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MAPPING THE AREAL PRECIPITATION OVER ARIZONA -  
USING KRIGING TECHNIQUE

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**Abstract**

The classical methods for interpolating and spatial averaging of precipitation fields fail to quantify the accuracy of the estimate. On the other hand, kriging is an interpolation method for predicting values of regionalized variables at points (punctual kriging) or average values over an area (block kriging).

This paper demonstrates the use of the kriging method for mapping and evaluating precipitation data for the state of Arizona. Using 158 rain gage stations with 30 years or more of record, the precipitation over the state has been modeled as a realization of a two dimensional random field taking into consideration the spatial variability conditions.

Three data sets have been used: (1) the mean annual precipitation over the state; (2) the mean summer rainy season; and (3) the mean winter rainy season. Validation of the empirical semi-variogram for a constant drift case indicated that the exponential model was appropriate for all the data sets. In addition to a global kriging analysis, the data have been examined under an anisotropic assumption which reflects the topographic structure of the state.

**Introduction**

Several interpolation techniques such as arithmetic mean, linear interpolation or the nearest neighbor (Thiessen Weight) method, have been widely used for areal mapping of precipitation fields (Hall and Barclay, 1975). Other techniques have been reviewed by Creutin and Obled (1982). A relatively new technique is presented and evaluated in this paper referred to as kriging (after D. G. Krige). This technique was originally developed for geoscience applications. It has been

applied recently in a few cases to the mapping of precipitation fields (Delfiner and Delhomme, 1975; Montmollin et al., 1980; Chua and Bras, 1982; Bastin et al., 1984; Obley and Creutin, 1986).

Matheron (1971) coined the term "regionalized variable" to describe variables which can be characterized from a certain number of measurements which identify spatial structure. The optimal estimator (in the current case for the average areal precipitation) is a linear minimum variance unbiased estimator which requires knowledge of the variogram of the random variable (precipitation) as a function of space. Therefore a theoretical variogram model must be chosen and its parameters have to be estimated prior to the interpolation.

The kriging technique, which was adapted from various resources (Delfiner and Delhomme, 1975; Journel and Huijbregts, 1978; Delhomme, 1978), is briefly introduced below.

### Theoretical Background

Let  $x_1, x_2, \dots, x_n$  be the sample locations with given precipitation values of  $Z(x_1), Z(x_2), \dots, Z(x_n)$  and  $x_0$  is the unsampled location. Then the value of precipitation in the unsampled location,  $Z(x_0)$ , is estimated as a linear weighted combination of  $n$  known surrounding data, depending on distance from the unsampled location:

$$Z^*(x_0) = \sum_{i=1}^N \lambda_i Z(x_i) \quad i = 1 \dots n \quad (1)$$

where the weights  $\lambda_i$  are determined such that  $Z^*(x_0)$  is an unbiased estimate of  $Z(x_0)$ :

$$E \left[ Z^*(x_0) - Z(x_0) \right] = 0 \quad (2)$$

and the estimation variance is minimum:

$$E \left[ Z^*(x_0) - Z(x_0) \right]^2 \quad \text{minimum} \quad (3)$$

where  $E[\cdot]$  is the expectation. Substituting equation (1) into equations (2) and (3) yields:

$$E \left[ \sum_{i=1}^N \lambda_i Z(x_i) - Z(x_0) \right] = 0 \quad (4)$$

$$E \left[ \sum_{i=1}^N \lambda_i Z(x_i) - Z(x_0) \right]^2 \text{ minimum} \quad (5)$$

which leads to the following system:

$$\sum_{i=1}^N \lambda_i C(x_i, x_j) + \mu = C(x_0, x_j) \quad \sum_{i=1}^N \lambda_i = 1 \quad (6)$$

where  $C(x_i, x_j) = E[Z(x_i)Z(x_j)]$  is the covariance and  $\mu$  is a Lagrange multiplier which was employed to obtain the weights.

In the kriging system the estimation variance is written in terms of differences between two sample locations. The minimization yields the replacement of  $C(x_i, x_j)$  by  $\nu(x_i, x_j)$ :

$$\sum_{i=1}^N \lambda_i \nu(x_i, x_j) + \mu = \nu(x_0, x_j) \quad \forall j \quad (7a)$$

$$\sum_{i=1}^N \lambda_i = 1 \quad (7b)$$

which yields the semi-variogram equations:

$$\nu(h) = \frac{1}{2} E \left[ \left( Z(x+h) - Z(x) \right)^2 \right] \quad (8a)$$

or:

$$\nu(h) = \frac{1}{2} \text{var} \left[ Z(x+h) - Z(x) \right] \quad (8b)$$

where  $\nu(h)$  is the semi-variogram function,  $h$  is the distance between sample locations (also called the lag) and  $\text{var}(\cdot)$  is the variance. The semi-variogram  $\nu(h)$  is a graph which relates the differences or increments of the regionalized variable  $Z$  to the distance  $h$  between the data points. When there is a trend or drift in the data set, the residuals,  $R(x)$ , are used in Eq. 8 instead of the realizations,  $Z(x)$ , to estimate the semi-variogram. An empirical semi-variogram,  $\nu_e$ , can be calculated from the given set of observations by using the following numerical approximation:

$$\nu_e = 1/ \left[ 2N(h_e) \right] \sum_{i=1}^N \left( Z(x_i+h) - Z(x_i) \right)^2 \quad (9)$$

where  $N(h_e)$  is the number of pairs of points a distance  $h_e$  apart (Olea, 1975).

In order to solve Eq. 7, one of several common theoretical forms of Eq. 8 must be used in order to visually fit  $\nu$  to  $\nu^e$  (Delhomme, 1979). Once the theoretical semi-variogram has been chosen, four criteria can be used to determine the correctness of the model and to adjust its parameters:

(1) mean kriged estimation error:

$$1/n \sum_{i=1}^n [Z(x_i) - Z^*(x_i)] = 1/n \sum_{i=1}^n \epsilon_i \approx 0 \quad (10)$$

where  $\epsilon_i$  is the difference between the kriged and the known point value (this term should approach 0).

(2) mean standardized squared estimation error:

$$1/n \sum_{i=1}^n \left[ \frac{Z(x_i) - Z^*(x_i)}{s_i^*} \right]^2 = 1/n \sum_{i=1}^n [\epsilon_i/s_i^*]^2 \approx 1 \quad (11)$$

where  $s_i^*$  is the estimation standard deviation (this term should approach 1).

(3) sample correlation coefficient between the estimation values,  $Z^*$ , and the standardized estimation values,  $(Z - Z^*)/s^*$ .

This term should approach 0.

(4) sample correlation coefficient between the estimation values,  $Z^*$ , and the known values,  $Z$  (this term should approach 1).

### Data Collection

Annual average, summer and winter averages of rainfall depth over the state of Arizona has been adapted from Sellers et al. (1985). The summer rainy season includes the months of May to September and the winter rainy season includes the months of October to April.

Each of the three data sets are based on 158 rain gage stations having more than 30 years of record. This reduces the time variability of the precipitation record which is assumed to be very large in Arizona.

Figure 1 illustrates the location and the spatial distribution of the 158 raingage stations over the state.

# STATE OF ARIZONA: LOCATION OF RAINGAGES

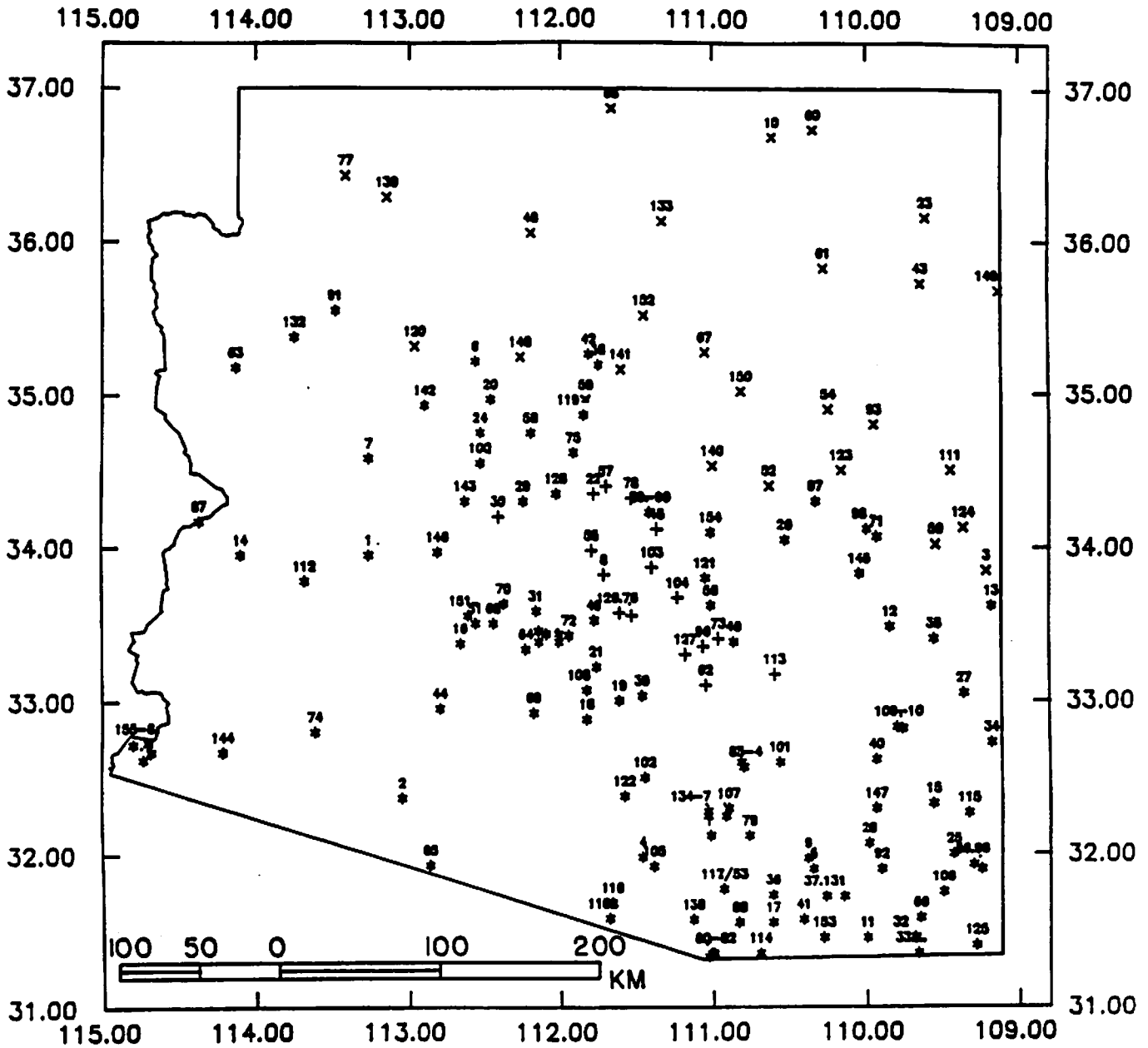


Figure 1: The State of Arizona - location of raingages.

## Methods

All the variogram and kriging calculations under were computed by using the BLUEPACK-3D software package implemented on a VAX 11/780 at the University of Arizona Computer Center. The BLUEPACK-3D is an integrated geostatistic program, written in FORTRAN, which was developed jointly by the Center de Geostatistique - Fontainebleau, France and the BRGM (French Geological Survey).

The variograms were fitted and plotted using VPLOT - a graphic package developed for the IBM-PC by D. E. Myers and G. J. Jalkanen, the University of Arizona, Tucson. The kriging maps have been produced by SURFER - a graphic computer package for two or three dimensional plotting.

## Structural Analysis and Results

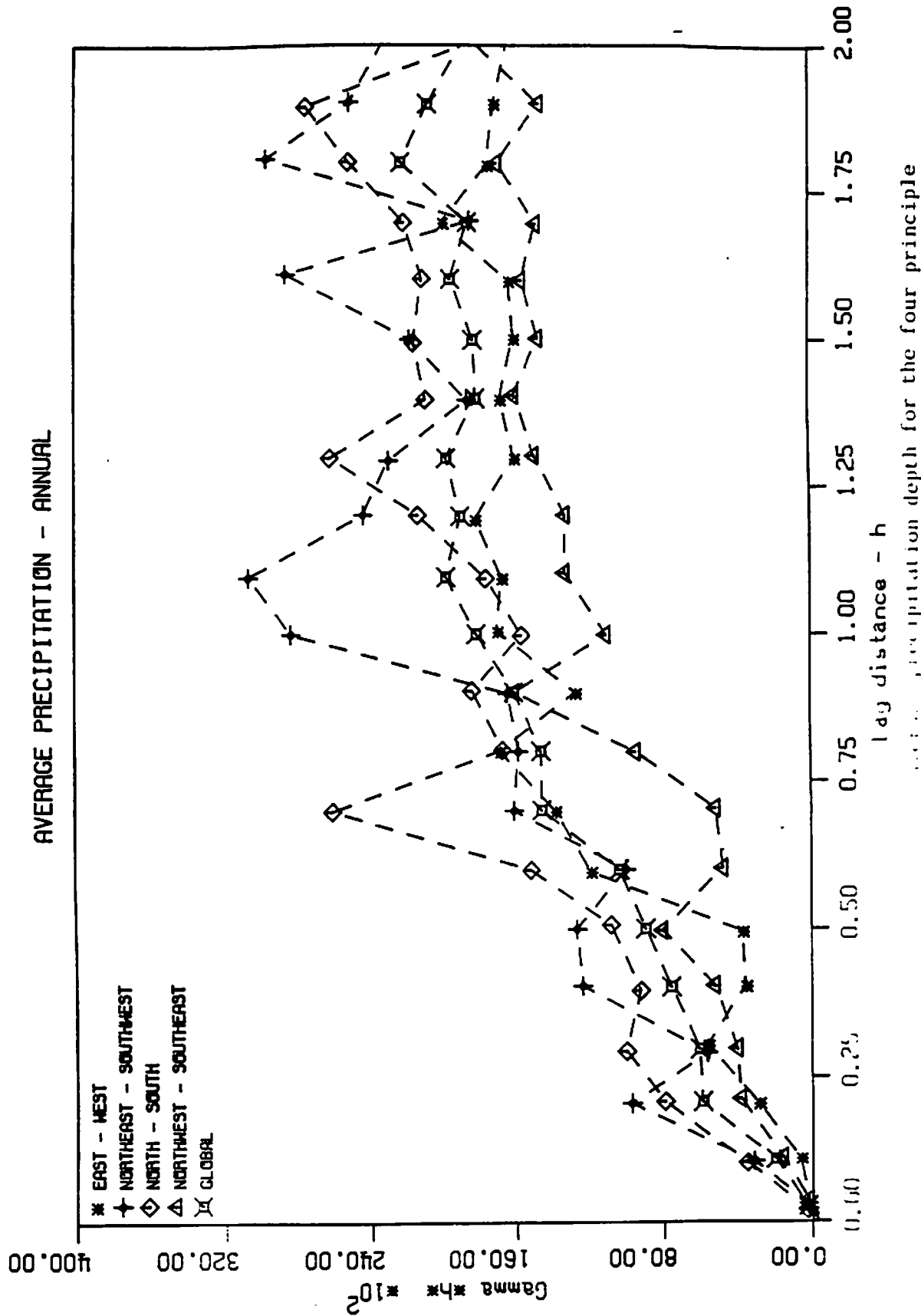
The first step in the kriging analysis was to establish the semi-variograms. The empirical variograms for all three cases (annual, summer and winter) are shown in Figure 2. Each plot includes the variogram for the four principal directions of the grid (North-South; East-West; NE-SW; and NW-SE) and the Onmi direction which is the average of the former four. For an isotropic phenomenon it is assumed that the variogram is not a function of the angle of the direction between the data points. As a result, the theoretical semi-variogram, assuming isotropy, is calculated only on the Onmi direction. From these plots the absence of detectable nugget variance can be recognized. All four semi-variograms start from the origin.

Eq. 9 was used to calculate the isotropic empirical semi-variogram for a constant drift case. Few theoretical models (Delhomme, 1978) have been examined. The final model was chosen as a result of the cross validation procedure. The exponential and the spherical models produce about the same results. All three empirical variograms were fitted with an exponential model of the form:

$$\nu(h) = C_0 + C_1 \left[ 1 - \exp(-|h|/a) \right] \quad (12)$$

where  $a$  is the range,  $h$  is the lag,  $C_0$  is the nugget variance and  $C_0 + C_1$  equals the sill. The fitted exponential models are illustrated in Figure 3 and the value of the parameters together with the cross validation results are presented in Table 1.

In the next step the kriging interpolation for the maps was performed. Figures 4, 5 and 6 show the final product of the analysis as isohyetal maps. One can find these maps very similar to other average precipitation maps of Arizona (e.g. the map in Sellers et al., 1985). However, the advantage of the proposed kriging technique is in its by product.



direct from and global - annual.



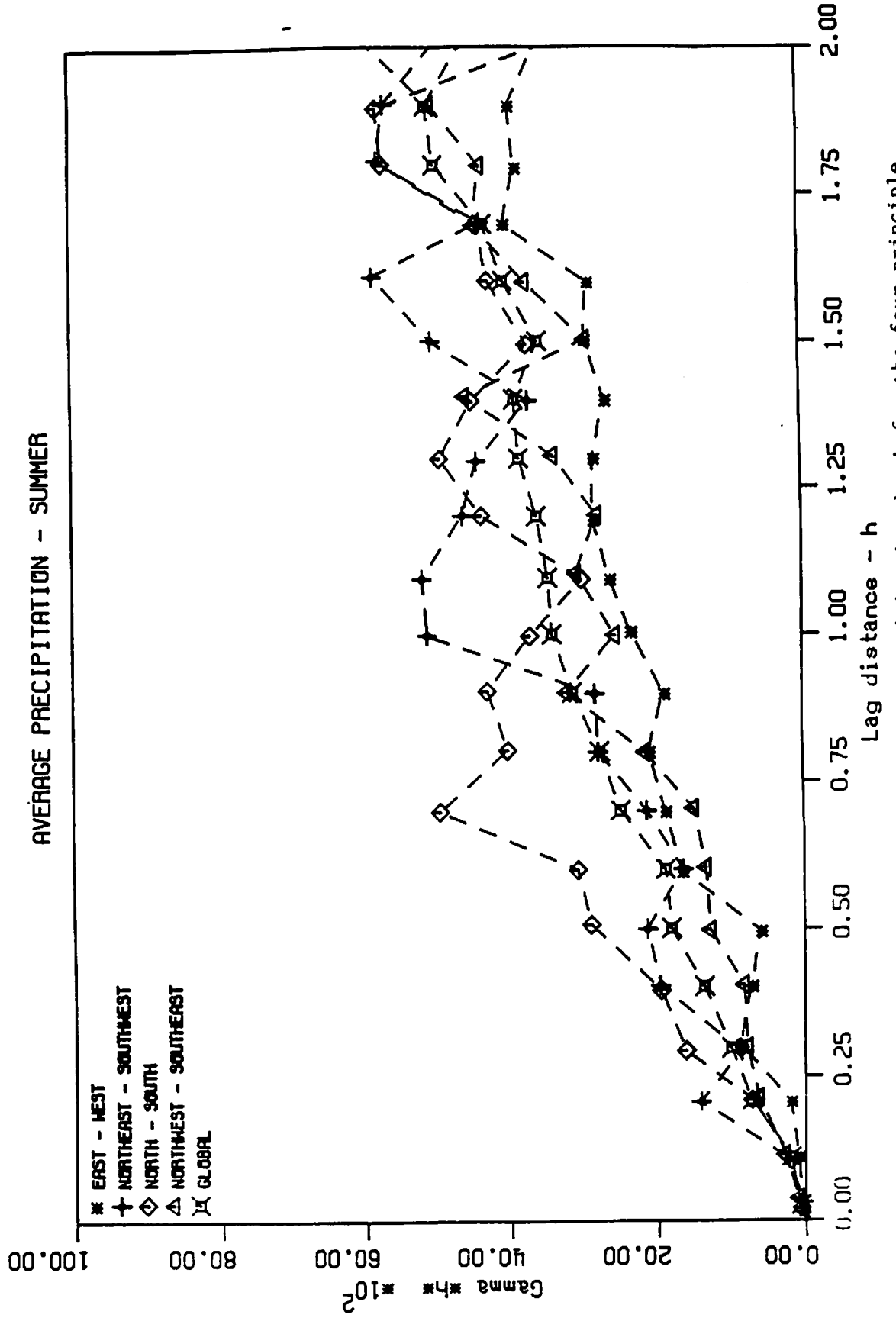


Figure 2b: Empirical semi-varigram of the average precipitation depth for the four principle

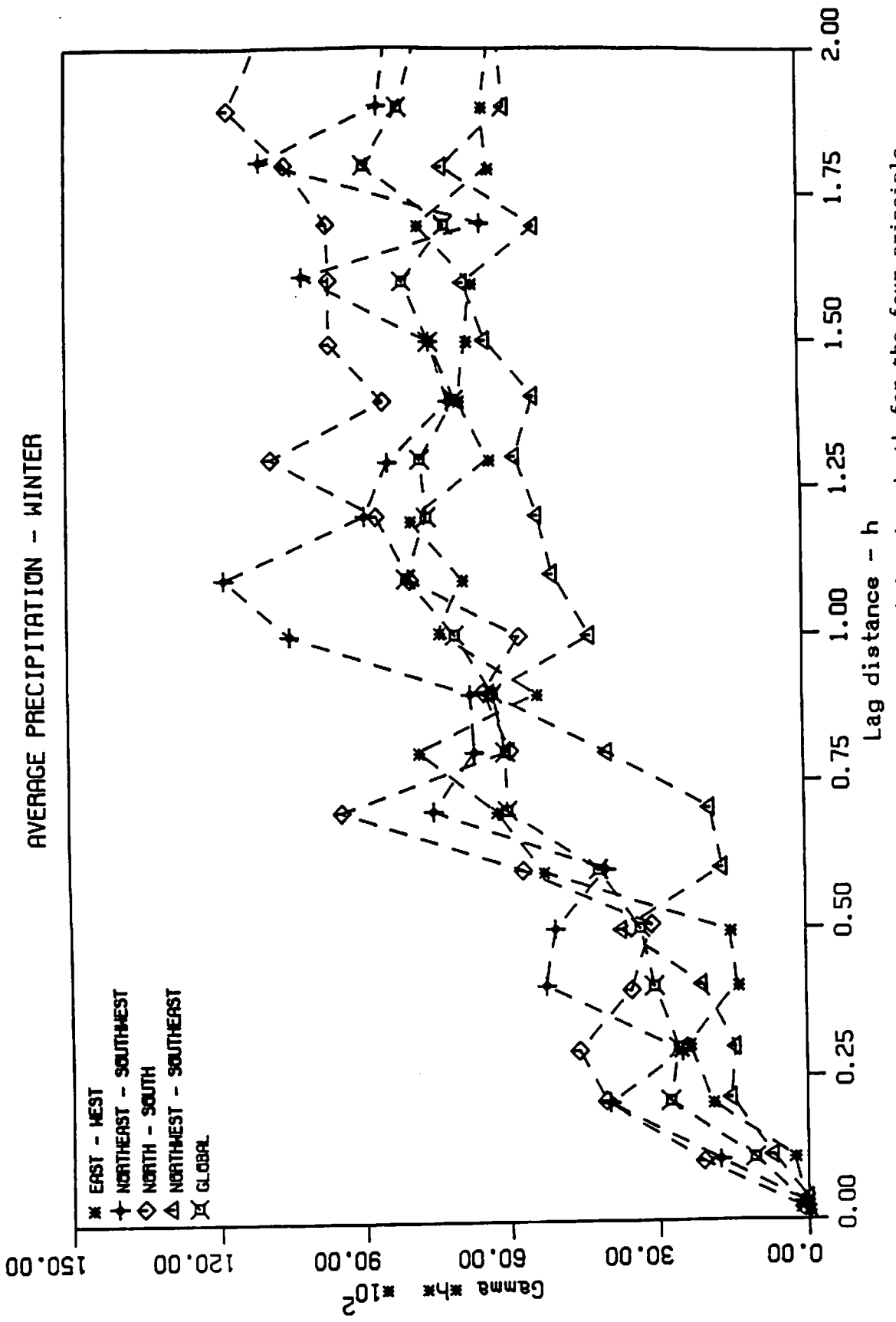
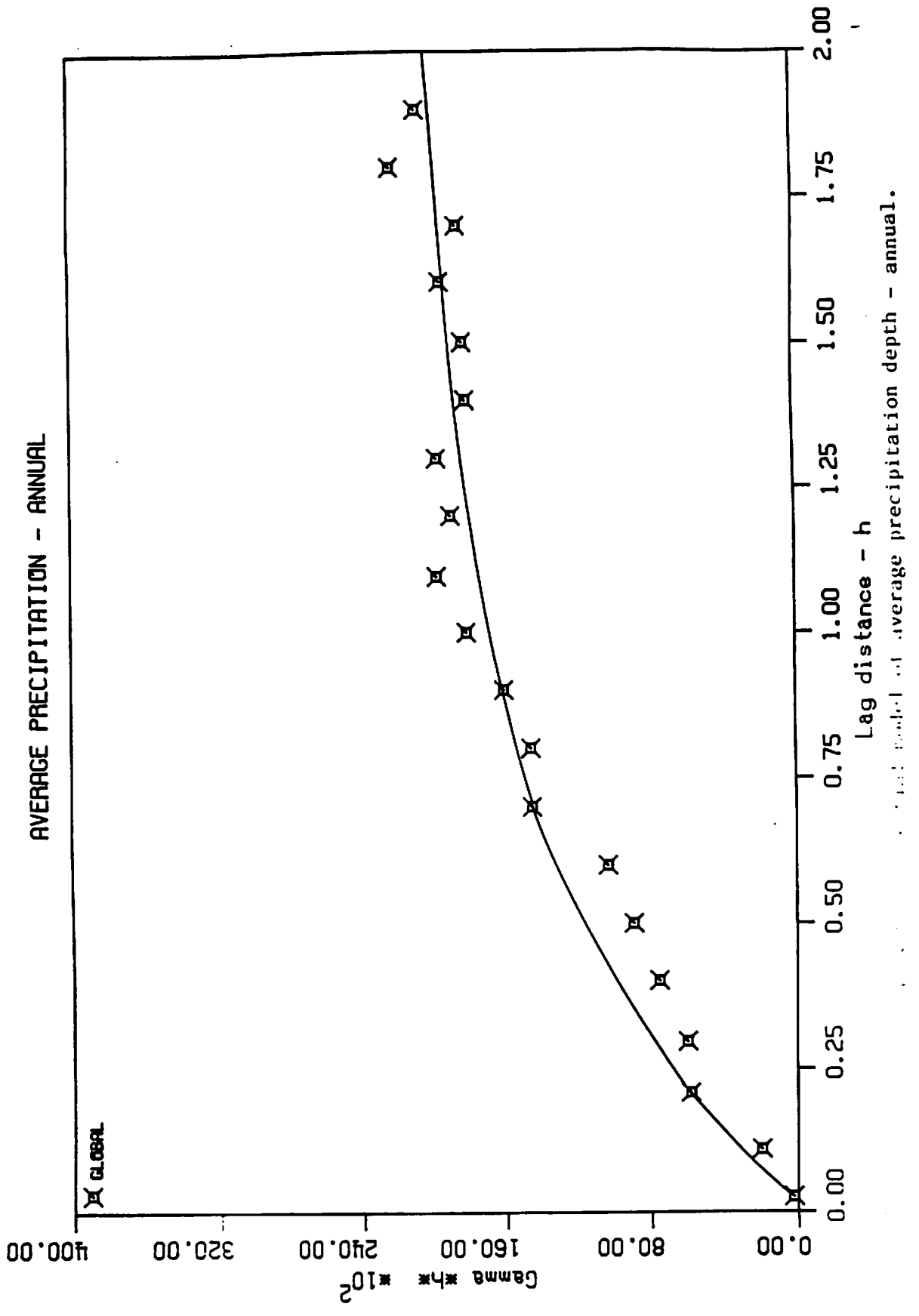


Figure 2c: Empirical semi-variogram of the average precipitation depth for the four principle directions and global - winter.



GLOBAL

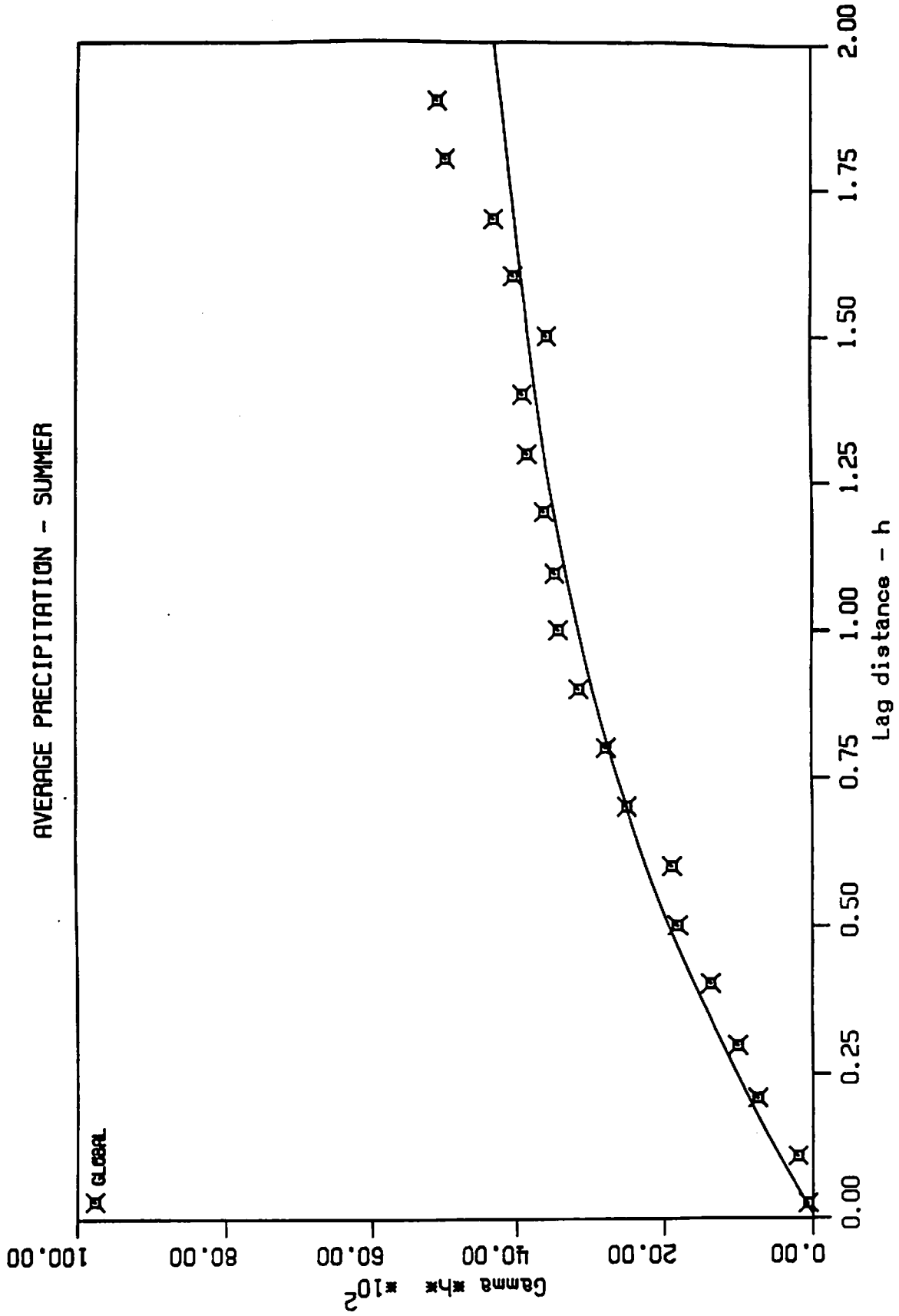


Figure 1. Comparison of actual precipitation with exponential model of average precipitation depth - summer.

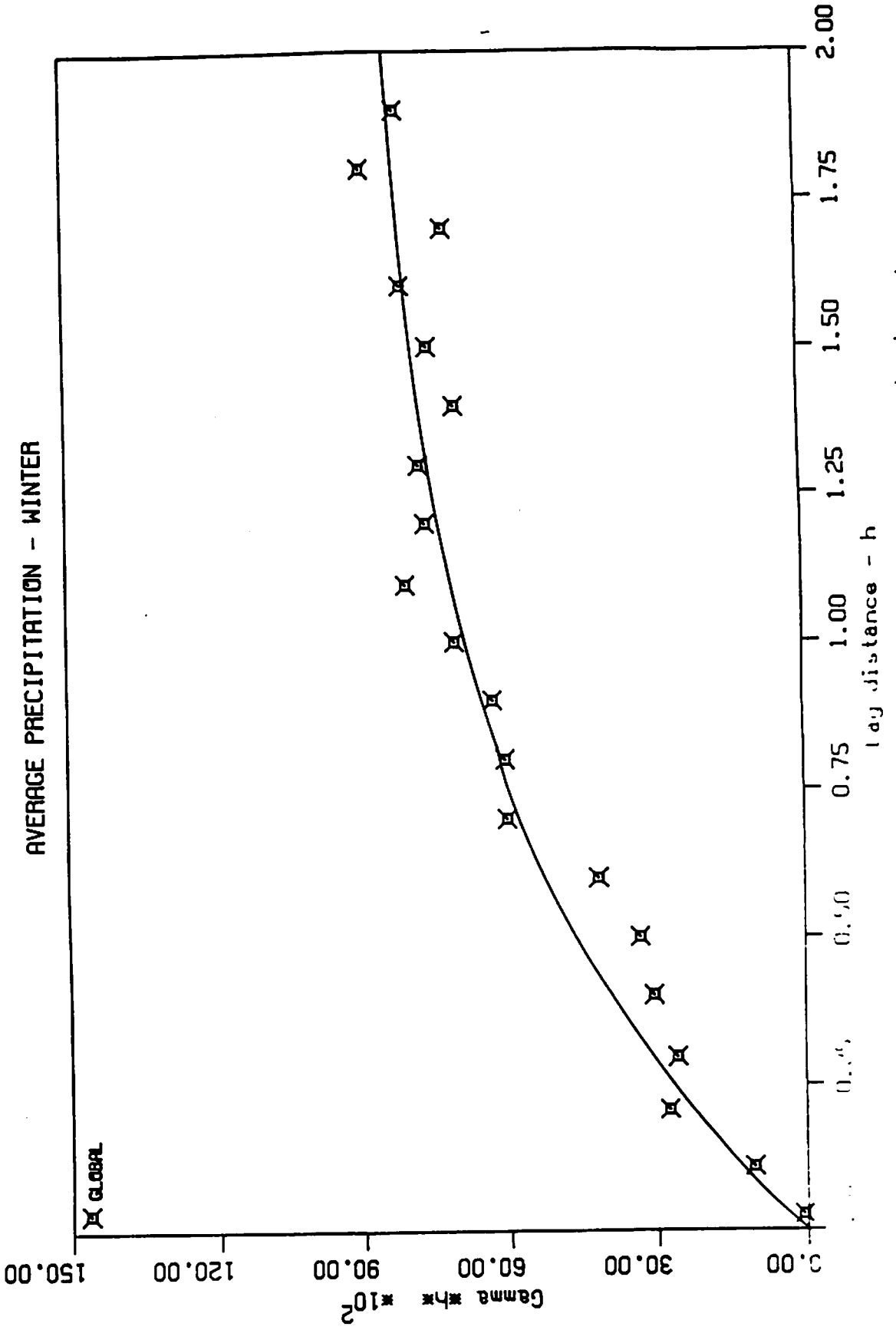


Figure 1. Comparison of the model of average precipitation depth - winter.

# AVERAGE PRECIPITATION — ANNUAL

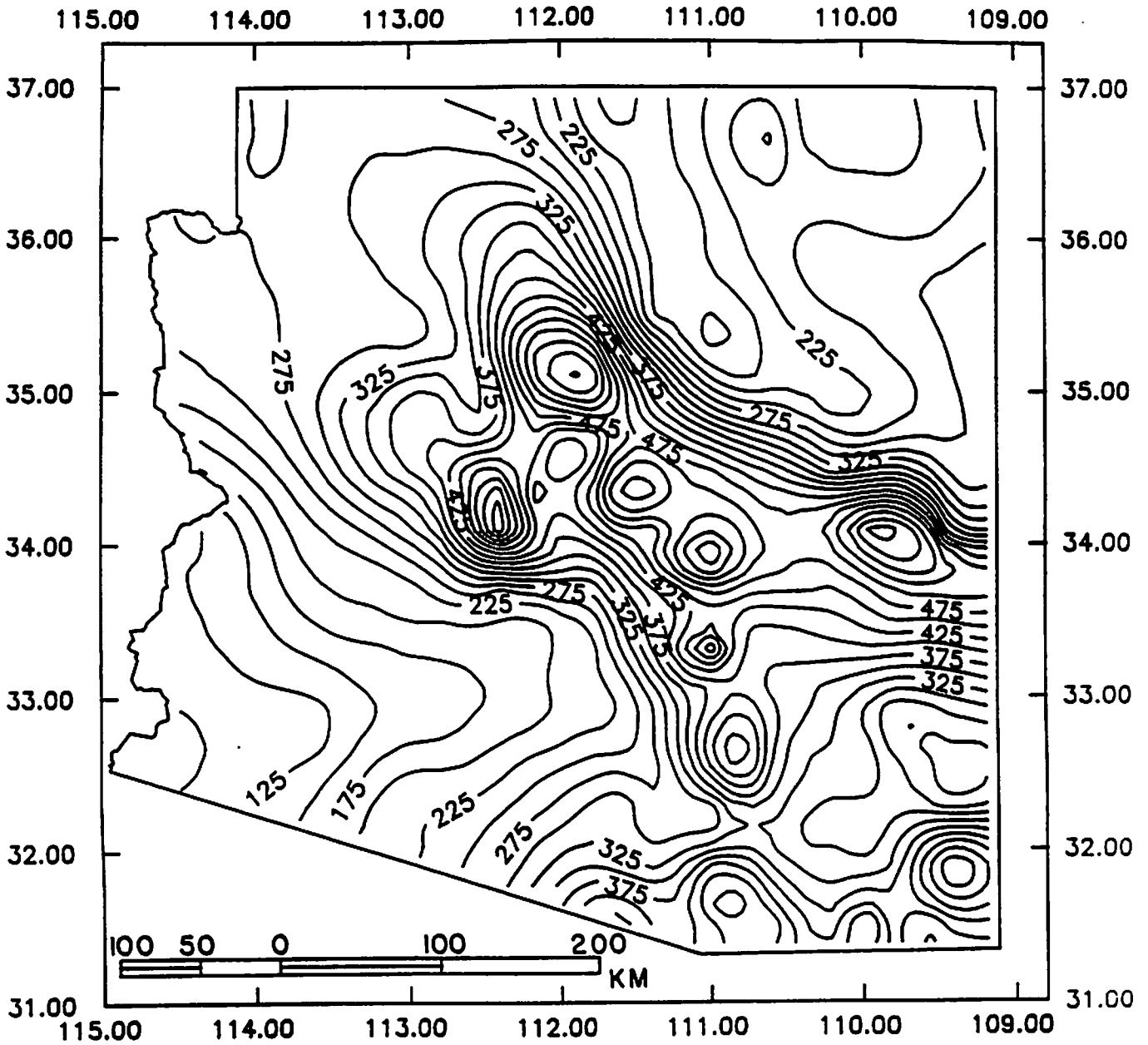


Figure 4: Isohyetal map of the annual average precipitation depth of Arizona produced by kriging technique.

# AVERAGE PRECIPITATION — SUMMER

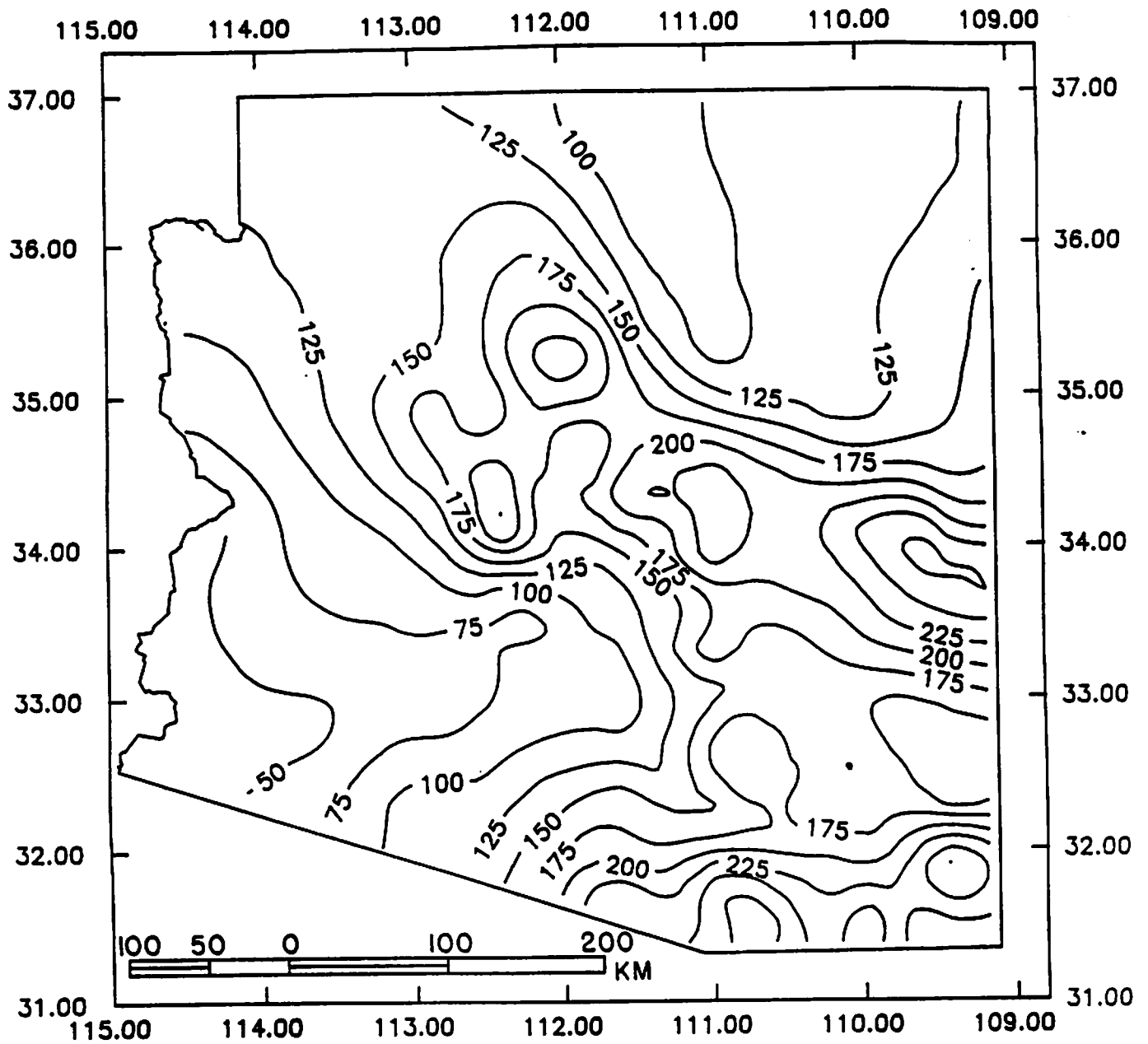


Figure 5: Isohyetal map of the summer average precipitation depth of Arizona produced by kriging technique.

# AVERAGE PRECIPITATION — WINTER

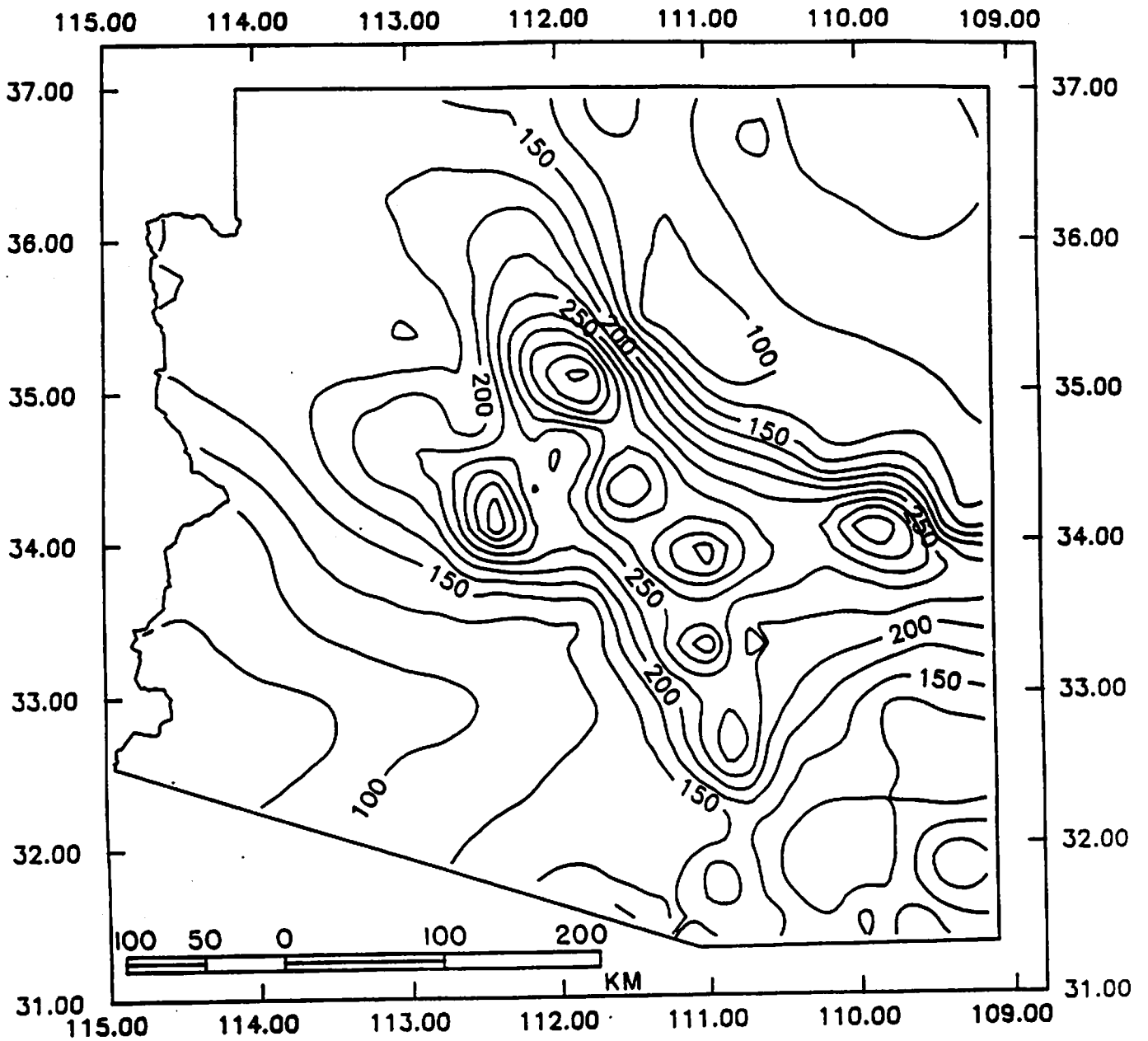


Figure 6: Isohyetal map of the winter average precipitation depth of Arizona produced by kriging technique.



TABLE 1: Parameters and cross validation results for exponential model semi-variogram fitting.

	MODEL PARAMETERS				CROSS VALIDATION RESULTS			
	NUGGET	RANGE	SILL	LAG	1	2	3	4
ANNUAL	0.0	0.8	0.18E5	0.1	0.02	0.61	0.03	0.84
SUMMER	0.0	1.0	0.50E4	0.1	0.01	0.76	0.08	0.91
WINTER	0.0	0.7	0.90E4	0.1	0.02	0.47	0.05	0.81

- 1 - mean kriged estimation error.
- 2 - mean standardized squared estimation error.
- 3 - correlation coefficient between the estimation values and the standardized estimation values.
- 4 - correlation coefficient between the estimation values and the known values.

Figure 7 presents the associated kriging errors map in terms of kriging variance. The kriging errors are a function of the sample site density and depend only on the geometrical location of the measured points. Errors are common when using an irregular grid such as rainfall gage stations and are a good measure of the precision of the interpolation (Delhomme, 1978). In this study the kriging variance is generally greater than 50 and less than 125. It can be seen that the kriged map is relatively precise in the middle of the state, however the errors become greater towards the edges of the map specially towards the north-west corner of the state as a result of fewer data points close to the state borders (refer to Figure 1).

Bastin et al., (1984) suggested to look on the variance as depending exclusively on the location of the rain gages. Thus, it is possible to compute the error variance associated with any set of hypothetical data points without getting actual data at these points. The above authors demonstrate the use of the kriging variance as an efficient tool for solving rain gage allocation problems.

So far discussed, it was assumed that the variation of the precipitation over the state was much the same in all directions. However, one of the features of the experimental semi-variograms presented in Figure 2, is evident anisotropy because of the semi-variograms sill. In all cases, annual, summer and winter, the sill differs appreciably within the four principal directions. When the variability is not the same in every direction and there is a greater spatial dependence in one direction the phenomenon is said to be directional (or zonal) anisotropic (Journel and Huijbregts, 1978).

Table 2 summarized the structural analyses of the anisotropic cases. Only those pairs of points lying within a particular interval

# KRIGING VARIANCE

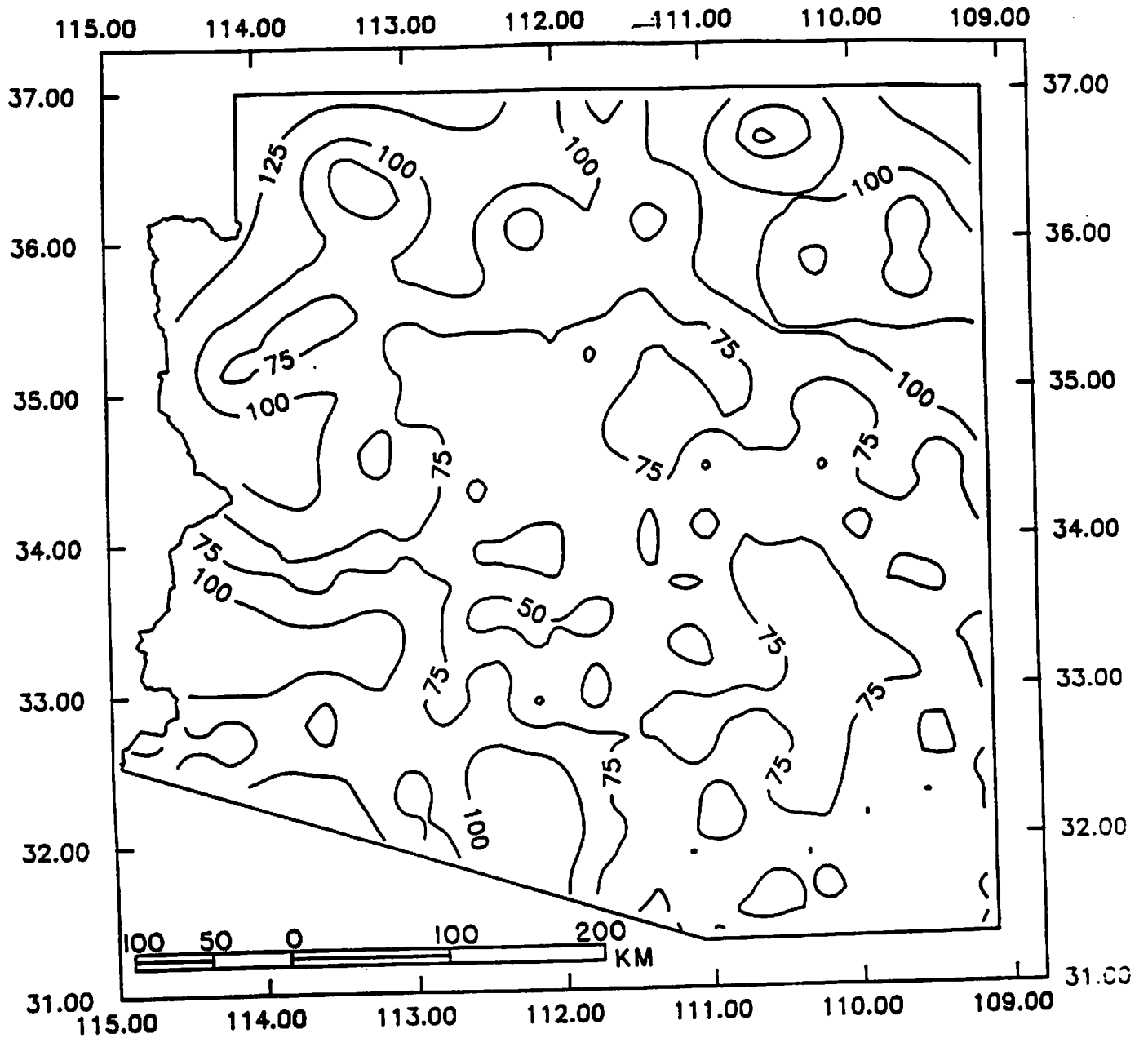


Figure 7: Error map in kriging variance of the average precipitation depth of Arizona.

are used in Eq. 9 to calculate empirical semi-variogram for that corresponding angle-of-direction interval. A separate theoretical semi-variogram is fitted for each direction. As can be recognized from Table 2, the semi-variograms can be grouped into two; the N-S and the NE-SW semi-variograms indicate lower sill, and the E-W and the NW-SE semi-variograms are characterized by higher sill. Note that in all the three cases the exponential model has been used and that all the other variogram parameters: the range, nugget and lag remain unchanged.

TABLE 2: Parameters of anisotropic semi-variograms.

DIRECTION	MODEL PARAMETERS			
	NUGGET	RANGE	SILL	LAG
<b>ANNUAL</b>				
EAST - WEST	0.0	0.8	0.18E5	0.1
NORTHEAST - SOUTHWEST	0.0	0.8	0.22E5	0.1
NORTH - SOUTH	0.0	0.8	0.25E5	0.1
NORTHWEST - SOUTHEAST	0.0	0.8	0.18E5	0.1
GLOBAL	0.0	0.8	0.21E5	0.1
<b>SUMMER</b>				
EAST - WEST	0.0	1.0	0.40E4	0.1
NORTHEAST - SOUTHWEST	0.0	1.0	0.60E4	0.1
NORTH - SOUTH	0.0	1.0	0.60E4	0.1
NORTHWEST - SOUTHEAST	0.0	1.0	0.50E4	0.1
GLOBAL	0.0	1.0	0.50E4	0.1
<b>WINTER</b>				
EAST - WEST	0.0	0.7	0.07E4	0.1
NORTHEAST - SOUTHWEST	0.0	0.7	0.10E4	0.1
NORTH - SOUTH	0.0	0.7	0.12E4	0.1
NORTHWEST - SOUTHEAST	0.0	0.7	0.07E4	0.1
GLOBAL	0.0	0.7	0.09E4	0.1

Figure 8 presents the theoretical semi-variogram for both the N-S and E-W directions. The unisotropic phenomenon of the precipitation fields in Arizona can be explained by the topographic structure of the state disregarding the storms origin and direction (for more detailed discussion see Karnieli and Osborn in this issue). This can be concluded also by the similar unisotropic structure for the annual, summer and the winter cases. The Mogollon Rim which stretches in the middle of the state, oriented from NW to SE provides a significant orographic effect on the precipitation. Consequently, the variation of the precipitation is greater in the N-S and NE-SW directions (perpendicular to the Mogollon Rim) than in the E-W and NW-SE directions.

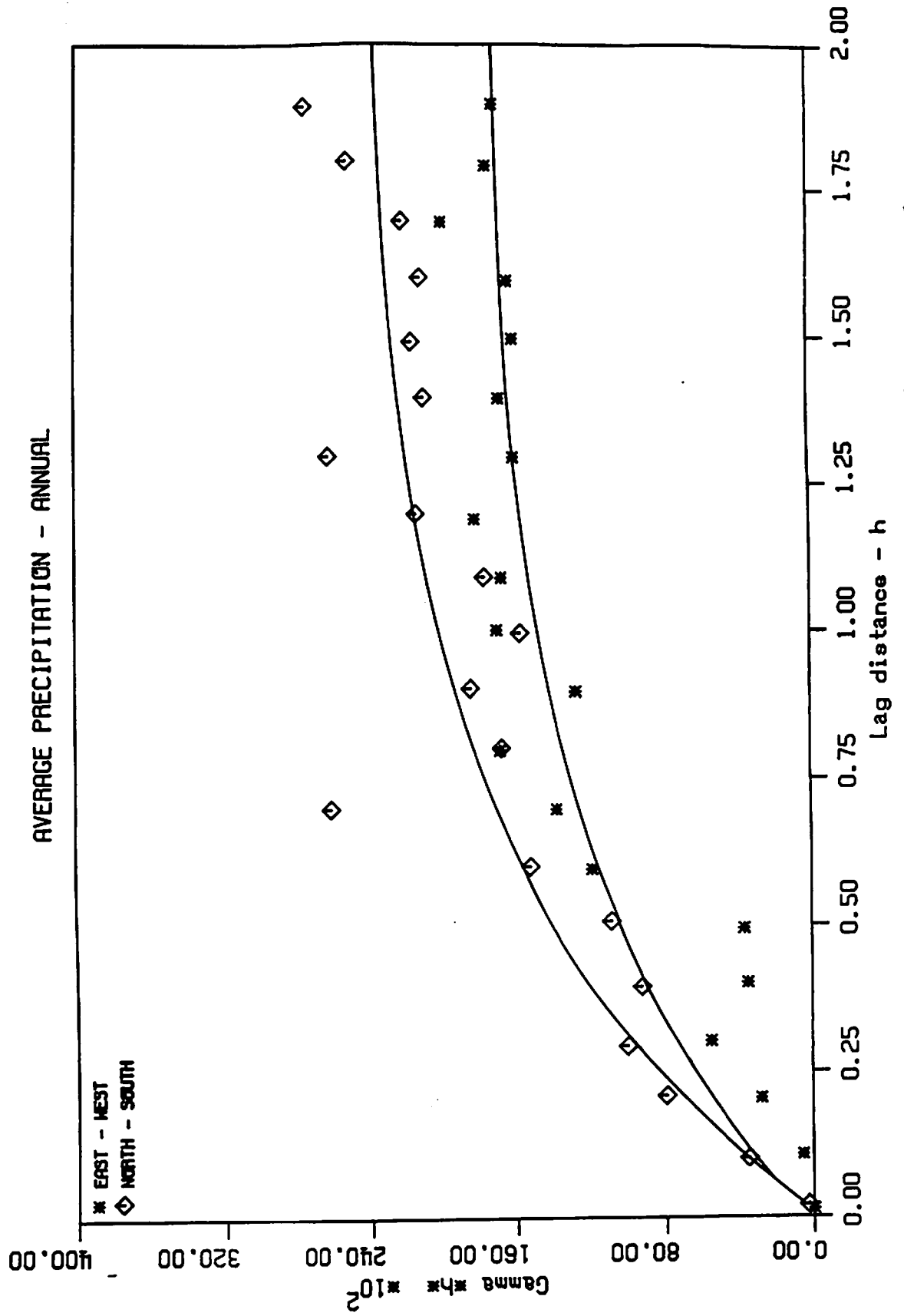


Figure 8.1: Zonal (directional) anisotropic of the average precipitation in Arizona - annual.

AVERAGE PRECIPITATION - SUMMER

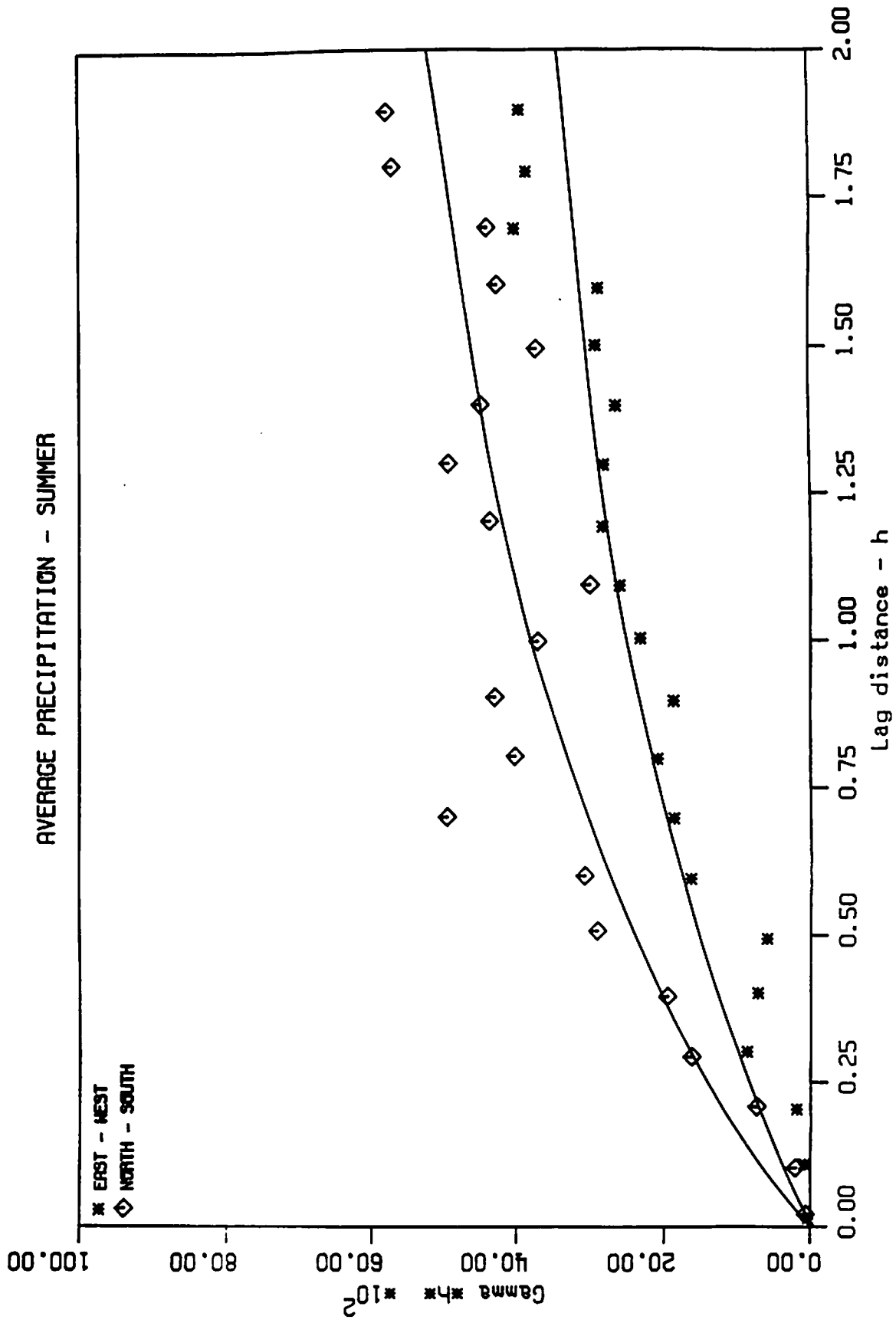


Figure 1. Relationship between lag distance and gamma for the average precipitation in Arizona - summer.

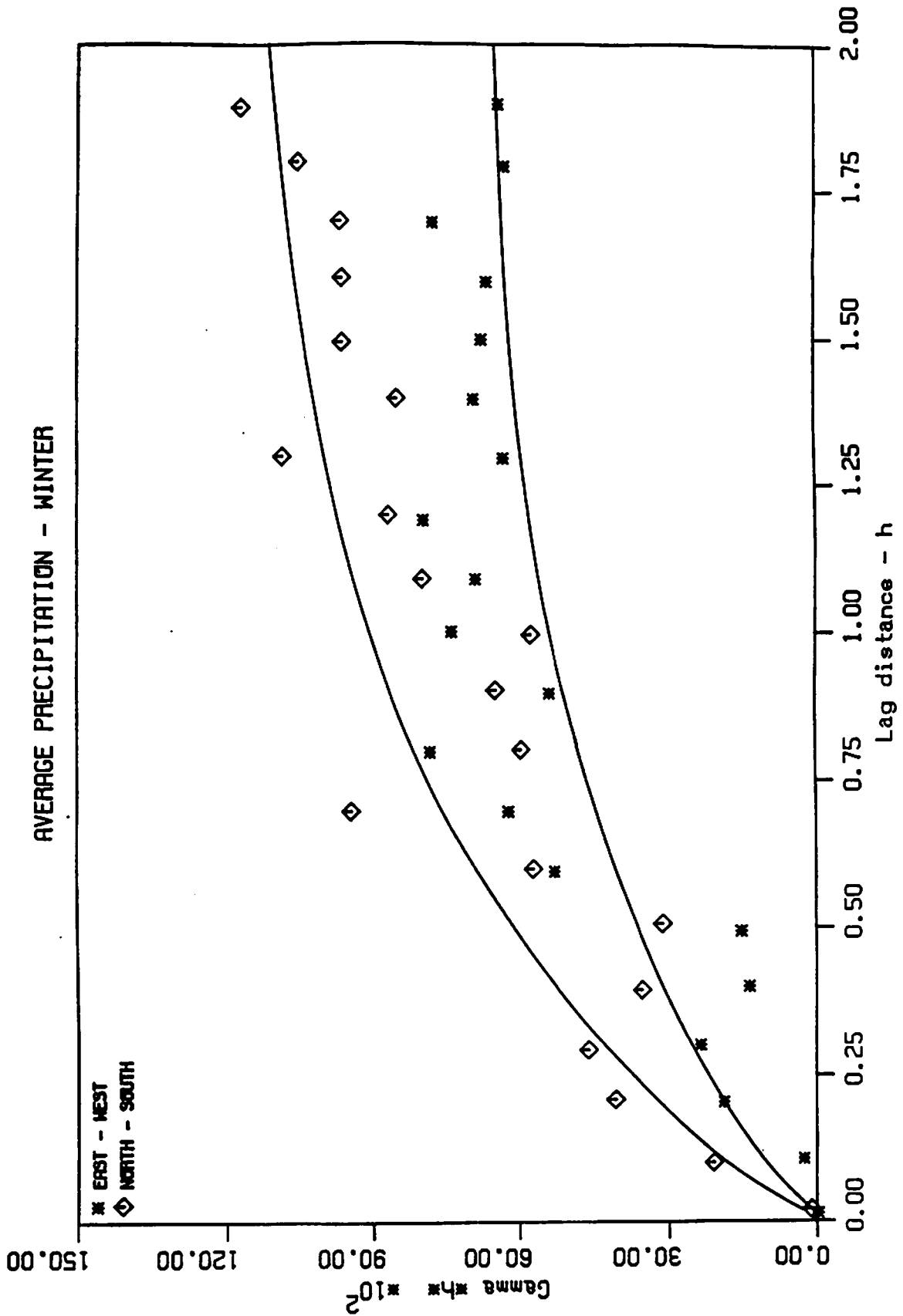


Figure 1: Total (directional) anisotropic of the average precipitation in Arizona - winter.

On the other hand, the presented zonal unisotropic can be interpreted as a spatial drift which have not been observed by the current computer package. In this case a polynomial drift should be fitted in order to be eliminated from the kriging algorithm. Chua and Bras (1982) and Neuman and Jacobson (1984) describe various of methods for dealing with this problem.

### Conclusions

Kriging is an advanced interpolation technique in which the estimator is a linear minimum variance unbiased estimator. This paper has proposed the application of kriging method for contour mapping as well as for estimating the average areal rainfall over large regions such as the State of Arizona with irregular rain gage network.

The kriging variance contour map (Figure 7) indicates that the predicted spatial structure agrees fairly well with the actual spatial structure. However, for better results stations surrounding the state borders, have to be taken into consideration. Furthermore, the error map can help the National Weather Service in selecting the optimal location of additional rain gages in order to increase the network accuracy.

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