MULTIVARIATE ANALYSIS OF HYDROLOGIC PROCESSES

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POINT STORM DISAGGREGATION - SEASONAL AND REGIONAL EFFECTS

By

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ABSTRACT

The variation of parameters of a stochastic dimensionless model of rainfall due to seasonal and regional climatic differences is examined. For the model under examination, the accumulated storm precipitation process is nondimensionalized by dividing the precipitation at any time by the total storm precipitation, and the elapsed time by the total duration. The process is divided into 10 equal dimensionless time increments, and the depth increments are rescaled to range between 0 and 1 by dividing each increment by the fraction of the precipitation that occurred between the beginning of that time period and the end of the storm.

Regional variations of model parameters were examined by identifying the parameters for data from Walnut Gulch Experimental Watershed near Tombstone, AZ, and Alamogordo Creek Experimental Watershed near Santa Rosa, NM for the summer season. Seasonal differences were detected by also identifying model parameters for winter rainfall at Walnut Gulch Experimental Watershed. The effects of storm amounts and durations were also examined by dividing each data set into four subsamples based on storm duration and amount.

The short duration summer storms from Alamogordo Creek, NM were similar to the summer thunderstorms at Walnut Gulch, AZ, while the long-duration storms were similar to the winter storms at Walnut Gulch.

Simulation techniques and a deterministic rainfall-runoff model were used to examine the sensitivity of derived variables such as storm runoff volumes and peak runoff rates to variations in rainfall disaggregation model parameters due to season and location. The Kolmogorov-Smirnov statistic, D*, is used as an index of goodness of fit between empirical distributions of peak rate simulated using observed rainfall data and those obtained using rainfall input simulated with the disaggregation model and two simpler approximations.

INTRODUCTION

The adoption of physically based infiltration models as components of watershed models has been hampered because of the requirement for high temporal resolution rainfall data. Recording raingage data can be readily used in such models if they are available for the location under consideration, and are in computer-compatible form, i.e., on magnetic tapes or disks. However, such data are relatively scarce, and even when they are available in computer-compatible form, they are cumbersome to work with.

Daily rainfall data, on the other hand, are plentiful, and recent work promises that very good simulation models for daily rainfall will soon be readily available for mainframe computers (Richardson and Wright, 1984) and for microcomputers (Woolhiser, Hanson, and Richardson, 1985).

It has been suggested (Woolhiser and Osborn, 1985) that if parameter-efficient techniques can be developed to disaggregate daily rainfall into the intermittent rainfall process within the day, and further disaggregate significant storms into short-period rainfall intensities, and if the disaggregation model structure and parameter values are (relatively) spatially invariant, simulated short-period rainfall data could be provided as input to physically-based runoff models.

A pilot investigation of the daily disaggregation problem has been completed with promising results (Hershenhorn, 1984). Woolhiser and Osborn (1985) proposed a multivariate model for dimensionless thunderstorm rainfall to disaggregate individual storms into short-period rainfall intensities.

The criteria for determining if there are significant seasonal and spatial differences between disaggregation model structures and parameter values are not readily apparent. Woolhiser and Osborn (1985) used the likelihood ratio test to test the hypothesis of common parameter sets for different data subsets based upon storm intensity and duration. This same approach could be used to test hypotheses regarding common parameter sets between seasons or locations. However, in hydrology, we are frequently interested in differences between derived distributions of runoff volume and peak discharge rather than statistical characteristics of rainfall. These derived distributions will be affected by the infiltration characteristics and the runoff dynamics of the watershed under consideration. The importance of the rainfall time distribution on runoff from small catchments has been recognized for a long time, and several investigators have developed design hyetographs for engineering purposes (see Yen and Chow, 1980; Pilgrim and Cordery, 1975, for examples and for brief reviews of other published work).

Several investigators have examined the sensitivity of overland flow hydrographs to rainfall hyetographs. Hjelmfelt

(1981) compared dimensionless peak rates of overland flow for constant rainfall intensity and for an average thunderstorm rainfall hyetograph published by the United States Weather Bureau (1947). He concluded that the constant intensity approximation is valid for rainfall durations approximately equal to, or less then, the kinematic time to equilibrium at the average constant intensity.

Engman and Hershfield (1981) pointed out the effect of rainfall intensity pattern on the infiltration process, and demonstrated the differences that could occur in calculated runoff if clock hour rainfall was used, rather than 5-min rainfall amounts, as input to a kinematic model with a parametric infiltration model. They also developed a triangular approximation of the intensity distribution within storms of one hour or less duration.

The objective of this paper is to examine the seasonal and spatial variability of parameters for the model proposed by Woolhiser and Osborn (1985), and to investigate the sensitivity of derived distributions of storm runoff volumes and peak rates to variations in storm rainfall disaggregation model parameters.

THE MULTIVARIATE RAINFALL INTENSITY MODEL

The multivariate rainfall intensity model, proposed by Woolhiser and Osborn (1985), can be specified by the joint density function

$$f(z_1, z_2, \cdots z_9) = f_1(z_1) \prod_{k=2}^{9} g_{k-1,k}(z_k|z_{k-1}) \qquad (1)$$

where $z_1, z_2, \cdots z_9$ are dimensionless rescaled increments for the ν^{th} storm defined by the following expression:

$$z_\nu(t_*) = \frac{\overline{U}_\nu^*(t_*) - U_\nu(t_*-1/m)}{1 - \overline{U}_\nu^*(t_* - 1/m)} \quad ; \quad 0 \leq z_\nu(t_*) \leq 1 \qquad (2)$$

where $t_* = k/m$; $k = 1, 2, \cdots m$, and $m = 10$. $f_1(z_1)$ is the marginal density function of the first dimensionless increment, and $g_{k-1,k}(z_k|z_{k-1})$ is the conditional density function of the k^{th} increment given the $k\text{-}1^{st}$ increment. The dimensionless accumulated rainfall process $\overline{U}_\nu^*(t_*)$ is obtained by normalizing the rainfall depth by dividing by the total storm rainfall and normalizing the time by subtracting the time of beginning of the storm and dividing by the storm duration. A definition sketch of the process $\overline{U}^*(t_*)$ is shown in Fig. 1.

Woolhiser and Osborn (1985) analyzed data for storms greater than 6.4 mm (0.25 inches) that occurred on the Walnut

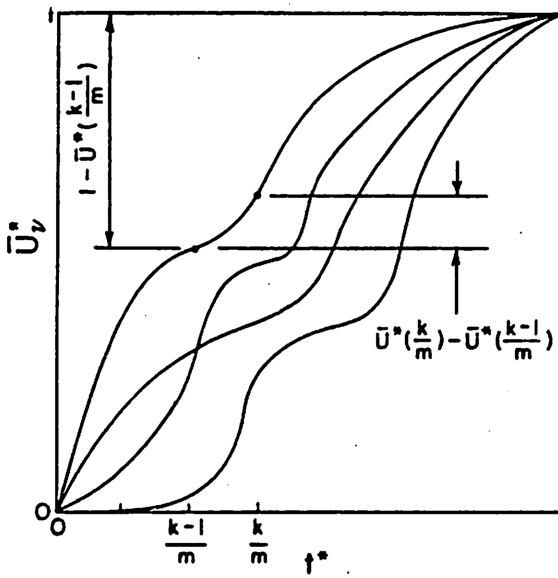


Figure 1. Definition sketch of dimensionless rainfall process $\overline{U}^*(t_*)$

Gulch Watershed, in southeastern Arizona, during the summer thunderstorm season from July through September. They found that the marginal distribution of increment z(0.1) could be described by a beta distribution, and that the conditional distributions $g_{k,k-1}(Z_k|Z_{k-1})$; k = 2, 3 • • • 9 could also be dedescribed by beta distributions with expected value function:

$$E\{Z(\frac{k}{m})|Z(\frac{k-1}{m}) = z_{k-1}\} = a_k + b_k z_{k-1} \; ; \; k = 2, 3, \cdots 9 \quad (3)$$

Z(1.0) is, by definition, equal to 1.

The conditional distributions $g_{k,k-1}(z_k|z_{k-1})$ are completely specified by the parameters a_k,. b_k, and a third parameter, α_k, which is related to the dispersion of the conditional distribution. In general, the joint distribution of equation (1) requires 26 parameters, i.e., a parameter set $\underline{\theta} = (\alpha_1, \beta_1, a_2, b_2, \alpha_2, \cdots a_9, b_9, \alpha_9)$. However, for the Walnut Gulch data, it was found that this number could be reduced by describing the parameters as functions of the index k:

$$\begin{aligned} a_k &= B_1 + B_2 k + B_3 k^2 \\ b_k &= B_4 + B_5 k + B_6 k^2 \\ \alpha_k &= B_7 + B_8 k + B_9 k^2 \end{aligned} \quad (4)$$

where k = 2, 3, • • • 9.

The parameter set $\underline{\theta}$, required for the distribution of z_1

and eq. (4), i.e., (α_1, β_1, B_1, B_2, • • .• B_9), was estimated by numerical maximum likelihood techniques.

For the Walnut Gulch data, Woolhiser and Osborn (1985) found that the parameter B_3 was zero, and that α was dependent on storm duration, and recommended a 13-parameter model, with the parameters B_6, B_7, and B_8, in equation (4), taking on different values, depending on whether or not the storm duration was greater or less than the median storm duration of 35 minutes.

SEASONAL AND REGIONAL VARIABILITY OF RAINFALL MODEL PARAMETERS

Parameter values were estimated for the previously described storm disaggregation model using winter rainfall data from the Walnut Gulch Experimental Watershed in southeastern Arizona, and data for 345 summer thunderstorms obtained at the Alamogordo Creek Experimental Watershed near Santa Rosa, New Mexico.

Statistical characteristics of these storms are compared with the Walnut Gulch summer thunderstorms in Table 1, and the

Table 1. Rainfall statistics.

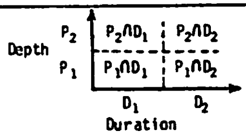
Location	Duration (minutes)			Depth (mm)		
	mean	std. dev.	median	mean	std. dev.	median
Walnut Gulch, AZ (summer)	39.78	21.94	35	17.12	10.44	13.34
Walnut Gulch, AZ (winter)	451.47	320.10	394	11.42	6.03	10.03
Alamogordo Cr., NM (summer)	78.80	64.10	59	16.90	12.10	12.70
Walnut Gulch, AZ (summer) 30 largest storms	47.27	20.70	44	25.90	13.30	22.10

disaggregation model parameters are shown in Table 2. The winter storms for Walnut Gulch were not divided into classes based upon amount and duration; consequently, the parameters identified represent all storms. The summer storms at Alamogordo Creek, NM were subdivided by amount and class based upon the medians. Likelihood ratio tests showed that there were significant differences in the parameter values among the 4 subsets; consequently, 4 parameter sets are shown for Alamogordo Creek. Parameters for the short duration, high intensity subset $P_2 \cap D_1$ were similar to the Walnut Gulch summer parameters, and were used in this study. The parameters for long duration, high intensity set $P_2 \cap D_2$, at Alamogordo Creek, were most similar to the winter Walnut Gulch parameter set. An inspection of Table 1 reveals that the mean storm durations, for both winter storms at Walnut Gulch and summer thunderstorms at Alamogordo Creek, are considerably greater than the duration of summer thunderstorm rainfall at Walnut Gulch, and the mean winter storm depths are lower.

Table 2. Disaggregation model parameters

	Location							
	Walnut Gulch (summer)			Walnut Gulch (winter)	Alamogordo Cr (summer)			
Parameter	D_1*	$D_1 \cup D_2$	D_2	$D_1 \cup D_2$	$P_1 \cap D_1$	$P_2 \cap D_1$**	$P_1 \cap D_2$	$P_2 \cap D_2$
α_1		2.058		1.458	2.159	2.431	2.167	1.303
β_1		12.981		7.612	13.00	14.997	11.781	6.212
B_1		-.028		-0.00981	-0.0022	-0.00037	0.00328	-0.00467
B_2		0.343		0.270	0.106	0.258	0.085	0.213
B_3		0		0	0.217	0.018	0.207	0.118
B_4		1.538		0.5257	1.477	1.461	1.509	1.358
B_5		-3.421		0	-2.967	-3.033	-3.308	-3.114
B_6		2.607		0	2.194	2.316	2.710	2.475
B_7	7.139		5.30	1.049	3.928	5.100	4.212	2.308
B_8	-20.179		-10.75	2.524	-8.288	-8.104	-12.300	-5.916
B_9	30.954		13.912	0	12.385	8.882	19.553	11.393

*Key



**Used in simulation

SENSITIVITY OF DISTRIBUTIONS OF PEAK RUNOFF RATES AND VOLUMES TO RAINFALL INTENSITY PATTERNS

The stochastic characteristics of rainfall simulated with a disaggregation model can be compared directly with similar characteristics of real data by standard statistical methods. Such comparisons, however, are often not very useful, because we are usually not interested in the rainfall statistics, per se, but rather are interested in derived distributions of peak rate and volume of runoff from a watershed. The response time and infiltration characteristics of the watershed affect the damping of the rainfall excess input and the volume of runoff. An alternative way to study the sensitivity of distributions of peak rates and volumes to the disaggregation model structure and parameters without an inordinate number of simulations is to devise dimensionless numbers which relate the basin runoff response and infiltration characteristics to certain rainfall characteristics. For the response of a single plane to complex rainfall intensity patterns, we propose the following two dimensionless ratios: a characteristic time ratio, T_* and an infiltration ratio, F_{p*}. We will define T_* as the the kinematic time to equilibrium on a plane at a rainfall excess rate equal to the mean intensity of a set of m storms divided by the mean storm duration, $\overline{D}$.

$$T_* = \frac{t_e}{\overline{D}} \tag{5}$$

where

$$t_e = \left(\frac{n\, L_o}{s^{1/2}\ 1.49\, \overline{q}^{2/3}}\right)^{3/5} , \tag{6}$$

n = Manning's roughness coefficient, L_o = length of plane, s = slope of plane, and $\overline{q} = \frac{1}{m} \sum_{i=1}^{m} \frac{P_i}{D_i}$. P_i and D_i are the total precipitation and duration of the ith storm, respectively. Also let

$$F_p* = \frac{F_p}{\frac{1}{m}\sum_{i=1}^{m} P_i} = \frac{F_p}{\bar{P}} \quad (7)$$

where F_p is the total rainfall infiltrated at ponding if the rainfall rate is $\bar{q}$. F_p can be calculated from the expression given by Smith and Parlange (1978):

$$F_p = A_p \ln\left(\frac{\bar{q}}{\bar{q}-K_s'}\right) \quad (8)$$

where A_p is a parameter related to sorptivity, and K_s' is the saturated hydraulic conductivity of the soil under imbibition.

The storms used in these simulation studies were the 30 largest storms (in terms of depth) for gage 5 in the geographically-centered data set used by Woolhiser and Osborn (1985). Some statistical characteristics of these storms are shown in Table 1.

Runoff peaks and volumes were simulated using the distributed, event-oriented model KINEROS (Smith, 1981). In this model, watersheds can be represented as a cascade of planes and channels. Surface runoff is generated only by the "Hortonian" mechanism, i.e., runoff occurs when the rainfall intensity exceeds the infiltration rate. A first order implicit finite difference kinematic model is used to route the runoff, and infiltration is described by the Smith-Parlange (1978) model. The infiltration and runoff components are interactive, so infiltration can occur when the rainfall rate is zero, provided the depth of water is greater than zero. The model has no component for evapotranspiration, so can consider only single events, although the initial soil water content may be specified by the user.

Watershed and infiltration characteristics used in simulation runs were chosen to obtain a substantial range of response time and infiltration characteristics. The time to equilibrium, t_e, varied from 1.63 minutes (ensuring that the maximum peak runoff rate with an impervious surface is the same as the maximum storm rainfall intensity) to 27.67 minutes. Infiltration ranged from zero (impervious plane) to that consistent with a sandy-loam soil. Infiltration parameter values were obtained from the table presented by Rawls et al. (1982) using the approximations

$$A_p \approx \frac{(2 + 3\lambda)}{(1 + 3\lambda)} \frac{P_b}{2}(S_{max} - S_i) \quad (9)$$

and

$$k_s' = K_s/2 \quad (10)$$

where λ is the pore size distribution index, P_b is the bubbling pressure, S_{max} and S_i are the maximum soil water content under imbibition and the initial water content, respectively, and K_s is the saturated hydraulic conductivity. See Smith and Parlange (1978) and Brakensiek (1977) for explanations of these approximations. The model parameters and dimensionless numbers T_* and F_{p*} are shown in Table 3. All planes were considered to have a width of one unit.

Table 3. Model parameters for infiltrating plane

Case No.	Length of plane (m)	Slope	Manning's n	t_e (min)	T_*	A_p (cm)	K_s' (cm/hr)	F_p (cm)	F_{p*}
1.	30.48	0.10	0.01	1.63	.0345	0	0	0	0
2.	152.4	0.005	0.05	27.67	.585	0	0	0	0
3.	30.48	0.10	0.01	1.63	.0345	27.35	0.035	0.271	0.104
4.	30.48	0.10	0.01	1.63	.0345	14.86	0.648	2.99	1.15
5.	30.48	0.10	0.01	1.63	.0345	7.46	0.340	0.75	0.290
6.	152.4	0.005	0.05	27.67	.585	27.35	0.035	0.271	0.104
7.	152.4	0.005	0.05	27.67	.585	7.46	0.340	0.75	0.290
8.	152.4	0.005	0.05	27.67	.585	14.86	0.648	2.99	1.15
9.	152.4	0.005	0.05	27.67	.585	11.88	0.518	1.87	0.723
10.	22.07	0.08	0.30	11.03	0.233	11.88	0.518	1.87	0.723
11.	22.07	0.08	0.30	11.03	0.233	0	0	0	0

The real "break point" data for each of the 30 storms were used as input to the KINEROS model, the runoff peak rate, Q_p, volume, V, and time to peak, t_p, were tabulated, and empirical distribution functions were calculated to serve as a basis for comparison. The depth and duration of each of these storms was then used to construct 5 additional input sets with the following time distributions:

1) Constant intensity.

2) Intensity distributed as an isosceles triangle.

3) Disaggregation model. Summer Walnut Gulch parameters.

4) Disaggregation model. Winter Walnut Gulch parameters.

5) Disaggregation model. Summer Alamogordo Creek parameters.

An example of the measured hyetograph and intensities obtained by the first 3 methods for a storm of 75.9 mm (2.99 in) in 61 minutes is shown in Fig. 2. The time resolution for the real storm was 2 minutes. Because the model breaks the storm down into 10 increments, the time resolution is 6.1 minutes for the disaggregated storm.

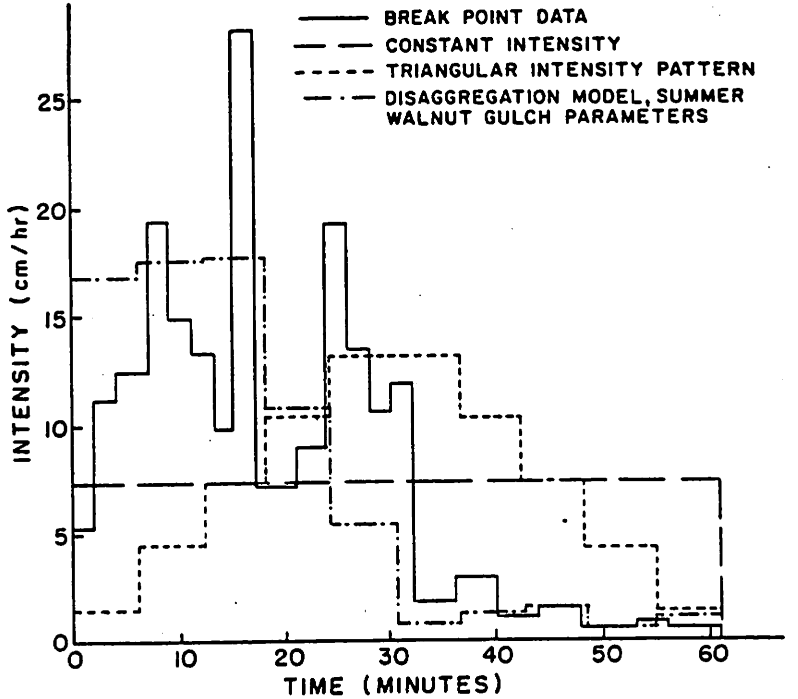


Figure 2. Comparison of hyetographs for largest storm.

Distribution functions of the peak discharge for the 30.48 m (100 ft) and 152.4 m (500 ft) impervious planes are shown in Fig. 3(a) and 3(b). Because the response time of the short plane is less than the rainfall time resolution, the distributions in Fig. 3(a) are those of the peak rainfall rate in each storm. The constant rainfall rate and isosceles triangle distribution clearly are a poor approximation of the peak rainfall rates, while the disaggregation model with the summer parameters for Walnut Gulch and Alamogordo Creek provides the closest fit (Fig. 3a). In fact, using the 2-sample Kolmogorov-Smirnov (K-S) test, we could not reject the null hypothesis that those distributions obtained with the disaggregation model rainfall input came from the same population as that obtained using the real breakpoint data with the probability of the type I error, $\alpha = 0.10$ (the null hypothesis is accepted for the sample obtained using the winter Walnut Gulch parameters at $\alpha = 0.05$!). The profound damping effect of increased time to equilibrium is demonstrated by the distribution functions in Fig. 3(b). The peak discharge rates are considerably reduced, and the distribution functions are much closer together. Only the distribution derived using the constant intensity rainfall as input is rejected using the K-S test. As Hjelmfelt (1981) has

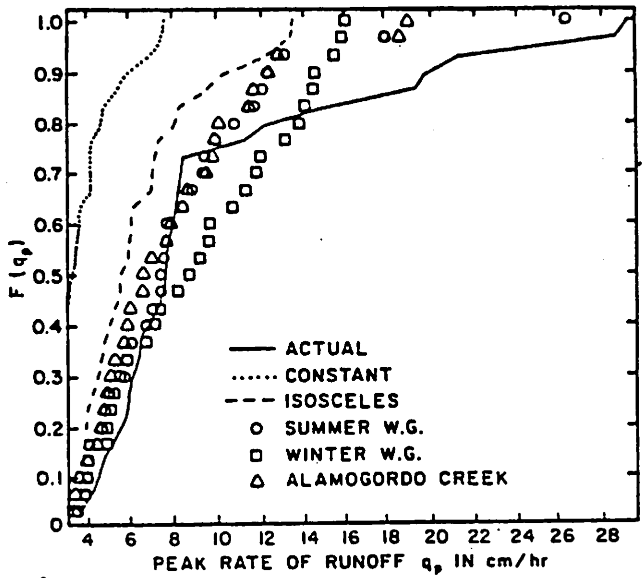


Figure 3a. Empirical distribution functions of peak discharge. $F_{p*} = 0$, $T_* = 0.0345$ (Case No. 1).

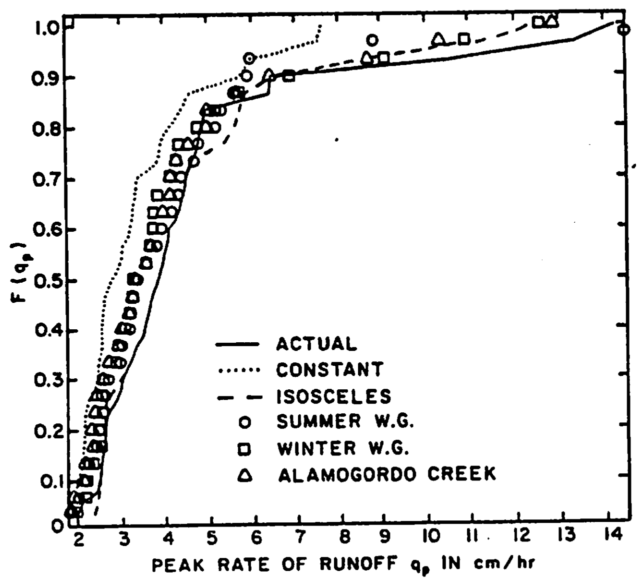


Figure 3b. Empirical distribution functions of peak discharge. $F_{p*} = 0$, $T_* = 0.585$ (Case No. 2).

demonstrated, as T_* gets larger, the distributions will eventually coincide, and the intensity representation becomes irrelevent. However, this conclusion is only true for impervious planes. When infiltration is taken into account, the volume of runoff will depend, to some extent, on the rainfall intensity distribution. For example, for Case 1 (shown in Table 3), the calculated mean runoff volume was 25.94 mm. For the silty clay loam soil (T_* = 0.0345) (Case 5), the calculated mean runoff ranges from 8.81 mm for a constant rainfall intensity, to 10.48 mm for the real breakpoint rainfall. For Case 7 (T_* = 0.585), runoff ranges from 6.94 mm to 8.67 mm, reflecting the greater amount of infiltration that occurs during recession after rainfall has stopped. These reduced volumes will have some effect on the distribution of peak rates, even though T_* is large. For Case 9 (F_{p*}= 0.72, T_* = 0.585), the distribution functions are even closer together than they are for Case 2 (see Fig 3b and 3c), demonstrating that the distribution func-

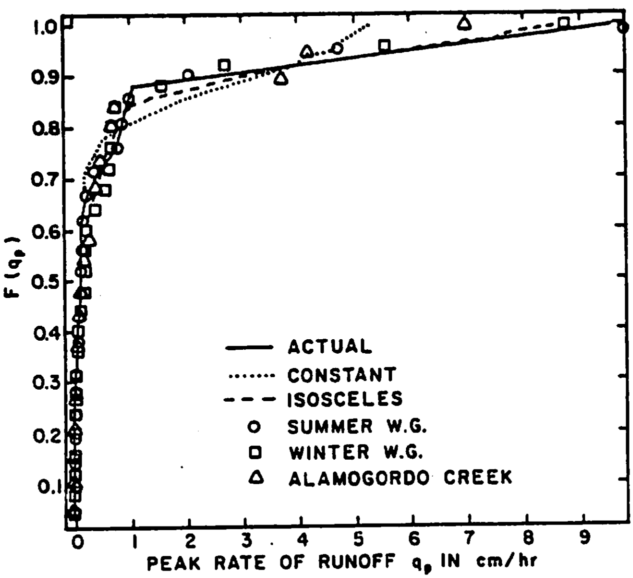


Figure 3c. Empirical distribution functions of peak discharge. F_{p*} = 0.290, T_* = 0.585 (Case No. 9).

tions are less sensitive to rainfall hyetograph shape when infiltration is present, as compared to the impervious case.

To illustrate the effect of infiltration characteristics and runoff response time characteristics on derived distributions of peak runoff rates, the Kolomogorov-Smirnov (K-S) statistic D_* was calculated.

$$D_* = \max \left[|F(q_p) - \tilde{F}_i(q_p)| \right] ; i = 1, 2 \cdots 5 \quad (11)$$

where $F(q_p)$ is the empirical distribution function of peak runoff rate obtained using real breakpoint rainfall data, $\tilde{F}(q_p)$ is the distribution obtained by approximations to the rainfall intensity while maintaining the real duration and total depth, and i refers to one of the 5 methods of hyetograph representation.

D_* was plotted on the F_{p*}, T_* plane for each case simulated and for each hyetograph approximation method, and approximate contours were sketched in. These plots are shown in Figures 4a through 4e. We can see that the distributions of peak

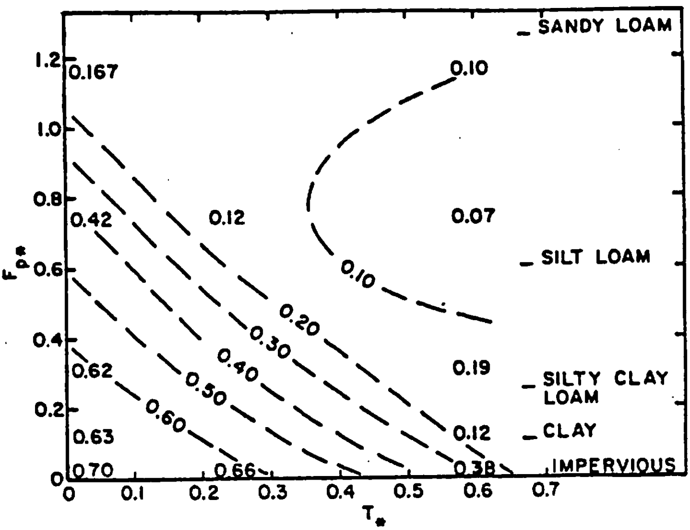


Figure 4a. Kolmogorov-Smirnov D* as a function of F_{p*} and T_* (constant intensity).

runoff obtained when the hyetographs were simulated using the disaggregation schemes provide values of D_* below the critical value (0.34 at α = 0.05), while the distributions obtained using triangular or constant intensity input have substantially larger values of D_* for impervious cases and for triangular regions near the origin. It should be noted that although all values of D_* obtained using disaggregated inputs were smaller than the critical level, this does not imply that there is no real difference between the $F_1(q_p)$ and $\tilde{F}_1(q_p)$. In general, all distributions $F_1(q_p)$ are biased low, especially at small values of T_*. As T_* becomes sufficiently large, all methods will be in perfect agreement at $F_p = 0$. As F_{p*} becomes large, a greater proportion of the storms result in no runoff, and as F_{p*} becomes sufficiently large, no runoff would occur, and all methods would be in perfect agreement.

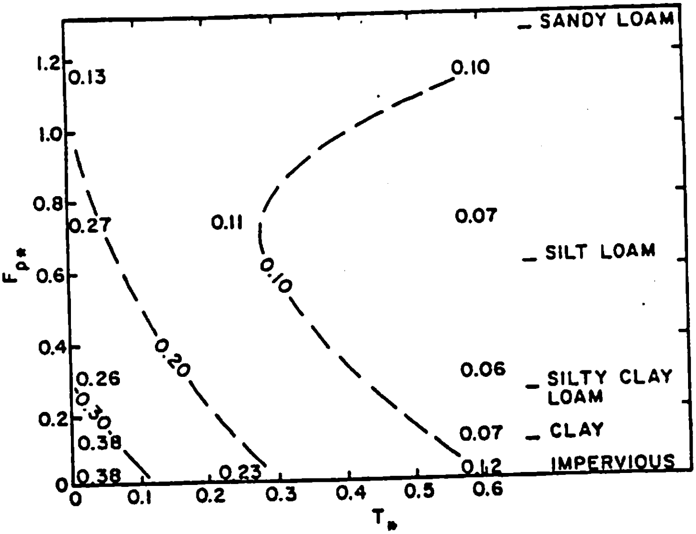


Figure 4b. Kolmogorov-Smirnov D* as a function of F_{p*} and T_* (isosceles triangle).

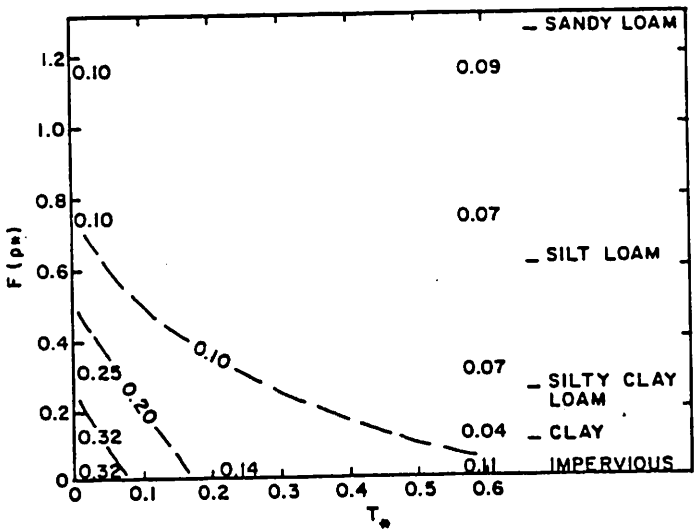


Figure 4c. Kolmogorov-Smirnov D* as a function of F_{p*} and T_* (Walnut Gulch (summer)).

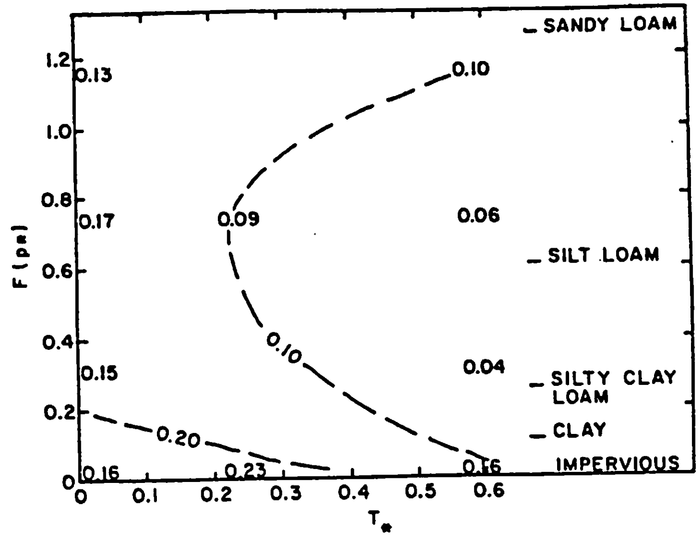


Figure 4d. Kolmogorov-Smirnov D* as a function of F_{p*} and T_* (Walnut Gulch (winter)).

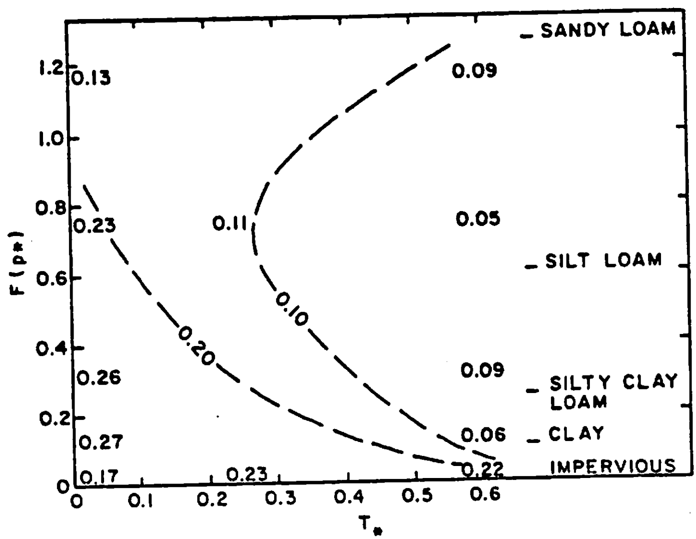


Figure 4e. Kolmogorov-Smirnov D* as a function of F_{p*} and T_* (Alamogordo Creek, NM (summer)).

All calculations were made with an initial water content of zero. If the soil were initially saturated, F_{p*} would go to zero, because the mean rainfall rate is greater than the saturated hydraulic conductivity for all infiltration parameters used in this study. To demonstrate the effect of an initial water content greater than zero, but smaller than saturation, F_{p*}, was calculated for $S_i = 0.5$ for Case 4, Case 5, and Case 3. An initial water content of $S_i = 0.5$ reduces F_{p*} to approximately 0.5 of its value with $S_i = 0$.

CONCLUSIONS

The following conclusions appear to be justified by this study:

1) The Woolhiser-Osborn (1985) disaggregation model provided a better approximation to natural rainfall hyetographs than did a triangular or constant intensity approximation.

2) For values of T_* greater than about 0.5 and $F_{p*} >$ 0.1, runoff is not very sensitive to differences in the parameters in the disaggregation model; thus seasonal and regional effects should be minor, provided the joint statistics of storm amount and duration are maintained.

3) For $T_* > 1.0$, the constant intensity approximation is valid for an impervious plane. Effects of infiltration require further examination.

ACKNOWLEDGEMENTS

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