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Sensitivity of calculated peak-runoff rates to rainfall-sampling frequency

Sensibilité des taux calculés d'écoulements maximaux à la fréquence d'échantillonage des précipitations

DAVID A. WOOLHISER
Research Hydraulic Engineer
Agricultural Research Service, USDA
2000 East Allen Road
Tucson, Arizona 85719, U.S.A.

ABSTRACT The sensitivity of calculated peak-runoff rates to rainfall-sampling frequency was investigated by simulation. The rainfall-runoff model included an interactive infiltration component and kinematic flow routing. The 30 largest storms, for a continuous-recording rain gage at the Walnut Gulch Experimental Watershed near Tombstone, Arizona, U.S.A., were taken as the correct input. These "correct" data were then sampled at intervals of 5, 10, 15, 20, and 30 minutes, and these new data were then used as input to the rainfall-runoff model. The peak-discharge rates for the 30 storms, with each sampling interval, are compared to those generated using the correct data by means of empirical quantile-quantile plots. Previously obtained sampling-interval criteria are verified for impervious basins. A 5-minute sampling interval is adequate for heavy-textured soils (silty-clay loam and silty clay), provided the basin equilibrium time is greater than 16 minutes. A shorter interval is required for lighter textured soils.

INTRODUCTION

The design of hydrologic networks requires a tradeoff between the marginal value of information obtained by increasing the spatial or temporal resolution of the measurements and the marginal cost of obtaining, processing, interpreting, and storing the additional data. Costs are relatively easy to assess, at least for the short run, but estimates of benefits depend on the potential use of the information. Both estimates are highly sensitive to technological change.

Sampling frequency for precipitation measurement ranges from days or months for manually read storage gages, to daily intervals for standard non-recording rain gages, to minutes for analog-recording rain gages or digital electronic systems. The basis for establishing the sampling frequency too often is based upon what can be accomplished with existing resources, rather than a careful analysis of what is required for the problem, or problems, at hand.

Eagleson & Shack (1966) carried out the first theoretical work on the problem of determining the necessary sampling intervals for rainfall and runoff. They used a lumped linear representation for the basin and examined the rainfall and basin-response characteristics in the frequency domain. They developed sampling-interval criteria for thunderstorm rainfall and cyclonic rainfall dependent on the relationship between the passband frequency of the drainage basin and the band width of the precipitation signal.

Harley et al. (1970) developed a criterion for precipitation-sampling interval based upon the spectral characteristics of the equivalent linear response of a kinematic surface-runoff model. The sampling interval they recommended was

$$\Delta t = t_e/3.2 \qquad (1)$$

where Δt is the sampling interval and t_e is the kinematic time to equilibrium for the basin.

In a review of sampling interrelated hydrologic random fields, Bras (1979) pointed out limitations in previous approaches with respect to the question of rainfall-sampling intervals. It appears that the most significant limitations of previous work are associated with the definition of storm duration and depth and the neglect of the effects of infiltration.

This paper is limited to a consideration of appropriate rainfall-sampling frequency for very small basins with differing infiltration and runoff-response characteristics. Only the overland-flow component is considered.

THE RAINFALL-RUNOFF MODEL

Overland flow will be described by the kinematic equations for flow over a plane of length L

$$\frac{\partial d}{\partial t} + \frac{\partial \alpha d^m}{\partial x} = p_i(t) - f_i(t) \qquad (2)$$

where d is the local flow depth (m) (perhaps better described as the average depth per unit area over some appropriate area), t is time, x is distance along the slope (m), $p_i(t)$ is the time-varying rainfall rate (ms^{-1}), $f_i(t)$ is the infiltration rate (ms^{-1}), and α and m are parameters. For the Manning resistance law, m is 5/3, and $\alpha = S^{1/2}/n$ where S is the slope of the plane, and n is the Manning coefficient.

Infiltration is described by the Smith-Parlange (1978) model, which is based upon an approximation to the Richards equation describing one-dimensional unsaturated flow in a porous medium. The Smith-Parlange model states that at the time of ponding, t_p:

$$\int_0^{t_p} p_i(t)dt = (A/K_s')\ln\{p_i(t_p)/[p_i(t_p) - K_s']\} = F_p \qquad (3)$$

where K_s' is the effective saturated hydraulic conductivity under imbibition, A is a parameter related to the sorptivity, and F_p is the volume of precipitation that has occurred from the time the rain started (t = 0) until ponding.

After ponding, $p_i(t)$, in equation (3), can be replaced with $f_i(t)$, and the following implicit relationship between the accumulated infiltration, F and time can be derived (Smith & Parlange, 1978).

$$K_s'(t - t_p) = F - F_p + A_p \exp(-F/A_p) - A_p\left(p_i(t_p) - K_s'\right)/p_i(t_p) \quad (4)$$

where $A_p = A/K_s'$.

The parameter A can be approximated from the soil sorptivity S:

$$A \simeq S^2/2 \quad (5)$$

Simulations in this study were performed with the distributed, event-oriented model KINEROS (Smith, 1981). In this model, watersheds can be represented as a cascade of planes and channels. Surface runoff is generated only by the "Hortonian" mechanism, i.e. runoff occurs when the rainfall intensity exceeds the infiltration rate. A first-order implicit finite-difference scheme is used to solve equation (2), and this difference scheme is coupled at each node with a finite-difference algorithm to solve the Smith-Parlange infiltration model. The infiltration and runoff components are interactive, so infiltration can occur when the rainfall rate is zero, provided the depth of water is greater than zero. The initial water content can be specified, but there is no evapotranspiration component, so only individual events can be considered.

SENSITIVITY ANALYSES

A simulation approach was used to analyze the sensitivity of peak storm-runoff rates and runoff volumes to rainfall-sampling frequency. Comparisons were made between the empirical distribution functions of peak-runoff rates calculated using the most accurate "breakpoint" precipitation data and precipitation data obtained from the breakpoint data by sampling at intervals of the Δt_j; $j=1, \ldots, n$.

These comparisons can be made only without an inordinate number of simulations by devising dimensionless parameters that relate the basin runoff response and infiltration characteristics to certain rainfall characteristics. The following two dimensionless parameters were used by Woolhiser & Osborn (1985b): a characteristic time ratio, T_*, and an infiltration ratio $F_p{*}$. T_* is defined as the kinematic time to equilibrium for a plane at a rainfall excess rate equal to the mean intensity of a set of m storms divided by the mean storm duration, $\overline{D}$.

$$T_* = t_e/\overline{D} \quad (6)$$

where

$$t_e = \left(\frac{nL}{S^{1/2}\bar{q}^{2/3}}\right)^{3/5} \quad (7)$$

where $\bar{q} = \frac{1}{m}\sum_{i=1}^{m} P_i/D_i$. P_i and D_i are the total precipitation and the duration of the i^{th} storm, respectively.

The infiltration ratio F_p^* is defined as the ratio of the infiltrated depth at ponding F_p to the mean storm depth.

$$F_p^* = \frac{F_p}{\bar{P}} = \frac{F_p}{\frac{1}{m}\sum_{i=1}^{m} P_i} \qquad (8)$$

where F_p is obtained from equation (3), with the rainfall rate assumed to be constant at the mean storm intensity $\bar{q}$.

The rainfall events used in these studies were the 30 largest storms for Walnut Gulch Experimental Watershed gage 5 in the geographically centered data set used by Woolhiser & Osborn (1985a). These storms were truncated by omitting intensities smaller than 3 mm per hour at the beginning or ending. This truncation resulted in a reduction of storm depth by less than 5%. Some statistical characteristics of these storms are given in Table 1.

TABLE 1 Statistical characteristics of rain events

	Depth (mm)	Duration (min)	Intensity (mm/hr)
Mean	25.9	47.27	35.51
Standard deviation	13.3	20.70	
Median	22.1	44.00	

Basin and infiltration characteristics were chosen to cover a substantial range of response time and infiltration characteristics. The time to equilibrium, t_e, varied from 1.63 minutes (ensuring that the maximum peak-runoff rate from an impervious surface is the same as the maximum intensity within each storm) to 81.4 minutes. Infiltration ranged from zero (impervious plane) to that consistent with a sandy loam soil. Infiltration parameter values were obtained from the table presented by Rawls et al. (1982) using the approximations

$$A_p \approx \frac{(2 + 3\lambda)}{(1 + 3\lambda)} \frac{P_b}{2} (S_{max} - S_i) \qquad (9)$$

and

$$K_s' = K_s/2 \qquad (10)$$

where λ is the pore size distribution index, P_b is the bubbling pressure, S_{max} and S_i are the maximum soil water content under imbibition and the initial soil water content, respectively, and K_s is the saturated hydraulic conductivity. See Smith & Parlange (1978) and Brakensiek (1977) for explanations of these approximations.

S_{max} was taken as 0.9, and S_i was set at 0 for all except one simulation.

The methodology employed in this study can be described by the schematic diagram in Fig.1.

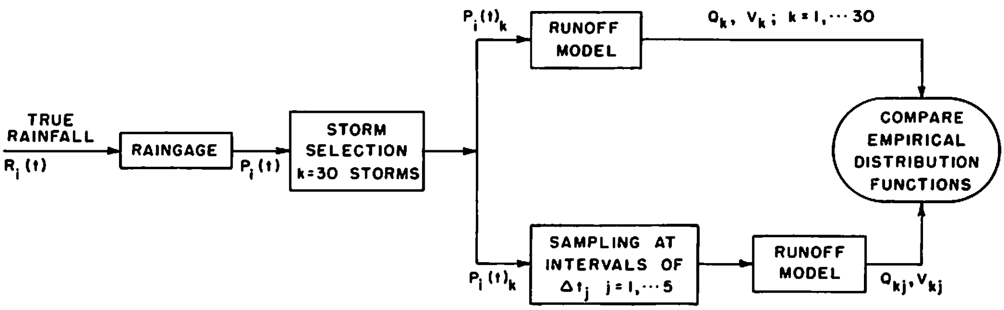


FIG.1 Schematic diagram of methodology.

The true rainfall intensity process at a point, $R_i(t)$, is measured in a weighing recording rain gage, and the accumulated depths at unequal time intervals are determined from the analog charts and digitized, resulting in the "breakpoint" data $p_i(t)$. Rain gage number 5 at the Walnut Gulch Experimental Watershed in southeastern Arizona, U.S.A.), is a dual-traverse, weighing, recording gage with a depth resolution of approximately 0.15 mm ± .30 mm and a time resolution of 2.5 min ± 5 min. Although it is known that there may be significant bias in the water caught in the rain gage due to wind effects, that factor is not considered here. The 30 largest storms that occurred in a 23-year period of record were used in this study. Each of the storms, as represented by the breakpoint data $p_i(t)_k$, were used as rainfall input to the simulation model, and for each event, k, the volume of runoff V_k, the peak rate of runoff Q_k, and the time to peak, t_{Qk}, were recorded.

The same 30 storms were then sampled at intervals Δt_j, j=1, 2...5 where Δt_1 = 5 min, Δt_2 = 10 min, Δt_3 = 15 min, Δt_4 = 20 min, and Δt_5 = 30 minutes. The sampling-interval grid was placed randomly over the breakpoint data by letting the starting time of the breakpoint record occur as a uniformly distributed random variable over the first interval of length Δt_j. Each of these 30 storms with each sampling interval was used as input to the runoff model. Note that the runoff volumes and peak rates obtained by this procedure are not independent, because they were obtained by different transformations of the same breakpoint rainfall series.

RESULTS

The model parameter values and the dimensionless numbers T_* and $F_p{}^*$ for all cases investigated are shown in Tables 2 and 3.

Findings from previous studies (Harley et al., 1970, Hjelmfelt, 1981) suggest that it is unnecessary to examine the effect of all five rainfall sampling intervals for all combinations of flow planes and infiltration model parameters. For the impervious surface, for

TABLE 2 Kinematic model parameters

Geometry no.	Length of plane, L (m)	Slope s	Mannings n	t_e (min)	T*
1	30.48	0.10	0.01	1.63	0.0345
2	91.44	0.03	0.10	18.0	0.382
3	152.4	0.005	0.05	27.7	0.585
4	91.44	0.01	0.30	48.6	1.03
5	152.4	0.005	0.30	81.4	1.72

TABLE 3 Infiltration model parameters

Soil texture	λ	P_b (cm)	A_p (cm)	K_s (cm/hr)	K_s' (cm/hr)	F_p (cm)	F_p^*
A Impervious			0	0	0	0	0
B Silty clay	0.127	34.2	29.5	0.10	0.05	0.375	0.145
C Silty clay loam	0.151	32.6	27.4	0.19	0.085	0.663	0.256
D Silt loam	0.211	20.8	16.7	0.68	0.34	1.52	0.586
E Sandy loam	0.322	14.7	11.1	2.18	1.09	3.64	1.406

TABLE 4 Sampling intervals investigated

Infiltration model parameter set	Kinematic model geometry (Table 2)				
	1	2	3	4	5
A (Impervious)	5	5,10,15	5,10,15	10,15,20	15,20,30
B (Silty clay)	5	5,10,15	5,10,15	5,10,15,20	10,15,20,30
C (Silty clay loam)	5	5,10,15	5,10,15	5,10	10,15
D (Silt loam)	5	5,10	5,10		
E (Sandy loam)	5	5	5,10		

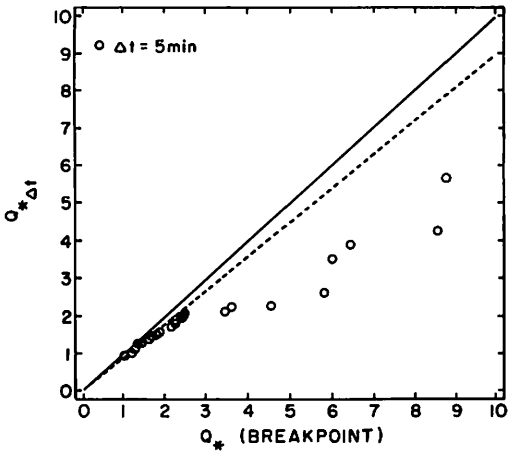
example, runoff volumes will be unaffected by the choice of Δt, and if the response time t_e is greater than the duration of all storms, the peak-runoff rates will be unaffected as well. Furthermore, the response time for geometry 1 is smaller than any of the sampling intervals, so it is apparent that peak-runoff rates will be reduced (in fact, the breakpoint data probably distort the rainfall data significantly for this case). The breakpoint data were used as input for all combinations of F_p* and T$_*$. Sampling intervals were chosen to bracket the Δt's obtained using the criterion recommended by Harley et al. (1970) as given by equation (1). The sampling intervals investigated for each combination of geometric and infiltration parameters are shown in Table 4.

The peak-runoff rates and volumes obtained for the 30 events with rainfall-sampling intervals of Δt_j may be compared with those obtained using breakpoint data by means of the empirical quantile-quantile plot (see Kolata, 1985). If the points on the quantile-quantile plot lie along the line y = x, then the two populations are identical. If they depart from the line in any systematic manner, additional information is obtained. The effect of the dimensionless time parameter T_* on the required Δt is shown in Fig.2, where quantile-quantile plots of the dimensionless peak discharges are shown for geometries 1, 3, and 5 for the impervious case. (Dimensionless peak discharges, Q_*, were obtained by dividing the peak discharges, Q, by the equilibrium steady-state discharges for each geometry with zero infiltration rate and lateral inflow equal to the mean intensity of the 30 storms, 35.51 mm per hour). Also shown on these plots are the lines y = x and y = 0.9x. Any points of the quantile-quantile diagram that plot below the second line indicate a difference of 10% or greater for that quantile. These plots show clearly the approximate validity of the criterion given in equation (1). In dimensionless terms, Δt_*, crit = Δt crit/$\bar{D}$ = $\frac{Tc}{3.2\bar{D}}$ = $T_*/3.2$. A Δt of 5 minutes is greater than the allowable Δt for geometry 1 (T_* = 0.0345), but is slightly smaller than the allowable Δt of 5.62 minutes for geometry 3. For geometry 5 (T_* = 1.72), the allowable Δt is 25.4 minutes, and the maximum deviation is less than 10% at Δt of 20 minutes.

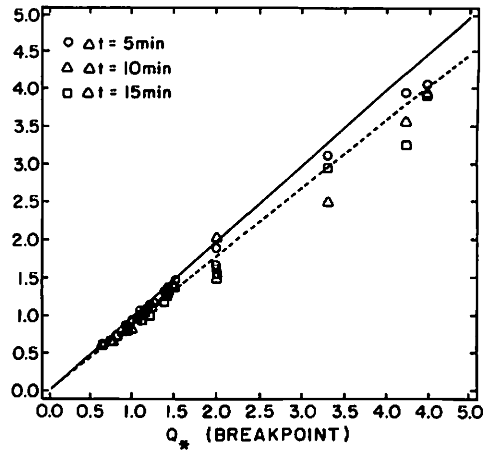
The effect of infiltration on the required Δt is shown in Fig.3, where quantile-quantile plots of peak rate are shown for infiltration parameter sets B, C, D, and E for geometry 3. From this figure, we see that the quantiles for increments of 10 and 15 minutes are more than 10% smaller than the quantiles for breakpoint input, while those for a Δt of 5 minutes are less than 10% smaller for the largest four storms for silty-clay and silty-clay-loam soils. For the sandy-loam soil, F_p* = 1.406, which suggests that, on the average, runoff is not expected for dry initial conditions. From Fig.3d, we see that, in fact, runoff is zero for all but six storms, and for these storms, only one point lies within the 10% error band for a Δt of 5 minutes. For the silt loam and sandy-loam soils, the 5-minute sampling interval appears to be too large, even though it is considerably smaller than the limit suggested by equation (1).

The percentage deviation from the line y = x is greatest for the smaller storms. Indeed, the deviation for the largest storm is less than 10% for a Δt of 10 minutes for most cases, indicating that the peak rate is governed primarily by the saturated hydraulic conductivity.

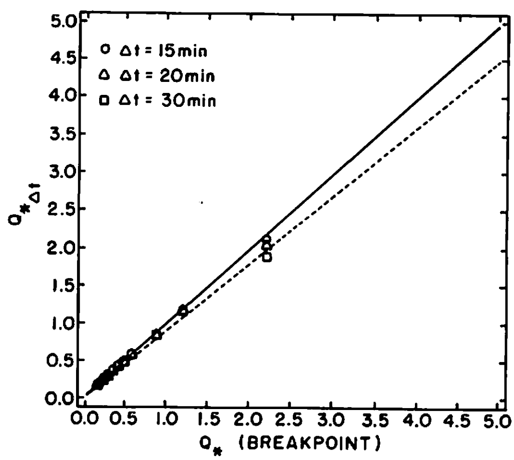
Similar results were found for geometry 4. For geometry 5, however, a sampling interval of 10 minutes gives errors smaller than 10% for the silty-clay and silty-clay-loam soils. According to the criterion of Harley et al. (1970), the sampling intervals for geometries 4 and 5 would be 15.2 minutes and 25.4 minutes,



(a) Geometry 1, impervious
(T_* = 0.0345, $F_p{}^*$ = 0.0).



(b) Geometry 3, impervious
(T_* = 0.585, $F_p{}^*$ = 0.0).

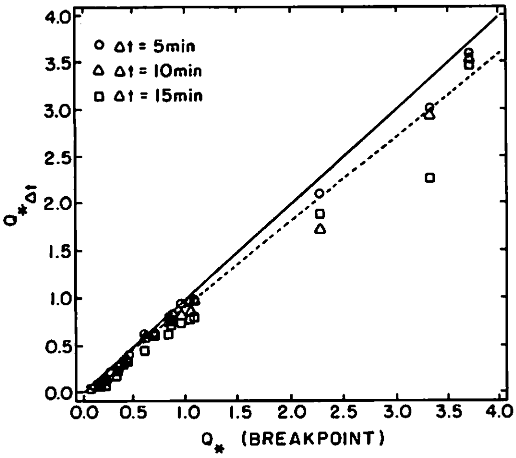


(c) Geometry 5, impervious
(T_* = 1.72, $F_p{}^*$ = 0.0).

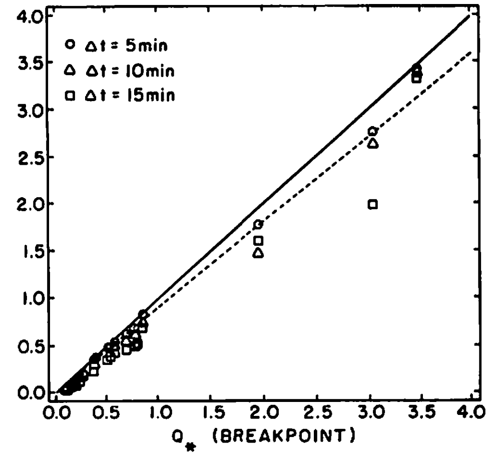
FIG.2 Quantile-quantile plots of dimensionless peak discharge rates.

respectively.

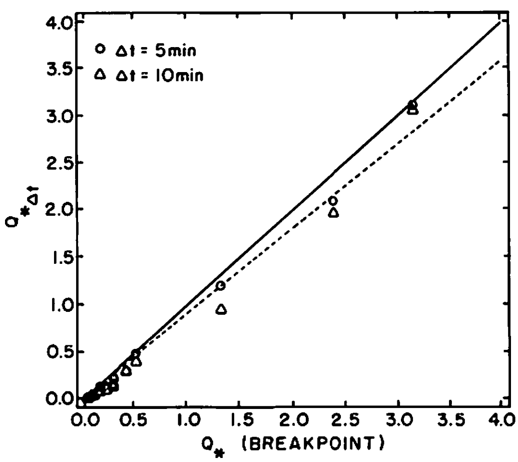
The effect of initial water content can be seen by comparing Fig.3c with Fig.4, which is the quantile-quantile plot for the same soil and geometry, but with an initial water content of 0.5 rather than zero. Peak rates for all storms are increased, and the peak-discharge rates for Δt = 5 minutes are now within 10% of those for the breakpoint data for the largest of four storms. Therefore, more stringent sampling intervals (smaller Δt) are required for small storms or initially dry conditions when the suction term has a major effect on runoff volumes.



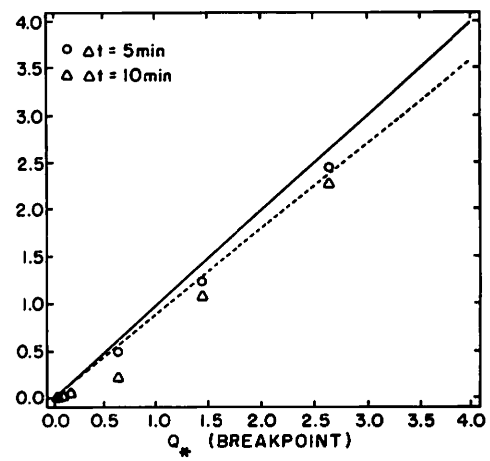
(a) Geometry 3, silty clay soil
($T_* = 0.585$, $F_p{}^* = 0.145$).



(b) Geometry 3, silty clay loam soil
($T_* = 0.585$, $F_p{}^* = 0.256$).



(c) Geometry 3, silty loam soil
($T_* = 0.585$, $F_p{}^* = 0.586$).



(d) Geometry 3, sandy loam soil
($T_* = 0.585$, $F_p{}^* = 1.406$).

FIG.3 Quantile-quantile plots of dimensionless peak discharge rates.

DISCUSSION AND CONCLUSIONS

The appropriate rainfall-sampling interval depends upon the within-storm variability of the rainfall intensity, the basin's hydraulic-response characteristics and infiltration dynamics. Rainfall-sampling-interval criteria, developed by Eagleson & Shack (1966) and Harley et al. (1970), were based upon the assumption that the basins were impervious, and more properly should be referred to as rainfall-excess-sampling criteria. The simulations performed in this study verified that the criterion of Harley et al. (1970) is appropriate

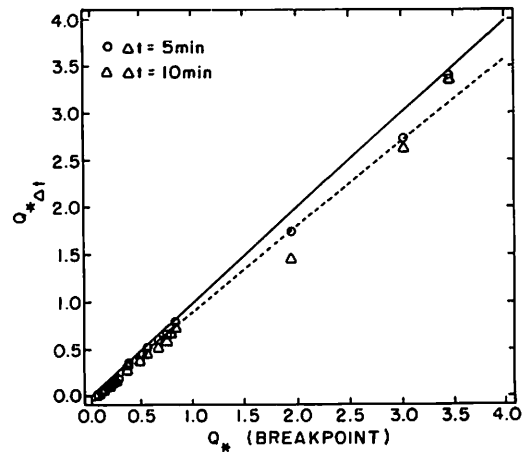


FIG.4 Quantile-quantile plot of dimensionless peak discharge rates. Geometry 3, silt loam soil, s_i = 0.5 ($T*$ = 0.585, F_p* = 0.260).

for impervious basins but is not adequate if infiltration is calculated using an interactive infiltration model. A fixed-sampling interval of 5 minutes appears to be acceptable for soils with textural characteristics of silty clay or silty-clay loam if the basin time to equilibrium is greater than about 16 minutes. For lighter textured soils and basins with shorter times to equilibrium, a smaller sampling interval is required. These conclusions are, of course, based upon basins with the Hortonian runoff-generation mechanism. If a substantial proportion of the runoff is generated by rain falling on saturated soils or water bodies, infiltration effects are less important, but the hydraulic-response criterion still holds.

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REFERENCES

Brakensiek, D.L. (1977) Estimating the effective capillary pressure in the Green and Ampt infiltration equation. Wat. Resour. Res. 13(3), 680-682.

Bras, R.L. (1979) Sampling of interrelated random fields: the rainfall-runoff case. Wat. Resour. Res. 15(6), 1767-1780.

Eagleson, P.S. & Shack, W.J. (1966) Some criteria for the measurement of rainfall and runoff. Wat. Resour. Res. 2(3), 427-436.

Harley, B.M., Perkins, F.E. & Eagleson, P.S. (1970) A modular distributed model of catchment dynamics. Report no. 133, Ralph M. Parsons Laboratory, Dept. of Civil Engineering, M.I.T., Cambridge, Massachusetts, U.S.A.

Hjelmfelt, A.T., Jr. (1981) Overland flow from time-distributed rainfall. J. Hydraul. Div. ASCE 107(HY2), 81-95.

Kolata, G. (1984) The proper display of data. Science 226(4671), 156-157.
Rawls, W.J., Brakensiek, D.L. & Saxton, K.E. (1982) Estimation of soil water properties. Trans. Am. Soc. Agric. Engrs. 25 SW, 1316-1320, 1328.
Smith, R.E. (1981) A kinematic model for surface mine sediment yield. Trans. Am. Soc. Agric. Engrs. 24 SW, 1508-1514.
Smith, R.E. & Parlange, J.Y. (1978) A parameter efficient hydrologic infiltration model. Wat. Resour. Res. 14(3), 533-538.
Woolhiser, D.A. & Osborn, H.B. (1985a) A stochastic model of dimensionless thunderstorm rainfall. Wat. Resour. Res. 21(4), 511-522.
Woolhiser, D.A. & Osborn, H.B. (1985b) Point storm disaggregation-seasonal and regional effects. Paper presented at 4th International Hydrology Symposium, Ft. Collins, Colorado, U.S.A., July.