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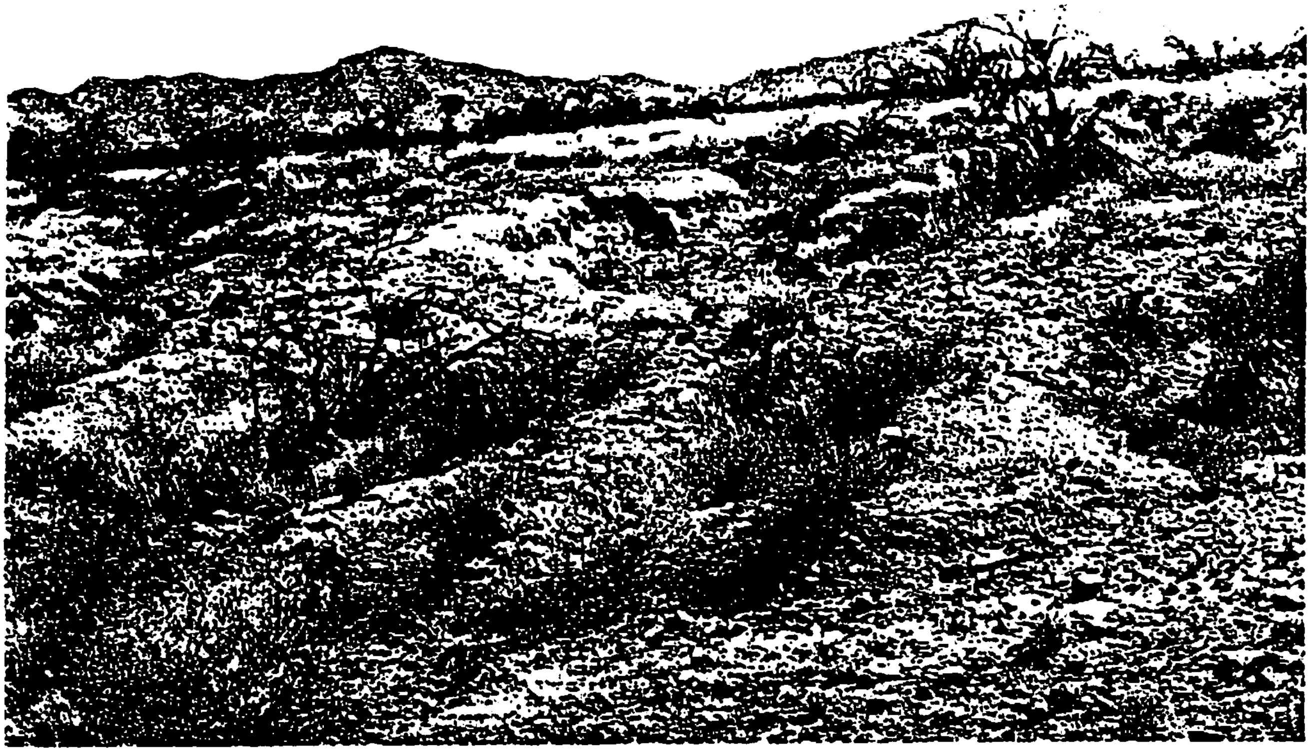

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MODELING EROSION IN OVERLAND FLOW

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INTRODUCTION

Overland flow on a plane is a function of time and space and is often modeled using the kinematic wave equations (Henderson and Wooding 1964; Wooding 1965a, 1965b, and 1966). A kinematic flow number, as a criterion for accuracy of the kinematic approximation to the unsteady flow equations, was developed by Woolhiser and Liggett (1967), who found that the approximation was accurate under conditions representative of many overland flow surfaces (Woolhiser 1974). The kinematic wave equations were derived for flow on smooth planes but have been shown to apply on many irregular surfaces where the mean velocity per unit width is proportional to the storage in an incremental area. Such surfaces include simple upland areas typical of many natural watersheds (Woolhiser, Hanson, and Kuhlman 1970).

Erosion on upland areas is conceptionalized as rill and interrill erosion (Foster and Meyer 1971; Foster, Meyer, and Onstad 1977). Interrill erosion is assumed due to impact of raindrops and associated transport overland. Rill erosion is assumed due to soil detachment and subsequent transport by flow in rills or small channels. Hjelmfelt, Piest, and Saxton (1975) give a partial solution to the coupled runoff and erosion equations that are described herein.

THE MODEL

The kinematic wave equations for overland flow on a plane are:

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = R, \tag{1}$$

and

$$q = Kh^m \tag{2}$$

where h, q, and R are, respectively, the depth of flow, runoff rate per unit width of the plane, and rainfall excess rate. The coefficient K is a parameter including slope and roughness, and m is an exponent reflecting the flow type (laminar or turbulent) and the roughness-velocity relationship (Manning or Chezy equation)

Given overland flow as described above, interrill erosion rate is assumed as

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$$E_I = K_I R \qquad (3)$$

and rill erosion rate is assumed as

$$E_R = K_R(Bh^a - q_s) \qquad (4)$$

where K_I, K_R, and B are, respectively, interrill coefficient and rill coefficients. The exponent a is usually assumed equal to m, which also facilitates solution of the equations. The sediment discharge per unit width of the plane is

$$q_s = cq \qquad (5)$$

where c is sediment concentration. Notice that the variables defined by Eqs. 1 thru 5 are functions of time, t, and distance, x, down the plane.

Using the above equations, Shirley and Lane (1978) derived a sediment yield equation by integrating, with respect to time, the sediment continuity equation

$$\frac{\partial(ch)}{\partial t} + \frac{\partial q_s}{\partial x} = E_I + E_R \qquad (6)$$

to produce a sediment yield equation as a function of position on the plane. The resulting equation for sediment yield per unit width of the plane, $Q_s(x)$, as a resultant of constant and uniform rainfall excess is

$$Q_s(x) = Q(x) \, [\frac{B}{K} + (K_I - \frac{B}{K})(\frac{1 - e^{-K_R x}}{K_R x})] \qquad (7)$$

where Q(x) is runoff volume per unit width of the plane, and the other variables are described earlier. Equation 7 expresses the influence of slope length (x) on sediment yield in overland flow.

PARAMETER ESTIMATION

The runoff model has two parameters, K and m. Procedures for estimating K, and thus determining m, are summarized in Table 1. Under circumstances where observed runoff data are available, optimal parameters can be determined by fitting simulated runoff rates to corresponding observations. The parameters to be determined for the erosion equations are K_I, K_R, and B. These could also be estimated using optimization and available sediment concentration data.

Concentration as a function of t and x has been derived and evaluated to produce three equations in three unknowns (Shirley and Lane, 1978, Eqs. 28 thru 30). Initial concentration $C_o = C(t = o,x)$ can be estimated by extending observed sediment concentration data back to the t = o axis on a plot of concentration versus time. Mean concentration can be estimated as the observed sediment yield divided by the observed runoff volume $\overline{C} = Q_s/Q$. Also, the final concentration, C_∞, can be estimated by extending the plot of observed sediment concentration through the hydrograph recession until the end of the event on a plot of concentration versus time.

Table 1.—Hydraulic resistance parameters for steady-state turbulent flow over the indicated surface (after Woolhiser, 1974 and Lane et al., 1975)

Overland flow surface		Approximate range in resistance parameters[1]	
Roughness condition	Type	Manning[2] n	Chezy[3] C
Very smooth	Concrete, asphalt	0.010 - 0.013	60 - 45
Smooth	Bare sand	0.010 - 0.016	60 - 37
	Eroded bare soil, small gravel	0.012 - 0.033	50 - 20
Moderate	Sparse vegetation, rangeland	0.050 - 0.130	14 - 5.7
High	Short grass prairie, good grass	0.100 - 0.200	7 - 4
Very high	Dense grass, sod	0.170 - 0.400	4.5- 2

[1] English units are used in this table and in the references cited.

[2] For turbulent flow, K becomes: $K = \frac{1.49}{n} S^{1/2}$, $m = 5/3$.

[3] For turbulent flow, K becomes: $K = CS^{1/2}$, $m = 3/2$.

The corresponding equations from the model are:

$$C_o = K_I, \tag{8}$$

$$\overline{C} = \frac{B}{K} + (K_I - \frac{B}{K})(\frac{1 - e^{-K_R x}}{K_R x}), \text{ and} \tag{9}$$

$$C_\infty = \frac{B}{K} + (K_I - \frac{B}{K})e^{-K_R x} \tag{10}$$

Given estimates of C_o, C, and C_∞ from the observed data, they are set equal to the corresponding values from Eqs. 8 thru 10, and the resulting equations are solved simultaneously for K_I, K_R, and B.

APPLICATION TO RANGELANDS

Data used are from the Walnut Gulch Experimental Watershed operated by the U.S. Dept. of Agriculture. A detailed description of this research facility is given by Renard (1970). Generally, surface runoff on Walnut Gulch results from short duration thunderstorms during the summer rainy season. The area is described as semiarid rangeland.

To satisfy the model assumptions, a rainfall simulator was used to obtain runoff and sediment concentration data from a small plot. In addition, data were selected from a 1.3-ha watershed on the Walnut Gulch Experimental Watershed (Shirley and Lane 1978; Smith 1976). The plot data were used to test the derived solutions for consistency and reasonableness with observations. The watershed data were used to test the solutions for consistency and to determine if the model might have applications for natural watersheds.

The rainfall simulator used is a portable version of the Colorado State University apparatus (Dickinson, Holland, and Smith 1967). The portable simulator is described in detail by Lusby and Toy (1976). The artificial rainfall is produced at a rate of about 50 mm/hr. Analysis of drop-size distribution and raindrop velocities indicated that the artificial rainfall has about 30% to 40% of the kinetic energy of natural rainfall (Neff 1978). This reduction in rainfall energy is reflected in the interrill erosion parameter, as discussed later.

A 22.1 by 6.1 m plot (Lucky Hills Plot) was instrumented to obtain continuous runoff records and sediment concentration data at 1-min intervals throughout the overland flow hydrographs. The plot had a slope of 7% and closely approximated an overland flow plane. By making a series of closely spaced runs on the plot, it is possible to approximate the constant, uniform rainfall excess pattern assumed in obtaining solutions to the equations. This procedure was followed to obtain data from the runoff plots. The plot was established in an undisturbed area adjacent to Watershed 63.101, described below. In a preliminary effort to calibrate the rainfall simulator, a 22.1 by 9.1 m plot (Montijo Plot), with a slope of 2%, was instrumented. Limited runoff and sediment data were also obtained from this plot.

Examples of rainfall, runoff, and sediment concentration data for the Montijo and Lucky Hills plots are shown in Fig. 1. Also shown in Fig. 1 are the resulting simulated hydrographs and sediment concentration graphs for the runoff-erosion model. These two events were selected to show cases where the runoff peak rate was under and overestimated, and where there was a relatively poor and good fit, respectively, to the observed sediment concentration data.

Optimal parameters (K, K_I, K_R, and B) were determined for each of nine events from the Lucky Hills Plots, as summarized in Fig. 1. As stated earlier, the product of the runoff volume and mean concentration, $Q \bar{c}$, is the sediment yield for the individual event:

$$Q_s = Q \bar{c} = Q\left[\frac{B}{K} + \left(K_I - \frac{B}{K}\right)\left(\frac{1 - e^{-K_R x}}{K_R x}\right)\right] \qquad (11)$$

where Q_s is sediment yield in kg and Q is runoff volume in m^3. The regression equation relating computed sediment yield, Y, and observed sediment yield is:

$$Y = -0.007 + 1.09 Q_s \qquad (12)$$

with $R^2 = 0.99$. Equation 12 represents a very good fit, although the optimal parameters were determined from fitting sediment concentration data instead of total sediment yield. Using the mean values of the optimal parameters ($K = 1.66$, $K_I = 0.87$, $K_R = .19$, and $B = 0.027$) results in the regression equation:

$$Y = -0.017 + 1.58 \, Q_s \qquad (13)$$

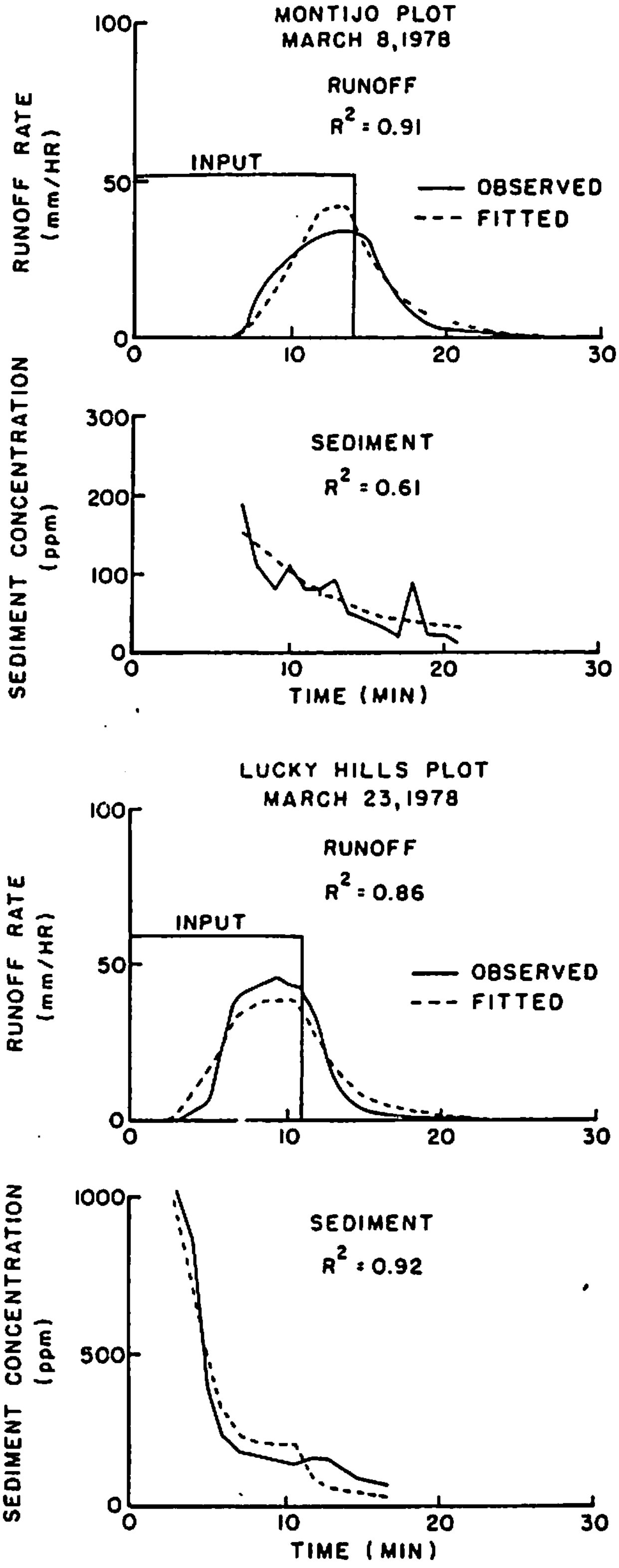

Figure 1.--Examples of observed and fitted data, rainfall simulator.

with $R^2 = 0.99$. The coefficient in this equation represents a significant bias in the computed sediment yields. Therefore, the means of the optimal parameters from fitting individual events produced larger errors than were obtained by letting the parameters vary from event to event.

Simulated concentration data matched the observed sediment concentration data quite well (Fig. 1) and, using optimal parameter values, computed sediment yields compared favorably with observed sediment yields. Thus, the runoff-erosion model appears to adequately simulate overland flow and erosion on the experimental plots.

A small (1.3-ha) watershed, called Lucky Hills Watershed 1 (63.101), was selected for additional analysis. This watershed is instrumented with a recording raingage, broad-crested v-notch weir, and a water-level recorder. During periods of ephemeral flow, pump-type (suspended sediment) samples are taken at 3-min intervals throughout the duration of runoff. This 1.3-ha watershed was approximated as a plane of length 194 m, width of 67 m, and a total relief of 7.8 m. A more complex representation of this watershed was presented by Smith (1976) wherein the watershed was represented by overland flow planes contributing to a small channel. Smith's simulation results agreed quite well with measured runoff and sediment yield data. However, his simulated raindrop splash detachment rates exceeded the amounts estimated from observed data, especially later in the storm events.

During the period 1973 to 1975, rainfall, runoff, and concentration data were obtained from 13 runoff events. Of these, eight events with single- peaked hydrographs were selected for analysis. In addition, a ninth event with a small secondary peak was included, because it was the largest event of record, and it provided an extreme.

As stated above, simulated hydrographs were computed for each of the nine events. The sum of squared deviations in runoff rate was used as the objective function. Optimal runoff parameters (K, R, t_*) were determined. Rainfall excess rate is R, and t_* is the duration of rainfall excess. Values of R and t_* were computed to reproduce the observed volume of runoff for each event. Optimal concentration parameters (K_I, K_R, B) were also determined (using the optimal runoff parameters as fixed values).

The equation (corresponding to Eq. 12) relating observed and fitted sediment yield is:

$$Y = 8.2 + 0.89\ Q_s \tag{14}$$

where $R^2 = 0.99$. The equation using mean values of the optimal parameters ($K = 3.69$, $K_I = 4.39$, $K_R = 0.032$, and $B = 1.31$) is

$$Y = 54.7 + 0.90\ Q_s \tag{15}$$

with $R^2 = 0.98$. Again, using mean rather than individual values for the parameters resulted in reduced fitting accuracy.

Optimization results for the Lucky Hills Plot and for Watershed 63.101 are summarized in Table 2.

Table 2.—Mean values of optimal parameters for the Lucky Hills Plot and for Watershed 63.101 (Note: $m = a = 3/2$)

Watershed	Drainage Area	K	K_I	K_R	B
	(ha)	($m^{1/2}$/sec)	(kg/m^3)	(m^{-1})	(kg/sec-$m^{2.5}$)
Lucky Hills Plot	0.014	1.66	0.87	.19	0.027
63.101	1.30	3.69	4.39	.032	1.31

The natural watershed had a hydraulic resistance parameter of $K = 3.69$, while the plot had a value of $K = 1.66$ for an average increase in flow velocity coefficient of $3.69/1.66 = 2.2$. The interrill parameter, K_I, increased by a factor of 5 from the plot to the natural watershed. As discussed previously, we might expect a 2- to 3-fold increase due to rainfall energy considerations. The product $K_R B$ represents a rill erosion parameter. This product increased by a factor of 8 from the plot to the watershed. Interpretation of the changes in these parameters from the plot to the watershed suggest that : (1) flow velocities increased, (2) interrill erosion rates increased, and (3) rill erosion rates increased. Since we were not modeling the channel network on Watershed 63.101 and the simulated rainfall had significantly less energy than natural rainfall, these parameter changes are in the direction expected. However, since they are mean values of parameters determined from limited data, the changes should be given only qualitative interpretations.

SUMMARY AND CONCLUSION

Runoff from upland areas can be accompanied by substantial erosion. We modeled overland flow on upland areas as overland flow on a plane. Erosion on upland areas is conceptualized as consisting of rill and interrill erosion. Interrill erosion is assumed due to rainfall impact, and rill erosion was defined as erosion due to tractive forces and transport capacity in flow as it occurs in rills or small channels. The combined runoff-erosion process is called overland flow with rill and interrill erosion.

Partial differential equations have been formulated for the above runoff-erosion process. Solutions had been developed for the specific cases of the rising and equilibrium hydrographs (Hjelmfelt, Piest, and Saxton 1975). We developed analytic solutions for the general case of rising, equilibrium, and recession hydrographs and for the entire partial-equilibrium hydrograph (Shirley and Lane 1978).

The runoff-erosion model was tested using rainfall simulator data. Optimal values of the model parameters were determined for 9 runoff events. Simulation results with the optimal parameters seem to be reasonable approximations (good fit) to observed runoff and concentration data. Sediment yield values computed by the model also seem to be reasonable approximations to observed data.

To determine if the coupled runoff-erosion equations might have applications for natural watersheds, data from a small, natural watershed on Walnut

Gulch were analyzed. The computed sediment concentration and sediment yield data were consistent with observations on this watershed. The parameter values were logically related to parameters from the experimental plots, and thus, the procedure may have application to small watersheds.

The major result of this research is the derivation and testing of analytic solutions for sediment concentration and sediment yield in overland flow. Based upon our analysis of the properties of these solutions, we conclude that the runoff-erosion model used in this study produces reasonable results for erosion on upland areas. Limited testing with observed data supported this conclusion.

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