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## TECHNICAL NOTES

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# TECHNICAL NOTES

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# BOUNDARY LAYERS IN DEVELOPING OPEN CHANNEL FLOW

By Edward Silberman,[1] F. ASCE

## INTRODUCTION

Developing flow in open channels may be analyzed using the Bernoulli and continuity equations, and by taking boundary layer growth into account. However, the most general and useful analysis does not seem to appear in the literature. Absence of a general analysis has led some investigators to apply boundary layer calculations in greater detail than is supported by physical reality, and to make unwarranted assumptions in transferring data from physical model to prototype.

## GENERAL ANALYSIS

**Role of Boundary Layer.**—Following flow contraction near a channel entrance, fresh boundary layers form around the channel perimeter (Ref. 7, pp. 14–17, and Ref. 9, chapter 2). Outside the boundary layers, the flow may be assumed to be nearly potential. In this nearly potential region, the Bernoulli equation is valid; Bauer (2) commented that this was true in his experimental data and it is also true in Kindsvater's data (8). From Fig. 1, the Bernoulli equation may be written for steady flow

$$\frac{V^2}{2g} + D\cos\alpha - Z = H = \text{constant} \quad \text{(1)}$$

in which $V$ = magnitude of the velocity vector in the potential flow region. As flow progresses downstream, less of the cross section is fitted by Eq. 1 until, at some distance from the entrance, the boundary layer reaches the surface near the channel center. This analysis is applicable only until that point is reached; thereafter, fully developed flow occurs and the usual methods of analysis for fully developed flow are more suitable.

By assuming nearly straight and parallel streamlines, the discharge may be obtained by integrating the longitudinal component of velocity over the channel depth. Consider a wide channel so that the central part is two dimensional,

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and introduce the displacement thickness, $\delta_1$ (Ref. 7, p. 17, and Ref. 9, Eq. 2.6). Then

$$V = \frac{q}{D - \delta_1} \quad \text{and} \quad \frac{dV}{dx} = -\frac{q}{(D - \delta_1)^2}\left(\frac{dD}{dx} - \frac{d\delta_1}{dx}\right) \qquad (2)$$

in which $q$ = discharge per unit width.

**Critical Depth.**—From Eq. 1

$$\frac{dH}{dx} = 0 = \frac{V}{g}\frac{dV}{dx} + \frac{d}{dx}(D\cos\alpha) - \tan\alpha \qquad (3)$$

Introducing Eq. 2 and rearranging terms

$$\frac{dD}{dx} = \frac{\dfrac{d\delta_1}{dx} + \left(D\dfrac{d\alpha}{dx} + \sec\alpha\right)\left(\dfrac{D - \delta_1}{D_c - \delta_1}\right)^3 \tan\alpha}{1 - \left(\dfrac{D - \delta_1}{D_c - \delta_1}\right)^3} \qquad (4)$$

in which $D_c = \left(\dfrac{q^2}{g\cos\alpha}\right)^{1/3} + \delta_1$ (5)

Eq. 5 defines an amplified version of critical depth in the developing flow region that reduces to the conventional definition when $\delta_1$ and $\alpha = 0$.

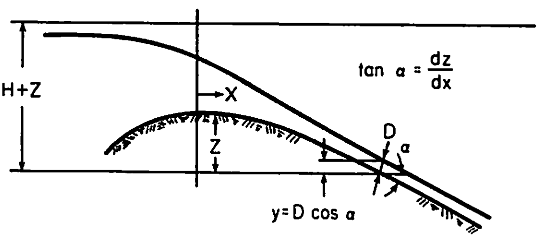


FIG. 1.—Definition Sketch

Eq. 4 demonstrates that for a nearly level channel entrance in which $\alpha \approx 0$, $D$ will drop when it is greater than $D_c$ and drop precipitously when $D$ comes close to $D_c$ (hydraulic drop); likewise, it will rise when it is less than $D_c$ and rise precipitously when it approaches $D_c$ (hydraulic jump).

**Basic Equation.**—Returning to Eq. 1, it may now be written

$$\frac{H + Z}{D_c\cos\alpha} = \frac{D}{D_c} + \frac{\left(1 - \dfrac{\delta_1}{D_c}\right)^3}{2\left(\dfrac{D}{D_c} - \dfrac{\delta_1}{D_c}\right)^2} \qquad (6)$$

Given $H$, $q$, and the bottom profile (which determines $Z$ and $\alpha$), Eq. 6, is a cubic equation for $D/D_c$ in terms of $\delta_1/D_c$ in which $D_c$ is given by Eq. 5. It is universal and its solution may be programmed to be called as a subroutine in a computer, or it may be solved once and for all and the results tabulated or graphed as in the sample in Fig. 2.

Observe that Eq. 6 does not involve head loss (except as head loss may be incidentally related to $\delta_1$). Nor does it depend on whether the surface is rough or smooth or the flow is laminar or turbulent. A somewhat more complicated equation may be written for the three-dimensional entrance region (6), but that will not be carried out here.

### Determining Displacement Thickness

The solution of Eq. 6 to obtain a water surface profile in the entrance region requires finding $\delta_1$ as a function of $x$. This is no simple task. When velocity

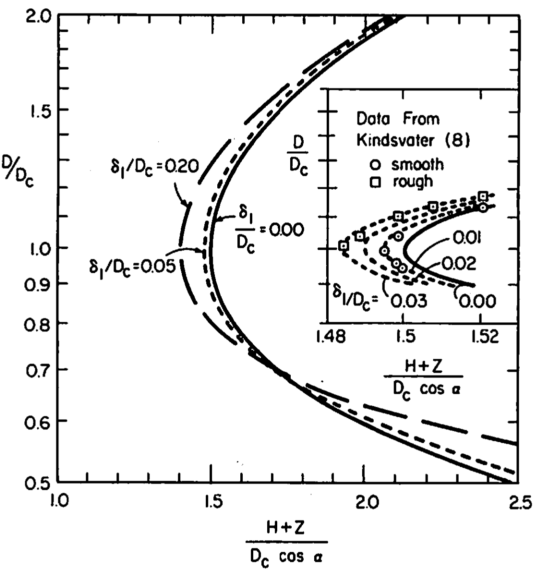


FIG. 2.—Influence of Boundary Layer Growth on Specific Energy

profiles have been measured in an experiment, the profiles can be integrated to obtain $\delta_1$ (Ref. 7, p. 17, and Ref. 9, Eq. 2.6). Several such sets of data have been reported. Kindsvater (8) made measurements in a model of flow of water over a highway embankment. His data have been reduced using Eqs. 5 and 6; some results are plotted on the inset in Fig. 2 (at an expanded horizontal scale). But the model results cannot be extended to prototype scale unless a separate determination of $\delta_1/D_c$ is made for the prototype.

Lacking measured velocity profiles, displacement thickness can only be estimated by calculation. On a solid boundary with well-defined leading edge, this can be done by boundary layer theory (9). Where the stream lines are nearly straight and parallel, as already assumed in writing Eq. 2, boundary layer theory may be simplified by assuming flat plate conditions. Harrison (5) analyzed channel entrances using this approach.

Unfortunately, the entrance to an open channel does not usually have a well-defined leading edge. The displaced layer may begin with separated flow, as was the case for Kindsvater's experiments (8). In that case, none of the velocity profiles were even close to the profiles commonly assumed in boundary layer theory, nor were they close for most profiles in Bauer's experiments (1). Even when separation does not occur and the usual theory applies, where is the origin of the boundary layer? Binnie (3), experimenting with laminar flows, chose to begin boundary layer calculations upstream of the entrance about nine displacement thicknesses, measured from the entrance. This was an entirely arbitrary choice in an attempt to fit experimental data, yet it fit poorly.

In view of the difficulty in defining entry conditions, it is futile to attempt a precise calculation for $\delta_1$. A good guess, guided by rough boundary layer calculations, is the most that can be hoped for; often, even that is impossible. Complicated boundary layer analysis is not warranted for a channel entrance.

## Conclusions

Estimating displacement thickness is the key operation in applying boundary layer calculations to the entrance region of an open channel. All other calculations are nearly exact by comparison. But displacement thickness is not really calculable because there may be separated regions in the flow near the entrance or, at best, the exact beginning of the boundary layer is not definable. Thus, only rough boundary layer calculations are warranted. High precision in prediction cannot be expected.

Can physical models be used to study flow in the entrance region? The answer is "yes" provided both gravity (Froude law) and boundary layer phenomena (Reynolds law and roughness) are simultaneously modeled. Only close to the entrance where boundary layer effects are small or separation is clearly geometrically induced would modeling by Froude law alone be adequate. The ship modeling procedure of working with the Froude law and calculating boundary layers is not available because of the difficulty in defining boundary layer origin.

Boundary layer thickness calculations are necessary to predict the place in which air entrainment begins (2,4). Thickness cannot be as precisely defined as $\delta_1$. Air entrainment prediction can hardly be expected to be precise under such circumstances, nor consistent between different experimental arrangements.

The drop in the specific energy line is another desirable calculation in the entrance region. It involves the energy thickness or, alternatively, both $\delta_1$, and the Coriolis coefficient (Ref. 7, p. 19) at each cross section. Thus, the calculation is even more imprecise than that for $\delta_1$ alone.

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## Appendix II.—Notation

*The following symbols are used in this paper:*

$c$ = critical value;
$D$ = depth measured normal to channel bottom;
$g$ = acceleration of gravity;
$H$ = total head in potential flow region;
$q$ = discharge per unit width;
$V$ = velocity in potential flow region;
$y$ = vertical component of channel depth;
$Z$ = elevation of channel bottom;
$\alpha$ = slope of channel bottom; and
$\delta_1$ = displacement thickness.

# RECIPROCAL-DISTANCE ESTIMATE OF POINT RAINFALL

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## INTRODUCTION

Accurate estimates of rainfall are most important in hydrologic modeling. Many methods have been suggested to estimate missing or ungaged point rainfall (2,3,4). A reciprocal-distance weighting technique appears to be one of the most accurate (1,4). In the reciprocal-distance method, the amount of rainfall at any ungaged point is a function of the measured rainfall and distance to nearby gages. Most earlier examples have been based on relatively widely spaced gages. In systems where thunderstorms dominate the hydrology, rainfall is difficult to define without dense rain-gage networks. Because of the limited areal extent of thunderstorm rainfall, estimates of rainfall are only accurate if gages are closely spaced. Dean and Snyder (1) found that for widely spaced gages in the Piedmont region of the Southeast, an exponent of 2 in the reciprocal-distance method gave best results. In this paper, the writers attempted to determine if an exponent of 2 would give best results for estimating thunderstorm rainfall in the Southwest and if there were definable relationships among gages, distance, and exponents in the reciprocal-distance method.

## PROCEDURE

The reciprocal-distance method gives the greatest weight to the nearest gage and reduces weight proportionally as distance increases and minimizes the smoothing of the rainfall distribution. The amount of estimated rainfall, $R$, at point $(x, y)$ is expressed by

$$R = \frac{\left(\dfrac{\sum_{i=1}^{n} P_i}{D_i^b}\right)}{\dfrac{\sum_{i=1}^{n} 1}{D_i^b}} \quad (1)$$

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**TABLE 1.—Simple Correlation Coefficients (*r*) for Predicted Versus Actual Storm Rainfall for Walnut Gulch Rain Gages**

| Average association distance, in kilometers | Standard deviation, in kilometers | Exponent | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| 1.09 | 0.296 | 0.935 | 0.943 | 0.950 | 0.955 | 0.959 | 0.962 | 0.963 | 0.964 | 0.963 |
| 1.30 | 0.471 | 0.937 | 0.945 | 0.951 | 0.957 | 0.961 | 0.963 | 0.964 | 0.964 | 0.964 |
| 1.74 | 0.629 | 0.900 | 0.901 | 0.901 | 0.899 | 0.897 | 0.894 | 0.890 | 0.887 | 0.884 |
| 2.37 | 1.158 | 0.860 | 0.909 | 0.933 | 0.941 | 0.941 | 0.938 | 0.935 | 0.932 | 0.929 |
| 3.02 | 1.334 | 0.775 | 0.820 | 0.858 | 0.883 | 0.896 | 0.900 | 0.901 | 0.899 | 0.898 |
| 3.39 | 0.972 | 0.786 | 0.805 | 0.814 | 0.817 | 0.815 | 0.808 | 0.800 | 0.789 | 0.777 |
| 3.52 | 1.121 | 0.722 | 0.718 | 0.714 | 0.709 | 0.705 | 0.701 | 0.698 | 0.694 | 0.691 |
| 3.61 | 0.637 | 0.725 | 0.731 | 0.736 | 0.741 | 0.744 | 0.746 | 0.747 | 0.746 | 0.744 |
| 4.00 | 1.123 | 0.682 | 0.719 | 0.748 | 0.771 | 0.786 | 0.796 | 0.800 | 0.800 | 0.796 |
| 4.50 | 3.102 | 0.797 | 0.838 | 0.846 | 0.842 | 0.833 | 0.821 | 0.809 | 0.796 | 0.782 |
| 5.06 | 2.093 | 0.733 | 0.764 | 0.777 | 0.773 | 0.756 | 0.735 | 0.715 | 0.697 | 0.684 |
| 5.08 | 4.277 | 0.744 | 0.857 | 0.889 | 0.886 | 0.870 | 0.851 | 0.833 | 0.819 | 0.808 |
| 5.21 | 4.521 | 0.857 | 0.873 | 0.870 | 0.863 | 0.855 | 0.846 | 0.837 | 0.828 | 0.818 |
| 5.65 | 4.364 | 0.620 | 0.803 | 0.870 | 0.875 | 0.862 | 0.844 | 0.829 | 0.816 | 0.807 |
| 5.72 | 5.517 | 0.692 | 0.845 | 0.887 | 0.886 | 0.870 | 0.851 | 0.833 | 0.819 | 0.808 |
| 6.01 | 3.936 | 0.549 | 0.689 | 0.762 | 0.790 | 0.799 | 0.801 | 0.800 | 0.799 | 0.798 |
| 6.08 | 5.179 | 0.631 | 0.760 | 0.805 | 0.818 | 0.820 | 0.819 | 0.817 | 0.814 | 0.812 |
| 3.96[a] | | 0.761[a] | 0.819[a] | 0.842[a] | 0.847[a] | 0.845[a] | 0.840[a] | 0.834[a] | 0.827[a] | 0.821[a] |

[a]Average.

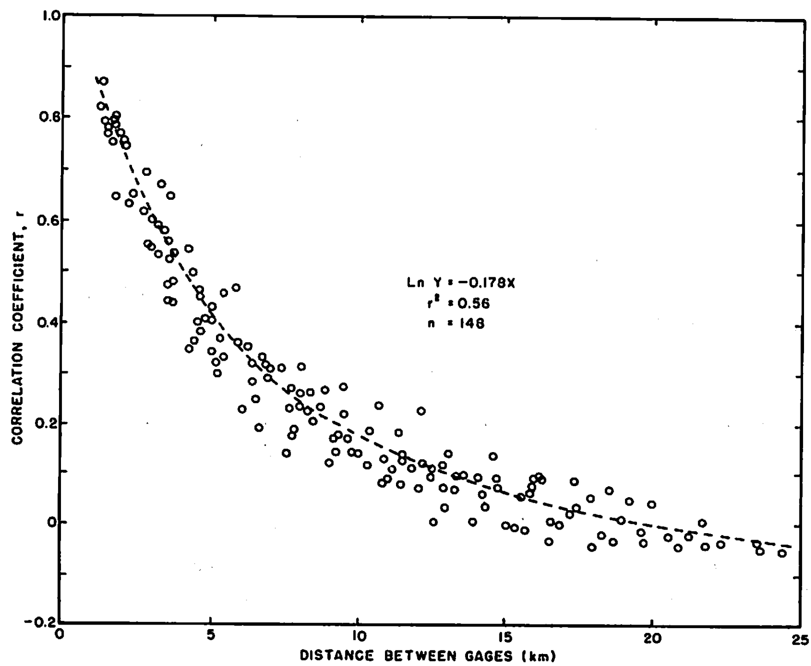


**FIG. 1.—Relation of Distance between Paired Rain Gages and Correlation Coefficient**

in which $P_i$ = measured rainfall at gage $i$; $D_i$ = distance between point $(x,y)$ and gage $i$; $n$ = number of rain gages used to estimate rainfall at point $(x,y)$; and $b$ = exponent by which the distances are weighted.

Rainfall data from the Walnut Gulch Experimental Watershed in southeastern Arizona were used in the analysis. The hydrology of Walnut Gulch is dominated by air-mass thunderstorms occurring mainly from July through September. On the average, about 70% of the 355 mm annual precipitation occurs during this rainy period. Precipitation over the 150-km$^2$ watershed is measured by 95 weighing-type recording rain gages spaced about 1,000 m apart.

Thirty rainfall events were selected for 17 raingage associations; each of which consisted of a base gage and four surrounding associated gages. The base gage record was estimated from the four associated gages. The events were selected

**TABLE 2.—Correlation between Predicted and Actual Rainfall Amounts for Associated Gages for Selected Durations Using Exponent of 1.5 in Reciprocal-Distance Method**

| Base gage identification (1) | Number of associated gages (2) | Average distance, in kilometers (3) | $r_{1.5}$ | | | |
|---|---|---|---|---|---|---|
| | | | 15-min depth (4) | 30-min depth (5) | 60-min depth (6) | Total depth (7) |
| 8 | 4 | 1.4 | 0.88 | 0.94 | 0.95 | 0.94 |
| | 1 | 1.3 | 0.75 | 0.85 | 0.88 | 0.89 |
| 12 | 4 | 1.4 | 0.93 | 0.95 | 0.96 | 0.96 |
| | 1 | 1.3 | 0.88 | 0.87 | 0.88 | 0.88 |
| 22 | 4 | 1.3 | 0.89 | 0.92 | 0.94 | 0.93 |
| | 1 | 1.0 | 0.82 | 0.89 | 0.92 | 0.91 |
| 32 | 4 | 1.4 | 0.91 | 0.92 | 0.93 | 0.93 |
| | 1 | 1.1 | 0.77 | 0.82 | 0.84 | 0.84 |
| 52 | 4 | 1.4 | 0.94 | 0.96 | 0.96 | 0.96 |
| | 1 | 0.9 | 0.79 | 0.80 | 0.79 | 0.77 |
| 92 | 3 | 1.1 | 0.95 | 0.97 | 0.98 | 0.98 |
| | 1 | 0.8 | 0.91 | 0.93 | 0.94 | 0.96 |
| 384 | 4 | 1.1 | 0.97 | 0.98 | 0.99 | 0.99 |
| | 1 | 0.3 | 0.96 | 0.98 | 0.99 | 0.99 |

Note: $t$-test indicates a significant (1%) difference between the reciprocal-distance estimate and the closest gage estimate.

on the basis of storm depth (>5 mm) and areal extent (all base gages recorded rainfall for each event). A square grid was superimposed over the watershed rain-gage network and $x$, $y$ coordinates determined for each rain gage. The grid distances, $D_i$, between gages were then determined from

$$D_i = [(x_i - x)^2 + (y_i - y)^2]^{1/2} \quad \quad (2)$$

Eq. 1 was then used to estimate rainfall for "missing" gages.

Rain-gage associations were selected by first choosing a base gage to be used as the "missing" record, and then using one gage each in the north, south, east, and west directions. Different average distances between the associated gages and the base or "missing" gage were obtained by selecting associated gages further and further away from the base. Using this procedure, 17 associations were used to estimate six base gage records for 30 storms using nine exponents in Eq. 1. The associations chosen contained both evenly and unevenly spaced associated gages. These spacings were used to determine if certain exponents worked better for different types of association geometries. The base gage estimate for each storm was then compared with the actual record, and a correlation coefficient was determined between actual and estimated depth.

## Results

Correlation coefficients ($r$) obtained from comparing actual and estimated total storm depths for 17 different associations are shown in Table 1. The zero exponent (average rainfall depth among the associated gages) produced the lowest average $r$ for the 17 associations. An exponent of 1.5 gave the highest average $r$, but a $t$-test indicated no significant differences between exponents 1 through 3. There were significant differences between exponents up to 1 and above 3.

For thunderstorm rainfall, the $r$ between rainfall depths of paired gages decreased rapidly with distance (Fig. 1). With this in mind, the writers determined if the reciprocal-distance weighting method was better than using the closest gage for estimating missing data. Results of this analysis indicated that the reciprocal-distance weighting method was significantly better than the closest gage for estimating missing data (Table 2).

The $r_{1.5}$ (correlation coefficient of actual versus estimated using an exponent of 1.5 in Eq. 1) for different incremental time depths are included in Table 2. The $r_{1.5}$ between estimated and actual rainfall increases as the storm duration increases. This is because thunderstorm rainfall is very intense, has limited areal extent, and is more often localized at the beginning of the event, thereafter becoming more generalized and less intense toward the end of the event. Spacing uniformity of associated rain gages had no effect on $r_{1.5}$ or any other exponent in the $r$ relationships (Table 1).

## Conclusions

The reciprocal-distance weighting method was found very useful in estimating missing thunderstorm rainfall data for distances of less than 10 km between gages. Exponents used in testing the weighting equation can range from 1 to 3 without significantly affecting the accuracy of the estimate. A zero exponent (actually indicating the average of the associated gages) was the least accurate, reaffirming the value of the reciprocal distance method. However, for areas where air-mass thunderstorms dominate there seems to be no reason for using an exponent other than 1 in the weighting equation. This is in contrast with indicated values of higher exponents in other regions. However, even using the reciprocal-distance weighting method will not lead to a good estimate at a single gage located at the maximum or minimum within a storm pattern.

## Appendix.—References

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