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Stochastic Characterization of Rainfall Events[1]

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ABSTRACT

One barrier to the widespread adoption of physically-based infiltration models as a component of watershed models is the scarcity of breakpoint rainfall data. It has been proposed that the development of procedures to disaggregate daily rainfall into short period intensities could overcome this problem. One disaggregation approach is outlined, progress on daily and storm disaggregation is summarized, and further research needs are identified.

INTRODUCTION

Watershed simulation models have proven to be useful tools in predicting the transport of chemicals, water, and sediment from field-size areas (c.f., Knisel, 1980), and in estimating runoff (Rovey et al. 1977; Alley and Smith, 1982; Correa and Morel-Seytoux, 1985) and sediment yield (Smith, 1981) from urban and rural watersheds. These models either use, or have options to use, physically-based infiltration equations to partition rainfall into surface runoff and infiltration. Such models require rainfall data with high temporal resolution (5 min or less), and this requirement has become a barrier to their widespread use. Rainfall intensities obtained from recording raingages can be used as input to these models, but such data frequently are not available for the location of interest. Daily rainfall data, on the other hand, are quite plentiful, and recent work promises that very good models for simulating daily rainfall will soon be readily available for mainframe or microcomputers (Richardson and Wright, 1984; Woolhiser et al. 1985).

Woolhiser and Osborn (1985a) suggested that if techniques could be developed to disaggregate daily rainfall into the intermittent rainfall process, and further disaggregate significant storms into short-period rainfall intensities, simulated short-period

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rainfall data could be provided at locations where only daily rainfall data are available. The purpose of this paper is to summarize progress on daily and storm disaggregation research, and to identify problem areas.

DAILY AND STORM DISAGGREGATION

Two quite different approaches have been used to disaggregate daily rainfall. One approach is to disaggregate the discrete daily rainfall into 24 hourly amounts, and to disaggregate the hourly rainfall into 6-minute intensities (Srikanthan and McMahon, 1985). The second approach is to disaggregate the daily rainfall into individual "storms" or showers defined by their time of beginning, depth and duration (Hershenhorn, 1984), and then to disaggregate the "storms" into shorter period intensities (Woolhiser and Osborn, 1985a). The second approach appears to be more parsimonius with regard to the number of parameters that must be estimated from breakpoint rainfall data, and will be briefly reviewed here.

The analysis for the disaggregation of daily rainfall into individual "storms" requires the following steps:

1. Develop the appropriate form and parameter values for the joint distribution of amount of rainfall per day for a particular season, and the number of individual "storms" or showers.

2. Devise a procedure to disaggregate the daily rainfall amount R into N storms (of depth Y_i and duration D_i), and estimate the required parameters.

3. Determine the form of the joint distribution of event amount Y_i and duration D_i, for each of the N storms, and estimate the required parameters.

4. Determine the form of the distribution function of time of storm beginning T_i, and estimate the required parameters.

5. Special procedures are required for partial storms, i.e., storms that continue over midnight.

The procedures developed by Hershenhorn (1984) were found to provide a satisfactory fit to data for a single raingage at the Walnut Gulch Experimental

Watershed near Tombstone, Arizona. The model structure and parameters identified at that gage were used in simulations for another gage at Walnut Gulch, and also for gages at Safford, AZ and the Santa Rita Range Experiment Station, AZ. It was found that simulated storm numbers, depths, durations, and time distribution compared closely with the observed distributions (Hershenhorn and Woolhiser, 1986). This finding suggests that this daily disaggregation procedure may be feasible for other locations, and we are currently analyzing breakpoint data for several U.S. locations.

Various approaches have been used to disaggregate storms into shorter period intensities. Several investigators have non-dimensionalized the accumulative rainfall process within a storm as a basis for simulating intra-storm intensity patterns. Bras and Rodriguez-Iturbe (1976) divided the duration of the storm into 8 dimensionless intervals, and assumed that the local intensity process in time could be represented by the relationship:

$$\Delta u^*_{k+1} = u^*_{k+1} - u^*_k = \overline{\Delta u}^*_{k+1} + \rho_1(\Delta u^*_k - \overline{\Delta u}^*_k) + \sigma_{k+1}(1-\rho_1{}^2)^{1/2} W_{k+1} \quad k = 0, 1, 2, \cdots m \tag{1}$$

where Δu^*_{k+1} = the nondimensionalized rainfall occurring in the interval (k, k+1) u^*_k is the normalized rainfall that has occurred in the interval (0, k), $\overline{\Delta u}^*_k$ is the mean depth increment for interval k, σ_{k+1} is the standard deviation, ρ_1 is the lag-one serial correlation between the residuals, and W_{k+1} is a normally distributed random variable with mean 0 and standard deviation of 1.

Wilkinson and Valaderes-Tavares (1972) also used a non-dimensional approach and simulated the octal depths u^*_1, u^*_2, $\cdots$ u^*_7 by simulating the dimensionless depth at median duration u^*_4, and then proceeding in both directions, allowing the process to reach zero or one before the dimensionless beginning or end. They did not account for dependence between increments.

Woolhiser and Osborn (1985a) rescaled the dimensionless increments (as did Wilkinson and Valaderes-Tavares) defining the rescaled increments as

$$z_{k+1} = \frac{u^*_{k+1} - u^*_k}{1 - u^*_k} \qquad k = 0, 1, 2, \cdots 10 \tag{2}$$

For thunderstorm rainfall data from southeastern Arizona, they found that the first increment could be described by the beta distribution $h_1(z_1)$, and the process could be described by a first order non-homogeneous Markov Chain with the joint density function

$$f(z_1, z_2 \cdots z_9) = h_1(z_1) \prod_{k=2}^{9} g_{k,k-1}(z_k|z_{k-1}) \quad (3)$$

where the conditional density function

$$g_{k,k-1}(z_k|z_{k-1})$$

is also beta distributed. The conditional expected value function for each increment was written in the linear form:

$$E\{Z_{k+1}|Z_k = z_k\} = a_{k+1} + b_{k+1}z_k \quad (4)$$

This model requires a total of 26 parameters for 10 dimensionless increments. However, there were certain regularities in the two dependence parameters, a_k and b_k, and the conditional beta parameter α_k such that the total number could be reduced to 10 by using the expressions

$$a_k = B_1 + B_2k$$

$$b_k = B_3 + B_4k + B_5k^2 \quad (5)$$

$$\alpha_k = B_6 + B_7k + B_8k^2$$

Woolhiser and Osborn (1985b) examined regional and seasonal differences in the stochastic dimensionless model of rainfall by identifying parameters for the summer season for data from Walnut Gulch Experimental Watershed near Tombstone, AZ and Alamogordo Creek Experimental Watershed near Santa Rosa, NM, and for the winter season at Walnut Gulch. They found both regional and seasonal differences, although the short-duration storms at Alamogordo Creek were similar to the summer thunderstorms at Walnut Gulch, while the long-duration storms were similar to the winter storms at Walnut Gulch. Simulation techniques and a deterministic rainfall-runoff model were then used to examine the sensitivity of derived variables such as storm runoff volumes and peak runoff rates to variations in rainfall disaggregation model parameters due to season and location. Constant intensity and triangular hyetographs were also used. They found that the Woolhiser-Osborn (1985a) disaggregation model provided a better approximation to natural rainfall hyetographs, as indicated by the empirical distribution functions of runoff peaks and volumes, than did a constant or triangular approximation. However, for slowly responding runoff systems and high infiltration rates, runoff peaks and volumes were not very sensitive to differences in disaggregation model parameters due to seasonal and regional effects. This finding suggests that disaggregation parameters identified for a particular station can be moved both in space and time without serious effects on derived peaks and volume distributions.

The storm disaggregation model of Woolhiser and Osborn (1985a) was based on storm data which had been truncated when intensities dropped below 3 mm/hr. Hershenhorn's (1984) daily disaggregation model, however, considered total storm duration regardless of intensity. The statistical characteristics of these two data sets are presented in Table 1.

TABLE 1.

Statistics for truncated and untruncated storm data, Gage No. 5, Walnut Gulch Experimental Watershed, AZ. (July-Aug)

Data set	Duration (min)		Depth (in)	
	mean	std.dev.	mean	std. dev.
Truncated data*	39.9	21.9	0.67	0.41
Truncated data* (30 largest storms)	47.3	20.7	1.02	0.52
Untruncated data	106.6	86.6	0.61	0.41
Untruncated data (30 largest storms)	147.5	97.6	1.21	0.48

*Composite record. Includes Gage No. 5 and 2 additional gages.

To overcome this inconsistency, we used Woolhiser and Osborn's (1985a) procedures to analyze untruncated data for the same raingages from the Walnut Gulch Experimental Watershed. To accommodate the longer mean durations of the untruncated storms, 20 dimensionless increments were used to describe the temporal variations in intensity. Although this did improve the temporal resolution as expected, deterministic relationships were introduced into the empirical joint distributions of z_{k+1} and z_k. These deterministic relations are evident in Figures 1a and 1b. It can be shown that the deterministic curvilinear relation in Figure 1a is represented by the equation:

$$z_{k+1} = \frac{z_k}{1-z_k} \qquad (6)$$

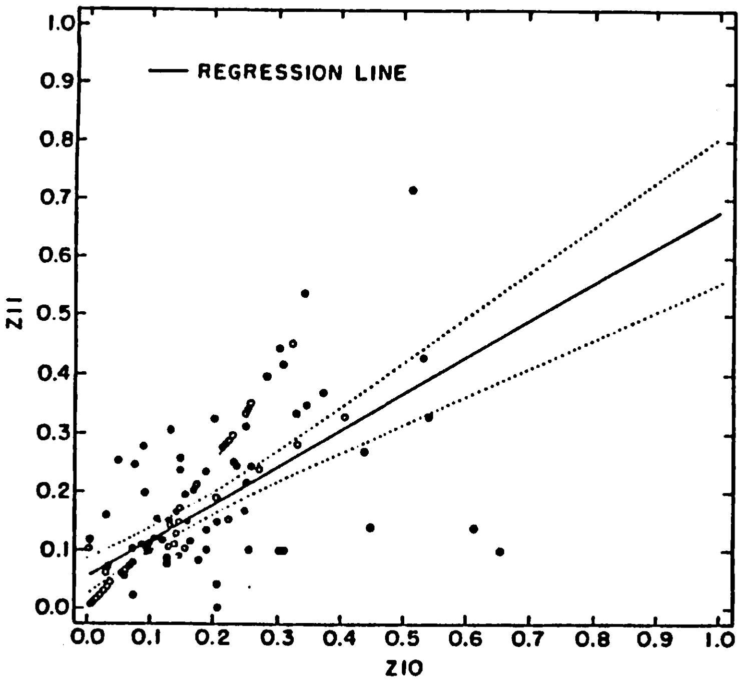

Figure 1a. Relationship between z_{11} and z_{10}

and is caused by sampling from regions of constant rainfall intensity within the breakpoint data. The discrete relationship $Z_{17} = 0.25$, evident in Figure 1b, is due to sampling from a constant intensity segment near the end of the storm. For this same reason, there are significant numbers of data points defined as $Z_{18} = 0.33$ and $Z_{19} = 0.5$. This problem is, of course, due to the time resolution of the original data and the criteria used to digitize the break point data, and cannot be avoided.

With 20 dimensionless increments, the model requires a total of 56 parameters. This number can be reduced to 11 by using quadratic functions for the parameters a_k, b_k, and α_k (see Eq. 5). The first increment requires two parameters, α_1 and β_1, to describe the beta distribution. The parameters were estimated by numerical maximum likelihood techniques, as described by Woolhiser and Osborn (1985a).

Model goodness of fit was first judged by simulating the sample number of storms (139) using the smoothed parameters and comparing the empirical distribution functions of the u_k^*, k = 1, 2, • • • 19. as shown in Figure 2. It is clear from this figure that the

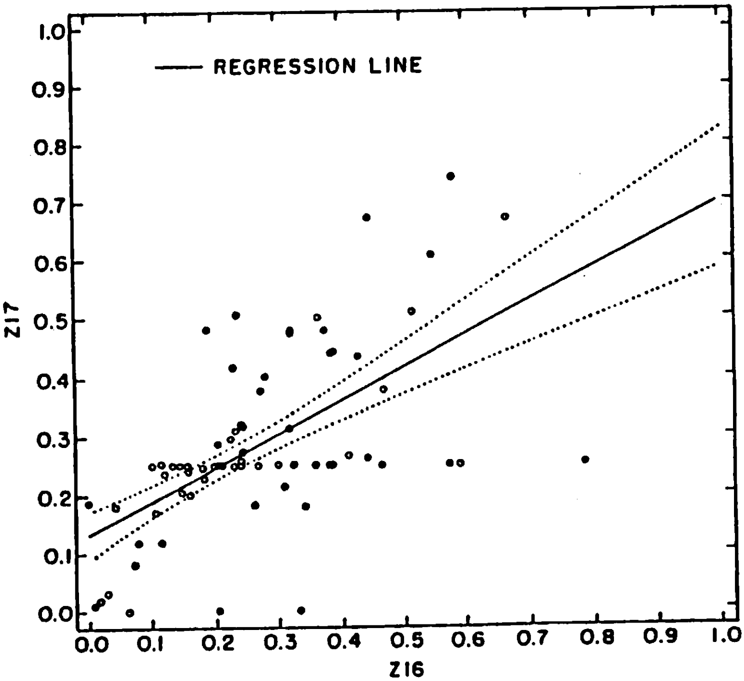

Figure 1b. Relationship between z_{17} and z_{16}

smoothing of parameters distorts the distributions of u_k^*, particularly near the midpoint of the storms. Although there are statistical differences between the observed and simulated data, it is useful to ask the following question: "Would distributions of runoff peaks and volumes, using the observed data and simulated rainfall intensities, be significantly different?" To examine this point, we selected the 30 largest storms from the data set and used them as input to a deterministic rainfall runoff model. The runoff model selected, KINEROS (Smith, 1981), utilizes the Smith-Parlange (1978) interactive infiltration model and kinematic runoff routing. A simple planar surface was used in this study.

Watershed response times were characterized by the ratio T_*, defined as the ratio of the kinematic time to equilibrium for a plane at a rainfall excess rate, $\overline{q}$, (equal to the mean intensity of a set of m storms) divided by the mean storm duration, $\overline{D}$. Infiltration was characterized by the dimensionless ratio F_p* defined as the ratio of the total rainfall infiltrated at ponding if the rainfall rate is $\overline{q}$,

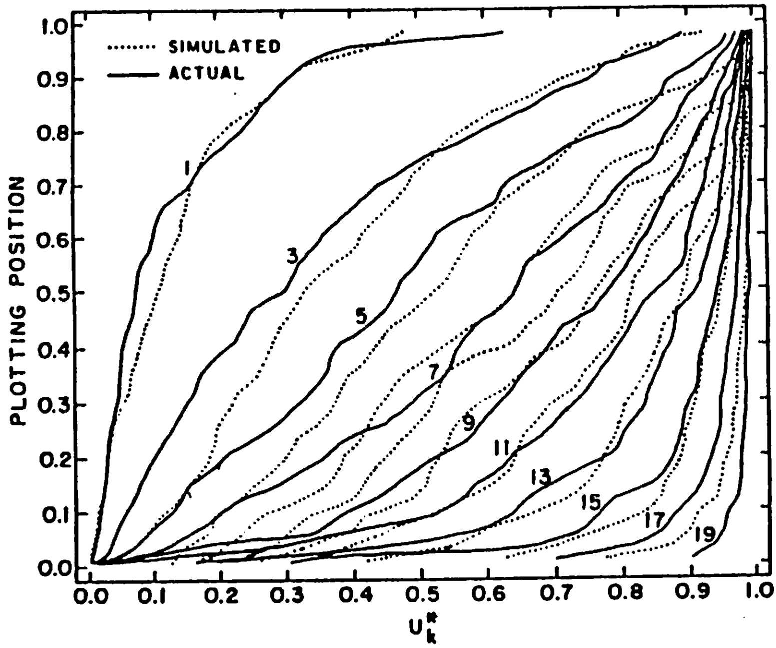

Figure 2. Empirical Distribution Functions of U_k^*. Parameter is Dimensionless Time Step Number, k.

and the initial water control is zero divided by the mean storm depth $\bar{P}$. Further details of these dimensionless ratios are given in Woolhiser and Osborn (1985b).

The depth and duration of each of the 30 observed storms (Table 1) were used with simulated dimensionless intensities to construct a set of 30 simulated storms which were also used as input to the deterministic rainfall runoff model. The peak flow rates, q_p, and runoff volumes, V, calculated by the runoff model, were ranked, and the empirical distributions, using real and simulated rainfall inputs, were compared by constructing quantile-quantile plots. Space limitations prevent showing all of the results. However, plots are shown for a rapidly responding geometry ($T_* = 0.014$) and a slowly responding system ($T_* = 0.24$), both with infiltration characteristics of a gravelly silt loam ($F_p^* = 1.22$) in Figures 3a and 3b, respectively. Ten different simulated sets of 30 storms each are shown in Figure 3b to demonstrate the effects of sampling variability. In general, the distributions of volumes followed the 1:1

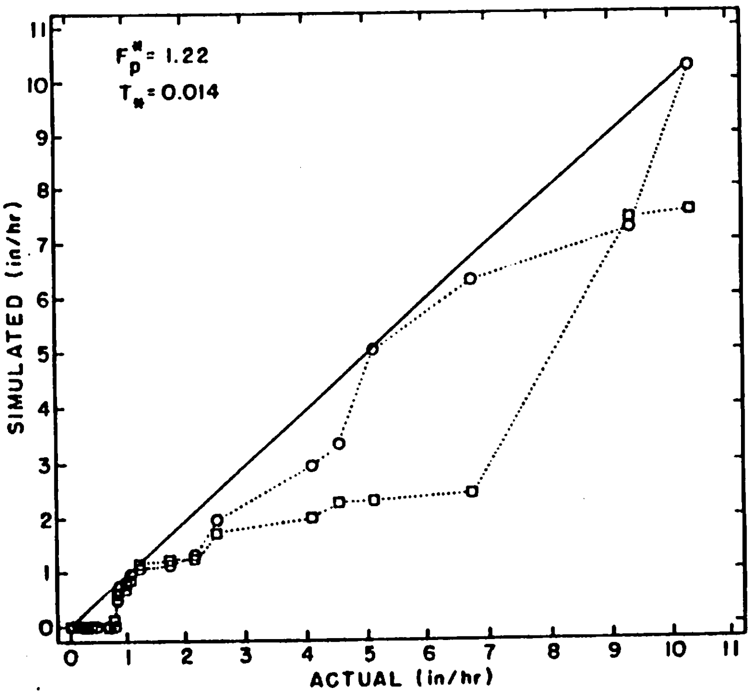

Figure 3a. Quantile-Quantile Plot of Peak Runoff Rates with Simulated and Measured Rainfall Inputs

line rather closely, as did distributions of peaks for the case $T_* = 0.24$. For the rapidly responding surface, the peaks with simulated rainfall intensities appear to be lower than those using the real data as input (Figure 3a).

DISCUSSION

Significant progress has been made in disaggregating daily and storm rainfall, and it appears that the approach outlined is feasible. However, considerable work must be done before simulated short period rainfall is generally available as input to watershed models. The analysis procedures must be tested with breakpoint data from different climatic regions, and the effects of seasonality must be identified and taken into account. As part of this testing phase, statistics obtained from coupled daily and storm disaggregation models must be compared with statistics obtained from rainfall data not used in the disaggregation analysis. These comparisons should be based both on rainfall statistics and

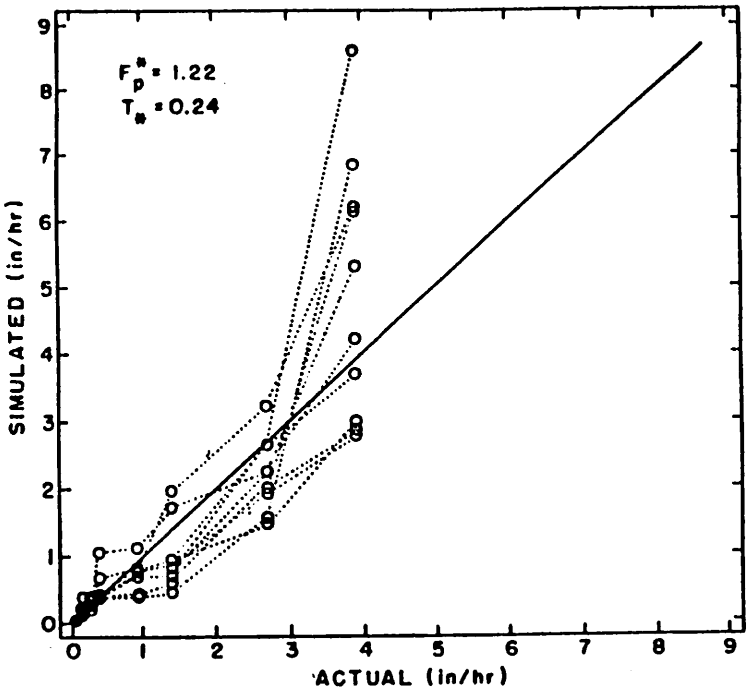

Figure 3b. Quantile-Quantile Plot of Peak Runoff Rates with Simulated and Measured Rainfall Inputs

derived statistics such as peak flow rates and volumes obtained from rainfall-runoff simulation models. Finally, objective criteria should be developed to establish daily and storm rainfall thresholds below which disaggregation does not provide useful information.

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