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DISAGGREGATION OF DAILY RAINFALL

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INTRODUCTION

Simulation models, including components describing the hydrologic cycle, soil erosion, sediment and chemical transport, crop productivity and the effects of various management activities on these processes, have become important tools in management of our land and water resources. The inputs driving these models include rainfall, solar radiation, maximum and minimum daily temperature and, in some cases, wind run. Although many models have options that allow the use of input information for time periods shorter than 1 day, these options are rarely used because of the difficulty in obtaining and using short-period or "break-point" rainfall data. This problem is also a major obstacle to adopting runoff estimation procedures based on infiltration theory (Brakensiek et al. 1981).

One practical way to provide short period rainfall data is to develop a parameter efficient method of disaggregating daily rainfall into individual storms and to disaggregate the rainfall from significant storms into short period intensities. This paper presents a brief progress report on a pilot project designed to assess the feasibility of daily rainfall disaggregation models.

THE PRECIPITATION PROCESS

The rainfall process is defined as the stochastic process $\xi(t)$, $0 < t < \infty$, where $\xi(t)$ is rainfall intensity. A storm period is defined as any time interval when $\xi(t) > 0$. Thus, for any day in which precipitation occurs, we can identify the possible cases shown in figure 1.

We will designate the total rainfall on day j as X_j and the amount of precipitation in the i^{th} complete storm on day j as Y_{ij}. The amount of the i^{th} partial storm on day j is Y_{pij}. The duration of the i^{th} complete and partial storm is D_{ij} or D_{pij}, respectively. The time of ending of the i^{th} complete storm is τ_{ij}. Obviously,

$$\sum_{i=o}^{C_j} \bar{Y}_{ij} + \sum_{i=o}^{N_j} Y_{Pij} = X_j \tag{1}$$

where $Y_{oj} = Y_{poj} = 0$, C_j equals the number of complete storms and N_j is the number of partial storms on day j.

A flow chart of the daily disaggregation process is shown in figure 2. The objective of this research is to devise a procedure to simulate the number, amounts, and durations of complete and partial storms, given only the amount of daily rainfall on day j and day j + 1. Further disaggregation is then used to determine the intensity patterns within significant storms.

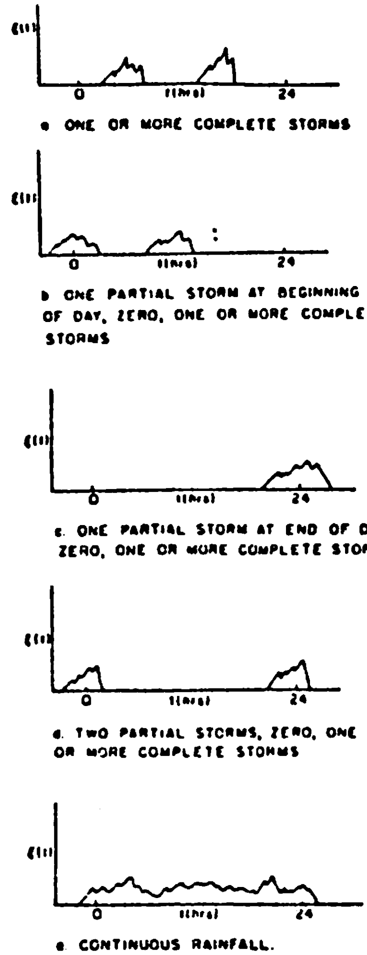

Figure 1. Possible realizations of rainfall process within a day.

ANALYSIS OF DATA

Breakpoint data for raingage 5, at the Walnut Gulch experimental watershed in southeastern Arizona, were used in this pilot project. The record length is 23 years, and only data from the months of July and August were used. We assume that the disaggregation process is stationary during this portion of the summer rainy season. Storms were defined as any period in which rainfall intensity was > 0 or did not equal zero for longer than 10 minutes.

The data were first separated into three files: (1) days with only complete storms, (2) days with one partial storm, and (3) days with two partial storms. Days were broken at midnight. As yet, we have studied only the first data set.

The first step in disaggregation for a day with only complete storms is to generate the number of storms, C_j, given the amount of rainfall X_j(see fig. 2). We found that the geometric probability mass function with a mean dependent on X_j gave a

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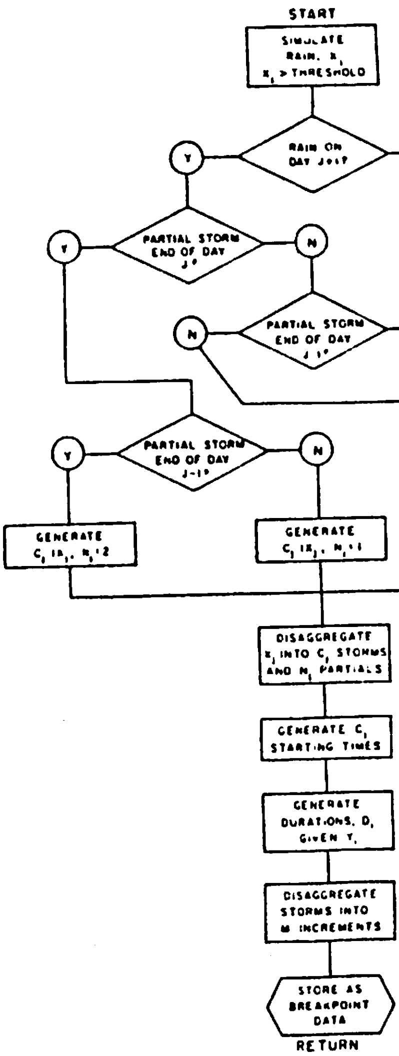

Figure 2. Flow chart of daily rainfall disaggregation.

very good fit to the data. Thus, the joint cumulative probability function can be written as

$$P\{C_j \le n, X_j \le x\} = F\{n,x\} = \sum_{i=1}^{C_j} \int_0^x (a_1 + b_1 x)^{-1}$$

$$[1-(a_1 + b_1 x)^{-1}]^{n-1} \{\frac{\alpha e^{-x/d}}{d} + \frac{(1-\alpha)e^{-x/\theta}}{\theta}\} dx \quad (2)$$

if the marginal distribution of daily rainfall is a mixed exponential with parameter set (α, d, θ) (Woolhiser and Roldan 1982).

Time Distribution of C_j Storms

Empirical distribution functions for the time of storm beginning, $\tau_{ij} - D_{ij}$, were calculated for days with one storm, two storms, three storms, four storms, and for all storms. If we assume that the storm times are independent and identically distributed, the distribution function of the time of the first storm, the time of the second storm, et cetera, can be obtained from the order statistics. For example, if $F(\tau^*) = P\{\tau - D \le \tau_*\}$, and if we consider days with two storms, the distribution function of the first storm is

$$F(\tau_1^*) = 2F(\tau^*) - F(\tau^*)^2 \quad (3)$$

The empirical distribution functions of the times of first and second storms during a day are compared with functions calculated using order statistics in figure 3. It appears that the assumption of independent, identically distributed storm arrival times is acceptable.

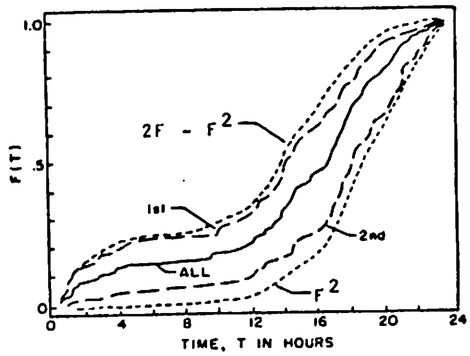

Figure 3. Distribution functions of time to beginning of storms (two storms per day).

Joint Distribution of Storm Duration and Amounts

After individual storm amounts have been obtained by disaggregating the daily total into C_j storms, it is then necessary to assign a duration, D_{ij}, to each storm amount, Y_{ij}. It has been found that D_{ij} and Y_{ij} are jointly dependent and, for 242 storms with $Y \ge 0.25$ inch, are best described by a joint distribution consisting of an exponential marginal distribution of storm amount, Y, and a log-normal conditional distribution of D, given Y. The dependence is specified by the linear relation

$$E\{\ln D \mid Y\} = a_2 + b_2 Y \quad (4)$$

For the marginal distribution of Y, the exponential distribution was superior to the mixed exponential, log normal, and Kappa III distributions. The log-normal conditional distribution was superior to the Weibull disbribution.

Disaggregation of Storm Rainfall

Let us consider the ν^{th} storm in a sequence, beginning at time τ_ν^*, and ending at τ_ν. The total amount of precipitation for this event is

$$Y_\nu = \int_{\tau_\nu^*}^{\tau_\nu} \xi(t)dt \quad (5)$$

and the duration $D_\nu = \tau_\nu - \tau_\nu^*$.

Let us define the dimensionless process

$$Y^*_\nu(t) = \frac{1}{Y_\nu}\int_{\tau^*_\nu}^{t} \xi(s)ds \; ; \; \tau^* \le t \le \tau \quad . \tag{6}$$

If ν is fixed,

$$t^*_\nu = \frac{t - t^*_\nu}{D_\nu} \; ; \; \tau^*_\nu \le t \le \tau_\nu \tag{7}$$

where $0 \le t^*_\nu \le 1$. Thus, the intensity pattern within a storm can be described by the dimensionless stochastic process $\{Y^*_\nu(t^*);\ 0 \le t^* \le 1\}$. Possible realizations of this process are shown in figure 4. Let us consider the discrete time process

$$\{Y^*_\nu(\tfrac{k}{m});\ k = 0,\ 1 \cdots m-1,\ m\}$$

and define the rescaled increments as

$$Z_\nu(t_*) = \frac{Y_\nu(t_*) - Y_\nu(t_* - \frac{1}{m})}{1 - Y_\nu(t_* - \frac{1}{m})} \; ;$$

$$t_* = \frac{1}{m} \cdots \frac{(m-1)}{m},\ 1 \ . \tag{8}$$

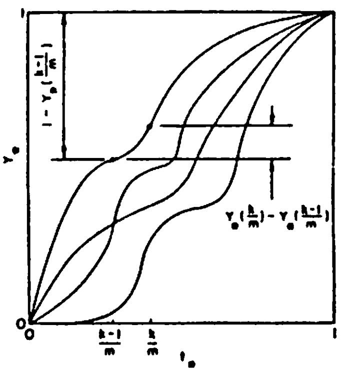

Figure 4. Sample function of the dimensionless rainfall accumulation process $Y^*(t_*)$.

This dimensionless process was divided into 10 equal time increments. An analysis of data for 275 thunderstorms greater than 0.25 inch, at the Walnut Gulch experimental watershed, showed that the sequence of rescaled increments Z_1, $Z_2 \cdots Z_9$ can be represented by a nonhomogeneous first-order Markov Chain. The expected value of Z_k, given Z_{k-1}, is a linear function of Z_{k-1}. The marginal distribution of Z_1, and the conditional distributions of $(Z_k | Z_{k-1})$ $k = 2, 3 \cdots 9$ can be adequately described by the beta distribution. The number of parameters in this model can be reduced from 26 to 10 by approximating the two parameters in the conditional expectation function and the conditional beta parameter as polynomial functions of dimensionless time. This represents a significant decrease in numbers of parameters compared with alternative methods, yet does an adequate job of simulating intra-storm intensities. An example of an observed storm hyetograph, along with two simulated hyetographs, is shown in figure 5.

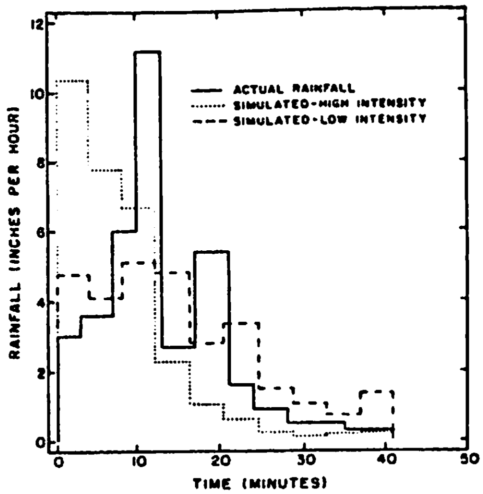

Figure 5. Actual storm of 2.02 inches in 41 minutes with two simulated storms with the same amount and duration.

SUMMARY

A pilot project designed to investigate the feasibility of a stochastic model to disaggregate daily rainfall is described. Significant progress has been made in disaggregating individual storms into rainfall intensities for periods of one-tenth of the storm duration. A logical procedure has been developed for disaggregating daily rainfall into storms, and an appropriate stochastic structure has been identified for several steps of the process. The research described is still underway, so it is possible that alternative distributions may yet be found that are superior to those described in this paper.

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