let Z_{ij} represent the total rainfall on day j of year i and let N_{ij} represent the number of showers on that day. Then:

$$N_{ij} = N_{cij} + N_{pij} \tag{1}$$

where: N_{cij} = the number of complete showers ($N_{cij} = 0, 1, 2, \ldots$), N_{pij} = the number of partial showers ($N_{pij} = 0, 1, 2$).

Let T_{ijk} represent time in hours from midnight to the beginning of the kth shower and let Y_{ijk} represent the depth of the kth shower (partial or complete). Thus, a shower which occurs over a two-day period will be separated into two partial showers with depths Y_{ijN} and $Y_{i,j+1,1}$.

By definition:

$$Z_{ij} = \sum_{k=1}^{N_{ij}} Y_{ijk} \tag{2}$$

Thus each day with an observable amount of precipitation is associated with random variables which describe the number of complete and partial showers (N_{ij}), shower depth (Y_{ij}) and duration (D_{ij}), and shower starting time (T_{ij}).

The rainfall process is assumed to be stationary on an annual basis but may have seasonal cycles.

DATA ANALYSIS

The Hastings and McCredie data were analyzed to determine if the model developed by Hershenhorn (1984) for southeastern Arizona could be used for the two time periods, May–June and July–August at Midwestern locations. The analysis for the disaggregation of daily rainfall into individual showers requires the following steps:

(1) Determine what special procedures are required for the partial showers (those showers which continue past midnight).

(2) Develop the appropriate form for the joint distribution of the number of showers per day and the daily rainfall depth. Estimate the values for the required parameters.

(3) Devise a procedure to disaggregate the daily rainfall amount Z_{ij} into N_{ij} showers (of depth Y_{ijk} and duration D_{ijk}), and estimate the values for the required parameters.

(4) Determine the form of the distribution function for the shower starting time, T_{ijk}, and estimate the values for the required parameters.

(5) Determine the form of the joint distribution of shower amount Y_{ijk} and duration D_{ijk}, for the complete and partial showers, separately. Estimate the values for the required parameters.

In the following sections we will omit the subscripts i and j.

Comparisons between partial and complete showers

A shower is defined as any period in which the rainfall is $\geqslant 0.01$ in. (0.254 mm) and includes no periods of zero intensity exceeding 10 min in duration. Partial

TABLE 2

Mean, standard deviation, and coefficient of skew for the duration of complete and partial showers

Location	Period	Shower type	Mean (min)	S.D. (min)	Coeff. of skew
Hastings	May–June	Complete	157.2	177.4	2.415
		Partial	175.4	203.2	2.568
	Jul.-Aug.	Complete	137.5	137.5	1.725
		Partial	137.9	156.5	1.713
McCredie	May–June	Complete	93.8	112.0	2.659
		Partial	186.3	218.6	2.336
	Jul.-Aug.	Complete	91.1	100.4	2.113
		Partial	150.8	148.7	1.221

TABLE 3

Mean, standard deviation, and coefficient of skew for the depths of complete and partial showers

Location	Period	Shower type	No. of Obs.	Mean (mm)	S.D. (mm)	Coeff. of skew
Hastings	May-June	Complete	540	8.43	11.02	2.68
		Partial	129	10.90	17.48	3.94
	Jul.-Aug.	Complete	432	8.08	10.46	3.08
		Partial	72	8.86	14.86	3.12
McCredie	May–June	Complete	1107	5.56	7.16	2.65
		Partial	129	8.20	10.82	2.19
	Jul.-Aug.	Complete	770	6.35	8.71	3.27
		Partial	69	7.80	9.91	1.61

showers usually begin late at night. If a location experiences an abundance of showers occurring near midnight, that location will also have a high frequency of partial showers. The mean, standard deviation, and coefficient of skew of the shower duration and depth for the complete and partial showers at Hastings and McCredie are listed in Tables 2 and 3. The partial showers have somewhat larger depths and considerably longer durations than the complete showers. However, it is possible that the differences in the mean and standard deviation in depths can be attributed to sample variation. Accordingly, we wish to test the hypothesis that the samples of partial shower depths and complete shower depths came from the same population. We found that the mixed exponential distribution provided a better fit than the exponential, gamma or Weibull distributions. For each station and for each season the mixed exponential parameters were estimated for the depths of complete and partial showers individually (6 parameters) and collectively (3 parameters). The likelihood ratio test (Hoel, 1971) was used to test the null hypothesis (Ho) that the distributions of depths of partial and complete showers may be described by one parameter set against the alternative hypothesis that two parameter sets are required. The likelihood ratio test indicated that the null hypothesis could not be rejected at the 0.01 level except for May–June of McCredie. Because the partial showers represent only 10% of the total showers for this period it appears that we will not introduce serious error if we assume that the depths of complete and partial showers have the same distribution. Note that throughout this analysis we have used midnight as the time of observation (TOB) for daily rainfall. Obviously the number and characteristics of partial showers may change if the TOB is changed.

Distribution of number of showers and daily rainfall depth

The joint distribution of the number of showers and the daily rainfall depth can be written as a product of the conditional distribution of number of showers given daily depth and the marginal distribution of daily depth:

$$H_{N,Z'}(n, z) = G_{N|Z'}(n|z)F_{Z'}(z) \tag{3}$$

where: N = the number of showers = $N_p + N_c$ and Z' = the daily precipitation depth minus a threshold.

Because the lower limit of observation is 0.01 in. (0.254 mm), the threshold was set to 0.009 in. (0.229 mm) to include all observations. Goodness of fit statistics were calculated for the exponential, gamma, mixed exponential and Weibull distributions for the marginal distribution of Z' for both time periods for each station.

The Mixed Exponential Distribution:

$$f_{z'}(z) = \frac{\alpha}{\beta}\exp\left(\frac{-z}{\beta}\right) + \frac{(1-\alpha)}{\theta}\exp\left(\frac{-z}{\theta}\right) \tag{4}$$

provided the best fit for three out of the four data sets. The one-sample Kolmogorov–Smirnov (KS) and the χ^2 goodness of fit tests were used to test the null hypothesis, (Ho), that each of the data sets were samples from mixed exponential distributions. According to the KS test at the 5% significance level, Ho could not be rejected for any of the data sets. According to the χ^2 test at the same significance level, Ho was accepted for both time periods at McCredie and rejected for both time periods at Hastings. It should be noted that with large sample sizes and with artifacts introduced by processing analog rainfall charts, it is very difficult to obtain fits that pass the χ^2 test. Based on these results we used the mixed exponential as the marginal distribution of daily rainfall, anticipating possible problems with χ^2 testing of the joint distribution of number of showers and daily rainfall depth for Hastings.

Hershenhorn (1984) found that the shifted negative binomial (SNB) distribution provided a good fit for the conditional distribution of the number of showers given daily depth. The probability mass function of the SNB can be written as:

$$P(N = n) = \binom{n+r-2}{n-1} p^r(1-p)^{n-1}; \qquad n = 1, 2, \ldots$$
$$r \geq 0; \qquad 0 < p \leq 1 \tag{5}$$

Hershenhorn (1984) allowed p and r to vary as functions of the daily rainfall depth. The southeastern Arizona (Walnut Gulch) and the Hastings data indicated that the expected number of showers given daily depth asymptotically approached a limiting value (Fig. 3). However, as also illustrated in Fig. 3, the McCredie data suggested that the expected number of showers asymptotically approached a straight line with a positive slope. Modifications were made in the functional forms of p and r used by Hershenhorn (1984), to allow for the expected number of showers given daily depth to approach a straight line with a positive slope or a zero slope. It is possible that with very large daily rainfall depths the expected number of showers might even decrease. However, the data for these locations did not support such a functional form. The modified functional forms for p and r are:

$$p = \exp(-A_1 Z') \tag{6}$$

$$r = (E\{N|Z'\} - 1.0)p/(1.0 - p) \tag{7}$$

$$E\{N|Z'\} = A_2 + A_3 Z' + (1.0 - A_2)\exp(-A_4 Z') \tag{8}$$

where: $E\{N|Z'\}$ is the expected number of showers given a daily precipitation

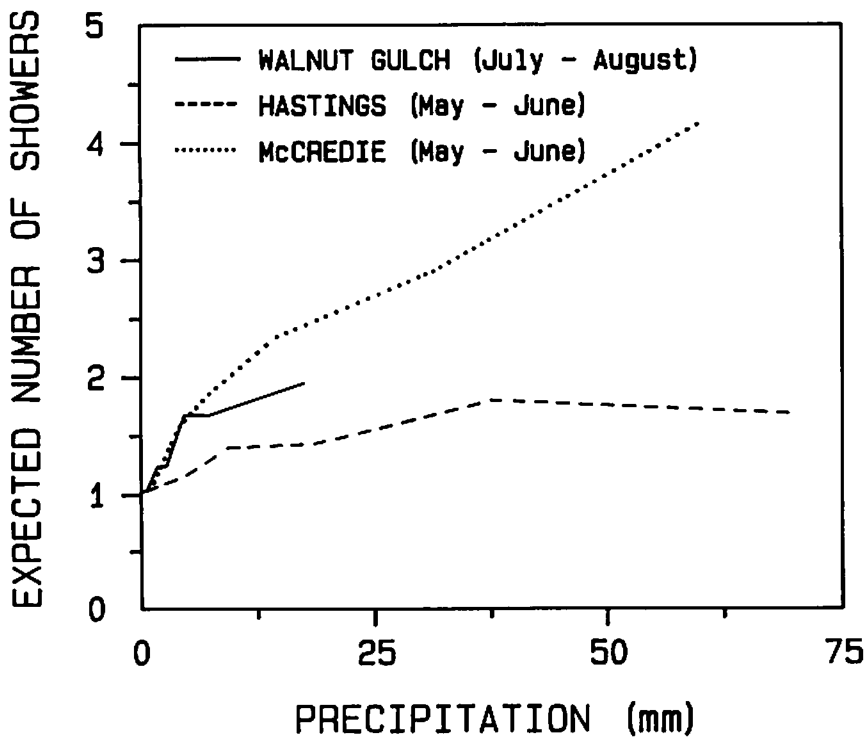

Fig. 3. Expected number of showers vs. daily rainfall depth.

TABLE 4

Parameter values for the shifted negative binomial distribution[a]

Station	Period	Parameters			
		A_1 (mm^{-1})	A_2 (mm^{-1})	A_3 (mm^{-1})	A_4 (mm^{-1})
Hastings	May–June	0.01057	1.387	0.00288	0.0492
	Jul.–Aug.	0.01417	1.623	0.00970	0.0459
McCredie	May–June	0.04472	4.162	0.0140	0.0620
	Jul.–Aug.	0.03201	4.677	0.00948	0.0590

[a] See text for functional form.

amount, and Z' is daily precipitation depth minus a threshold of 0.009 in. (0.229 mm). A_1, A_2, A_3 and A_4 are parameters.

Numerical maximum likelihood techniques were used to obtain the parameter values for the midwestern data shown in Table 4.

The bivariate distribution function may be written by utilizing the expressions for the mixed exponential and shifted negative binomial distributions and eqn. (3):

$$H_{N,Z'}(n, z) = \sum_{n=1}^{N} \int_0^z \left[\binom{n + r - 2}{n - 1} p^r (1 - p)^{n-1}\right]$$

$$\left[\frac{\alpha}{\beta} \exp \frac{-s}{\beta} + \frac{(1 - \alpha)}{\theta} \exp \frac{-s}{\theta}\right] ds \tag{9}$$

where: N is the number of events on a day with rainfall $Z' + 0.009$ in. (0.229 mm) and s is a dummy variable of integration.

A χ^2 goodness of fit test was used to test the hypothesis (Ho) that the sample data were taken from the identified bivariate distribution. At the 0.01 level, Ho could not be rejected for both time periods at McCredie; however, Ho was rejected for both time periods at Hastings. We conclude that the SNB with functional forms for the parameters r and p given by eqns. (6)–(8) is acceptable for the McCredie data. Because the mixed exponential was rejected as the marginal distribution of Z' for Hastings we cannot draw definite conclusions regarding the SNB and the functional forms for r and p for Hastings.

Individual shower depths

The daily depth Z must be distributed among N showers and the sum of the individual shower depths which occur within a day must equal the daily rainfall depth. The ratio technique developed for southwestern data (Hershenhorn, 1984; Hershenhorn and Woolhiser, 1987), also worked well with the Midwestern data. The ratios are defined in the following expressions:

(a) *Two showers per day*:

$$R1 = \text{(depth of first shower)/(daily total)} = Y_1/Z \tag{10}$$

(b) *Three showers per day*:

$$R2 = [Y_2 + Y_3]/Z \tag{11}$$

$$R3 = Y_2/[Y_2 + Y_3] \tag{12}$$

(c) *Four showers per day*:

$$R4 = [Y_3 + Y_4]/Z \tag{13}$$

$$R5 = Y_1/[Y_1 + Y_2] \tag{14}$$

$$R6 = Y_3/[Y_3 + Y_4] \tag{15}$$

Following Hershenhorn and Woolhiser (1987), we used the beta-Fourier distribution for the ratios $R1$ and $R2$ and the uniform distribution for $R3$–$R6$. The beta-Fourier consists of a beta distribution with a superimposed sine term:

$$f_R(r) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} r^{(\alpha-1)}(1 - r)^{(\beta-1)} + \theta \sin(2\pi r) \tag{16}$$

where $0.0 < r < 1.0$; $\alpha, \beta > 0$; and θ is constrained such that:

$$|\theta \sin 2\pi r| < \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} r^{(\alpha-1)}(1 - r)^{(\beta-1)} \tag{17}$$

Procedures described by Hershenhorn and Woolhiser (1987) were used to disaggregate daily totals for days with five and six showers. Although 1% of the wet days at McCredie had more than six showers, we used six as a practical maximum. The parameter values obtained for the beta-Fourier distribution for each time period at the two midwestern stations are listed in Table 5.

Starting times for complete showers

The starting time of a shower may be important because of its effect on antecedent moisture conditions. Only the starting times for the complete

TABLE 5

Parameter values for the beta-Fourier distribution for ratios R_1 and R_2[a]

Station	Period	Parameter		
		α	β	θ
Hastings	May–June	1.192	1.054	0.3003
	Jul.–Aug.	1.390	0.923	0.5228
McCredie	May–June	1.419	1.019	0.2959
	Jul.–Aug.	1.224	0.795	0.0490

[a] See text for functional form.

showers need to be generated because a partial shower starts either at midnight or at midnight minus the duration of the partial shower. Hershenhorn (1984) used the mixed beta distribution to describe normalized starting times for complete showers. The times were normalized by dividing the starting times, in hours from midnight, by 24.

The mixed beta distribution has several undesirable features. The distribution is not necessarily periodic; however, the process that we wish to describe is so. The distribution does not appear to be analytically integrable, and if numerical integration is used difficulties may arise at the bounds where the distribution may take on infinite values. To avoid the deficiencies of the mixed beta distribution, it was replaced by a Fourier density function with a mean of one and up to two harmonics. The Fourier is periodic and easily integrated. The Fourier distribution has the following form:

$$g_T(t) = 1.0 + \alpha_1 \cos(2\pi t + \gamma_1) + \alpha_2 \cos(4\pi t + \gamma_2) \quad (18)$$

$0 \leqslant t \leqslant 1$; and $\alpha_1 \cos(2\pi t + \gamma_1) + \alpha_2 \cos(4\pi t + \gamma_2) \leqslant 1.0$

Parameter values for the Fourier distribution were obtained by numerical maximum likelihood techniques and are listed in Table 6. The one-sample KS test was used to determine whether the fitted distributions individually described the subsets of the one-per-day showers, two-per-day showers, and up to six or more showers per day. Therefore, six subsets were tested for each period at each station. At the 0.01 level, the null hypothesis could not be rejected for any of the six subsets.

Although it is clear that the starting times of multiple showers within a day cannot be strictly independent because showers have finite durations and cannot overlap, we found that the approximation of independence used by Hershenhorn and Woolhiser (1987) gave quite acceptable results for both Hastings and McCredie. Therefore, if N showers are simulated for a day, N starting times are sampled independently from the Fourier distribution and are ordered from earliest to latest. The starting times for any simulated overlapping showers are adjusted to maintain a minimum of 10 min between showers.

TABLE 6

Parameter values for the Fourier distribution[a] of shower starting times

Station	Period	Parameter			
		α_1	γ_1	α_2	γ_2
Hastings	May–June	0.4986	−0.3784	−0.1180	0.1227
	Jul.–Aug.	0.6511	−0.3446	−0.1924	1.223
McCredie	May–June	0.0454	−0.3188	*	*
	Jul.–Aug.	−0.2551	1.563	−0.1236	0.9312

[a] $f(t) = 1.0 + \alpha_1 \cos(2\pi t + \gamma_1) + \alpha_2 \cos(4\pi t + \gamma_2)$.
* Not significant according to AIC (Akaike, 1974).

It has been demonstrated (Wallace, 1974) that in many parts of the United States, heavy precipitation displays a pronounced diurnal variation. Therefore, the distribution of time of occurrence should be conditioned on the shower amount. For days with one shower the CDF of the conditional distribution of time of occurrence given depth may be written as:

$$F_{T|Y}(t|y) = \int_0^t g_T(s) f_{Y|T}(y|s)/g_Y(y)\,\mathrm{d}s \tag{19}$$

Because $g_Y(y)$ is constant with respect to time, eqn. (19) may be rearranged to:

$$F_{T|Y}(t|y) g_Y(y) = \int_0^t g_T(s) f_{Y|T}(y|s)\,\mathrm{d}s \tag{20}$$

$g_Y(y)$ may be calculated by:

$$g_Y(y) = \int_0^{24} g_T(s) f_{Y|T}(y/s)\,\mathrm{d}s \tag{21}$$

The expression for $g_T(s)$ is the Fourier distribution (eqn. (18)) and the distribution for the conditional distribution of depth given time is the exponential-Fourier:

$$f_{Y|T}(y|s) = \lambda(s)^{-1} \exp[-y/\lambda(s)] \tag{22}$$

where:

$$\lambda(t) = \tau + \alpha_1 \cos[2\pi t + \gamma_1) + \alpha_2 \cos(4\pi t + \gamma_2) \tag{23}$$

According to eqn. (22), shower depths are distributed exponentially and the mean shower depth $\lambda(t)$ fluctuates diurnally according to eqn. (23).

Joint distribution of shower duration and depth

The joint distribution of shower duration and depth may be written as:

$$h_{Y',D}(y, d) = g_{D|Y'}(d|y) f_{Y'}(y) \tag{24}$$

where d is the shower duration and y is the shower depth minus a threshold amount. The shower depth is obtained from the shower ratio technique. Hershenhorn (1984) and Hershenhorn and Woolhiser (1987) used regression to predict expected log duration from log depth, and used the normal distribution with a mean equal to zero, and the standard deviation equal to the standard error of estimate, as the predictor of spread about the regression line.

Preliminary analysis indicated, for the midwestern stations, that a threshold amount of 0.099 in. (2.514 mm) subtracted from the complete shower depths facilitated fitting the regression line to the transformed data. The best linear relationship for complete showers occurred when the duration was transformed to its natural log and the depth was untransformed. The intercept, slope

TABLE 7

Regression statistics for ln (duration (min)) vs. depth (mm) relationshp for complete and partial showers[a]

Station	Period	Complete showers			Partial showers		
		Y-Inter.	Slope	SEE	Y-Inter.	Slope	SEE
Hastings	May–June	0.3656	0.0244	1.052	0.1774	0.0261	1.230
	Jul.–Aug.	0.4098	0.0124	1.056	−0.1734	0.0366	1.159
McCredie	May–June	0.04183	0.0319	0.9468	0.3087	0.0296	1.081
	Jul.–Aug.	−0.1183	0.0364	0.9643	0.1841	0.0183	1.228

[a]Threshold for the complete showers is 0.099 in. (2.514 mm), and the threshold for the partial showers is 0.009 in. (0.229 mm).

and standard error of estimate for the regression lines, are listed in Table 7. A correlation ratio test (Kendall and Stuart, 1979) showed no significant deviation from linearity at the 0.05 level for all four data sets. The residuals about the regression were examined for normality. Visual inspection revealed that the residuals appeared to decrease in magnitude with increased depth. To determine whether the standard deviation of the residuals was constant and whether the residuals were normally distributed, the χ^2 goodness of fit test was performed on four subsets of each data set (16 subsets in total). The subsets from each set had approximately the same number of observations. The hypothesis that the residuals came from a normal distribution with a mean equal to zero and the standard deviation equal to the standard error of estimate was rejected, at the 0.01 level, for four of the subsets. When the residuals from the four data sets, without breaking up into subsets, were tested for normality, Ho was rejected at the 0.01 level for only the McCredie, July and August data.

The log duration vs. depth relationship for partial showers was also tested for linearity and normality of the residuals. The threshold subtracted from the shower depth was at 0.009 in. (0.229 mm). The four data sets transformed in this manner, passed all the significance testing at the 0.05 level. The intercept, slope and standard error of estimate of the regressions fitted to the partial showers are listed in Table 7.

SIMULATION

A simplified flow chart of the daily disaggregation simulation model is shown in Fig. 4. The general approach is similar to that of Hershenhorn and Woolhiser (1987). The method examines rainfall amounts on days $j-1, j$, and $j+1$. If two successive days are wet a partial shower can be simulated with probability equal to the ratio of number of partial showers to the total number of wet–wet transitions observed in the record. Replacing the mixed beta distribution by the Fourier distribution should decrease simulation time and increase model accuracy. The functional relationship between depth and

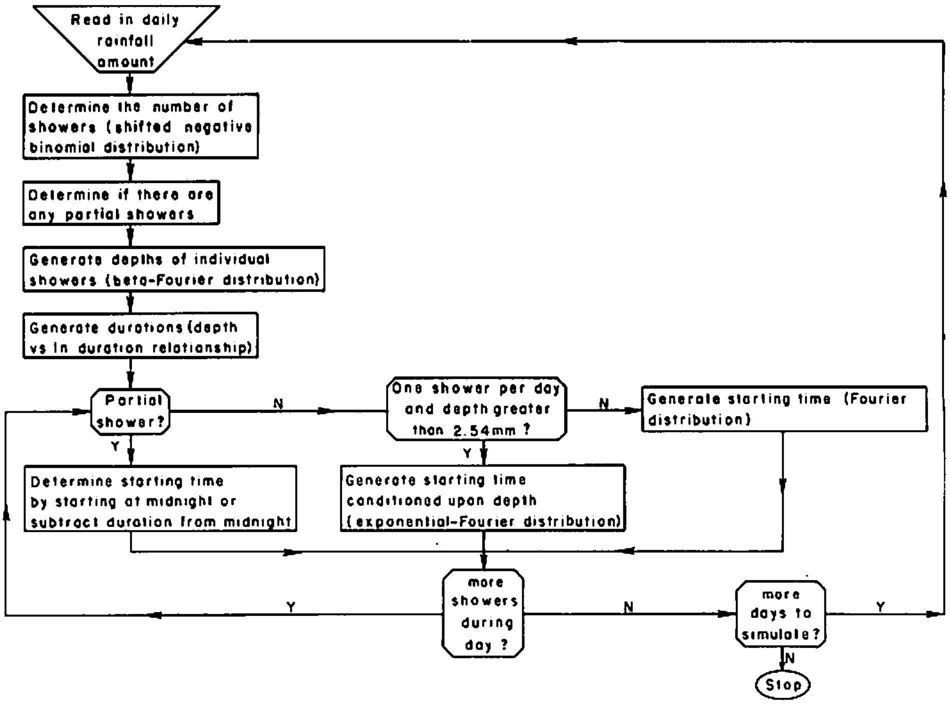

Fig. 4. Simplified flowchart of disaggregation model.

duration was changed from log duration vs. log depth to log duration vs. depth. The previous model simulated the durations of all showers; now the model does not simulate the durations of complete showers with less than 0.10 in. (2.54 mm) of rainfall. The durations for these showers were set equal to the mean duration of the showers from this class. This modification should increase simulation speed and should not adversely affect model applicability. Hershenhorn's (1984) model did not allow the duration of a shower to exceed 8 h. The modified model allows showers to be up to 24 h long. However, if the sum of the durations of two or more showers exceeds 24 h, the durations are reduced in proportion to their original duration so as to retain a minimum of 10 min between showers.

The modified model was used to disaggregate daily rainfall at Hastings and McCredie for the May–June and the July–August periods. A two-sample KS test was used to test the null hypothesis (Ho) that the simulated distributions of shower starting times, depths and durations came from the same parent distribution as the corresponding historical distributions. The χ^2 test was used to test Ho for the comparisons between the distributions of number of showers per day. In all cases the null hypothesis could not be rejected at the 0.01 level (Table 8). Although the distribution of the time interval between showers is not explicitly accounted for in the disaggregation model, a comparison of historical and simulated time intervals showed a remarkably good agreement of means, standard deviations and skew coefficients (Econopouly, 1987).

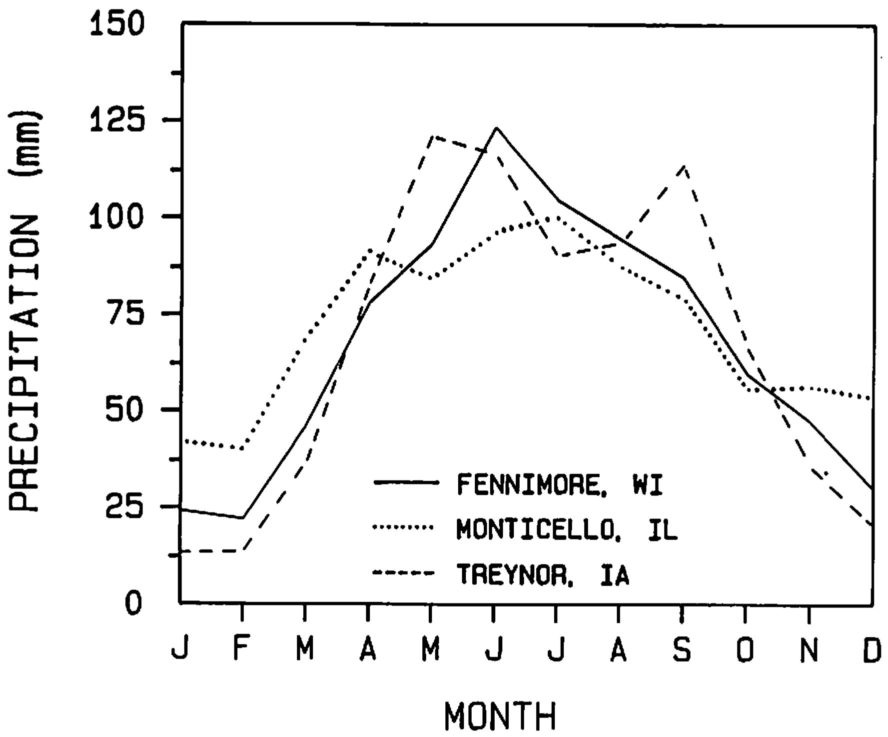

Fig. 5. Mean monthly precipitation at the test stations.

butions of starting times at Monticello for the July–August season. The historical distributions from Hastings were mostly different from those at the test stations. The historical distributions of shower starting times from Treynor and Hastings, for both time periods, could be considered to be from the same parent distribution at the 0.05 level. Also, at the same significance level, the distribution of shower depths from Monticello, July–August, compared favorably with showers from Hastings.

TABLE 9

Distances from test stations to primary stations

Station	Distance (km) to:	
	Hastings	McCredie
Fennimore	676	470
Monticello	772	322
Treynor	260	445

TABLE 8

Results of statistical tests between the distributions of historical and simulated showers at Hastings and McCredie

Station	Time period	Level which Ho[a] is not rejected for distributions of:			
		Starting times[b]	Depths[b]	Durations[b]	No. of showers per day[c]
Hastings	May–June	0.05	0.05	0.05	0.01
	Jul.–Aug.	0.05	0.05	0.05	0.05
McCredie	May–June	0.05	0.01	0.05	0.01
	Jul.–Aug.	0.05	0.01	0.01	0.05

[a] Ho is the hypothesis that the simulated and historical distributions come from the same parent distribution.
[b] Two-sample KS test was used.
[c] χ^2 test was used.

TRANSFERABILITY OF MODEL PARAMETERS

To be useful, the parameter values for the disaggregation model should be applicable over climatologically similar areas. In order to determine whether the parameters developed from the Hastings and McCredie stations could be used in other locations in the Midwest, daily rainfall data from Fennimore, Treynor and Monticello were disaggregated into showers using the Hastings and McCredie parameter sets. These locations will be referred to as the test stations, whereas McCredie and Hastings will be referred to as the primary stations.

The monthly profiles of precipitation for the test stations are illustrated in Fig. 5. The monthly precipitation patterns at the test stations resemble those at McCredie more than those at Hastings. The distances from each test station to the primary stations are listed in Table 9.

A comparison of historical data at the primary and test stations shows that stations in the same climatic zone (Fig. 1) have very similar characteristics. Hastings has fewer showers with larger depths and longer durations than the test stations. The shower statistics at McCredie are fairly similar to those at the test stations. The historical cumulative distribution functions (CDFs) of duration, depth and starting time of the showers from the test stations were compared to the corresponding historical CDFs from each of the primary stations. The two-sample KS statistic was used to test the hypothesis (Ho) that the historical CDFs came from the same parent distribution. The χ^2 statistic was used to test Ho for the comparison between the number of showers per day. When the comparisons were made with McCredie data, Ho was rejected at the 0.01 level only for the comparison between the historical and simulated distri-

Comparison of simulations

To determine whether the parameter sets developed for the primary stations were transferable, they were used to disaggregate daily rainfall data collected at the test stations. The two-sample KS statistic was used to test the null hypothesis (Ho) that the simulated and historical samples were from the same population. Test results are shown in Tables 10 and 11. Most of the comparisons between the simulated distributions obtained using parameters from Hastings and the historical distributions from the test stations did not pass significance testing, at the 0.01 level. The distributions for which Ho was not rejected were between the distribution of starting times at Treynor, May–June, and the distributions of duration for Treynor, July–August, and Monticello, both time periods. Recall that for most of the comparisons between the historical distributions at Hastings and the test stations Ho were also rejected. Most of the simulated distributions obtained using parameters from McCredie and the historical distributions from the test stations were not significantly different, at the 0.05 level. Ho was rejected at the 0.01 level for the distributions of shower depths from Fennimore for both time periods and the distribution of shower durations from Fennimore for July–August. Ho was also rejected for the distribution of shower starting times from Monticello, July–August. Recall that the historical and simulated distributions of shower starting times from Monticello and McCredie were also significantly different, at the 0.05 level. The greatest difference between the simulated mean and historical mean shower depths at Fennimore was only 0.021 in. (0.533 mm) and when showers

TABLE 10

Results of statistical tests between the simulated distributions using Hastings' parameters and the historical distributions at the test stations

Station	Time period	Level which Ho[a] is not rejected for distributions of:			
		Starting times[b]	Depths[b]	Durations[b]	No. of showers per day[c]
Fennimore	May-June	*	*	*	*
	Jul.-Aug.	*	*	*	*
Monticello	May-June	*	*	0.05	*
	Jul.-Aug.	*	*	0.05	*
Treynor	May-June	0.05	*	*	*
	Jul.-Aug.	*	*	0.05	*

[a] Ho is the hypothesis that the simulated and historical distributions come from the same parent distribution.
[b] Two-sample KS test was used.
[c] χ^2 test was used.
* Not accepted at the 0.01 significance level.

TABLE 11

Results of statistical tests between the simulated distributions using McCredie's parameters and the historical distributions at the test stations

Station	Time period	Level which Ho[a] is not rejected for distributions of:			
		Starting times[b]	Depths[b]	Durations[b]	No. of showers per day[c]
Fennimore	May–June	0.05	*	0.05	0.05
	Jul.–Aug.	0.05	*	*	0.05
Monticello	May–June	0.05	0.05	0.05	0.05
	Jul.–Aug.	*	0.05	0.05	0.05
Treynor	May–June	0.05	0.05	0.05	0.05
	Jul.–Aug.	0.05	0.05	0.05	0.05

[a] Ho is the hypothesis that the simulated and historical distributions come from the same parent distribution.
[b] Two-sample KS test was used.
[c] χ^2 test was used.
* Not accepted at the 0.01 significance level.

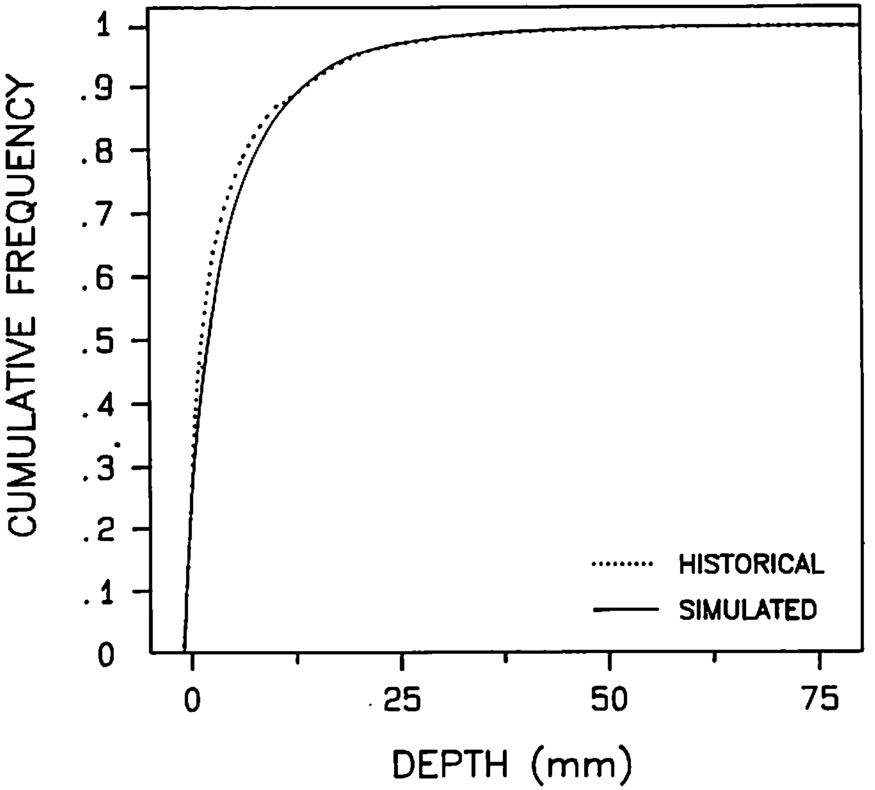

Fig. 6a. Observed and simulated distributions of shower depths, Fennimore, WI, May–June.

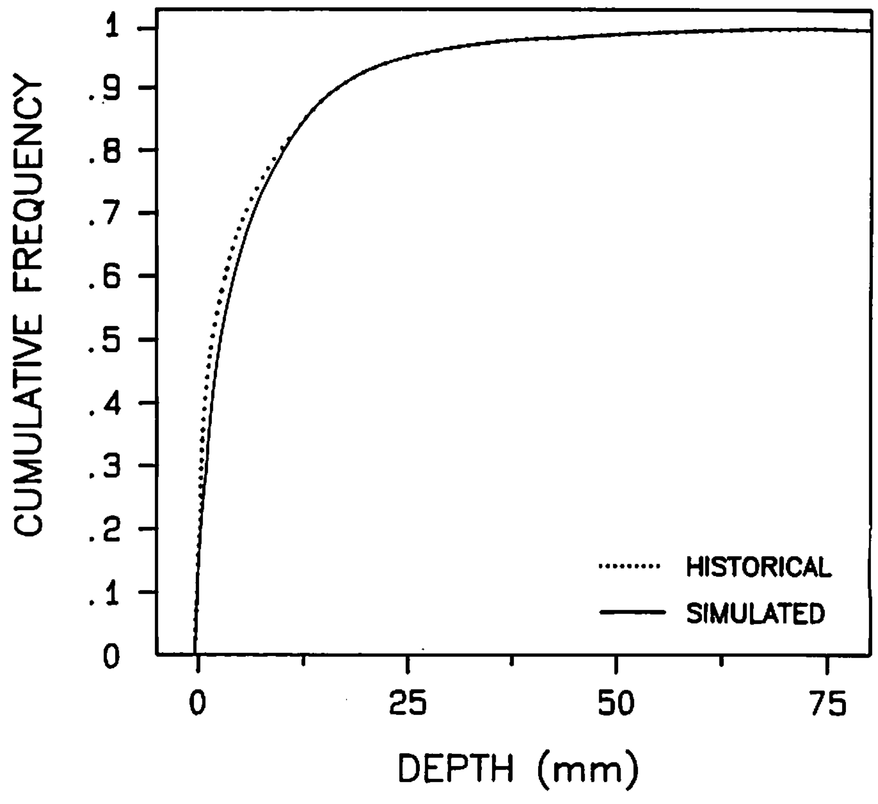

Fig. 6b. Observed and simulated distributions of shower depths, Fennimore, WI, July–August.

over 0.10 in. (2.54 mm) were compared, Ho could not be rejected at the 0.05 level. Observed and simulated distributions of shower depths for Fennimore, for the two periods are shown in Fig. 6. The curves virtually coincide at depths greater than 0.5 in. (12.7 mm), suggesting that the simulated values would be acceptable for most hydrologic applications.

DISCUSSION AND CONCLUSIONS

The technique for disaggregation of daily rainfall into the intermittent shower process reported by Hershenhorn (1984) and Hershenhorn and Woolhiser (1987) for southeastern Arizona required some modification for application in the midwestern United States. Specifically, the functional relationship between the expected number of showers per day given daily rainfall, was written in a more general form, and it was found that the best linear correlation between shower duration and depth was achieved with a log transformation of duration and no transformation for depth. The disaggregation procedure was also improved by using a Fourier distribution rather than the mixed beta distribution for shower starting time and by conditioning the distribution of the time of occurrence of single storms upon the storm (or daily rainfall) depth.

Analysis of rainfall data from Hastings, NE and McCredie, MO revealed that the same stochastic structure of the disaggregation process could be used but that the parameters of most distributions were different. However, parameters estimated at McCredie could be used to disaggregate daily rainfall at Treynor, Monticello and Fennimore, a distance of up to 470 km. Because Hastings and McCredie are in different climate zones it is possible that boundaries of precipitation regions based upon the annual march of precipitation (Trewartha, 1981) or other considerations can be used to determine the transferability of disaggregation model parameters. For use in obtaining input for infiltration models in areas where only daily rainfall is available, the model has been generalized to allow operation in the Midwest as well as in southeastern Arizona. Transferability of the disaggregation parameters over fairly large distances within climatologically homogeneous regions was also shown to be feasible in the Midwest as well as in southeastern Arizona. Further analysis will be required to establish limits on the applicability of this disaggregation model and the local transferability of its parameters.

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