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STOCHASTIC ASPECTS OF WATERSHED SEDIMENT YIELD

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INTRODUCTION

Sediment yield is defined as "the total sediment outflow from a watershed or drainage basin, measureable at a point of reference and a specified period of time" (ASCE Task Committee, 1970). It is the portion of the gross erosion within a watershed (sum of all erosion from land and channels) that is not deposited before being transported from the watershed.

The sediment yield process may be divided into two categories - - the upland phase and the lowland phase (Bennett, 1974). Sediment detachment and transport processes predominate in the upland phase and depend on individual rainfall (or snowmelt) events. The magnitude of erosion from upland phase depends on factors like slope steepness and shape, cover, soil type, land use, and rainfall characteristics.

In the lowland phase, sediment transport and deposition processes predominate, and the bed load transport capacity becomes a significant factor because channel flow is normally capable of transporting all the fine material available. Sediment transport rates are determined by hydraulic variables, like the flow depth and velocity, channel slope, water temperature, and physical properties of the sediments. The effect of individual precipitation events is highly damped.

Because sediment yield responds quite readily to man's activities, various types of nonstationary processes may be present in sediment yield data. For example, if the percentage of the land area in row crops is increasing or decreasing, it would be reflected in the sediment yield from upland areas. Construction of reservoirs or sediment basins may result in a sudden decrease in sediment yield immediately downstream. The reduction may be due partly to reduced runoff, and partly to the release of water relatively free of sediment from the reservoir. Urban development or surface mining may cause large increases in erosion rates, followed by a transient period as vegetation becomes reestablished and new surface drainage nets develop. Land treatment measures are basic

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elements of watershed projects developed under the U.S. Watershed protection and Flood Prevention Act of 1954 (P.L. 566) and will have a significant effect in reducing sediment yields from upland areas (ASCE Task Committee, 1969).

Many components of the sediment yield process are best described by their probabilistic structure because randomness is the very essence of the individual process. The most obvious source of randomness in sediment yield from upland areas is precipitation. Indeed, most investigators have considered precipitation as the only source of randomness and have coupled stochastic rainfall models with deterministic runoff and erosion models to develop simulation models of sediment yield. In the lowland phase, there have been many investigations of the relationship between river discharge and sediment discharge which indicate that it has a random component.

Other sources of randomness in the sediment yield process include fluctuations in rainfall drop size, wind effects, tillage operations, vegetative cover, soil infiltration rates and erodibility, gully headcutting and bank-caving, land slides and various random processes (both natural and man induced) that trigger potential instabilities in channels and valley floors. Fire, insect infestations, and earth movements provide additional sources of random variability with a relatively low frequency of occurrence.

Many of these stochastic processes have a complex dependency structure. For example, the probability of a forest fire 1 year after one has occurred is much lower than it would be 30 years later. A similar dependence structure is evident in insect infestations that kill perennial vegetation.

The designs of multiple-purpose reservoirs, flood detention reservoirs, and debris basins must include sediment yield of the contributing watershed and the size distribution of sediment. The current engineering practice is to provide additional storage for the expected accumulation of sediment in the reservoir over a selected design period. The expected accumulation of sediment is related to the expected sediment yield, the trap efficiency of the reservoirs, and the sediment size distribution (ASCE Task Committee, 1969). No provision is made for the random variability of sediment yield. Recent studies using Bayesian decision theory have shown that information on the distribution function of sediment yield is required to determine the worth of additional data in the Bayesian sense (Jacobi, 1971) and that reservoir designs based only on the expected sediment yield are often not optimal (Smith, Davis, Fogel, 1977). These findings suggested that all design procedures requiring estimates of sediment yield should be examined with respect to their sensitivity to the natural variability of sediment yield and to uncertainties in the parameters describing this variability.

STOCHASTIC DESCRIPTION OF SEDIMENT YIELD

Consider a generalized watershed with processes distributed in time and space. Such a system is bounded on the bottom by rock, by an imaginary side surface, s, and by the imaginary top surface, A. Input to such a system consists of the precipitation flux to the surface A, denoted as the stochastic process $\xi_1(x, y, t)$. Rainfall excess, $\xi_2(x, y, t)$, is the amount of precipitation greater than that which infiltrates soil and results in the surface runoff, $\zeta_1(t)$, at the watershed outlet. Besides the runoff and sediment yield, $\zeta_2(t)$, outputs from the system include the evapotranspiration, $\xi_3(x, y, t)$, and porous media flow through surface S, $\eta(x, y, z, t)$. Typical sample function of the processes $\xi_1(t)$, $\xi_2(t)$, $\zeta_1(t)$, and $\zeta_2(t)$ are shown in Fig. 1.

Definition and Notation

Suppose that we have accurate instantaneous measurements of total sediment transport at a stream draining a watershed with area A. We shall denote this instantaneous rate of transport as $\{\zeta_2(t), t \epsilon T\}$, where $\zeta_2(t)$ is the total sediment transport rate (MT^{-1}) and $T = \{t \geq o\}$. From physical considerations, we know that $\zeta_2(t)$ is dependent (among other things) on the stream discharge, $\zeta_1(t)$, and the precipitation rate, $\xi_1(x, y, t)$. The sediment-yield process, $Y_2(t)$, can then be expressed as:

$$Y_2(t) = \int_o^t \zeta_2(s)ds \tag{1}$$

It is evident that $Y_2(t+\Delta t) \geq Y_2(t)$; therefore, the sediment yield process represents a stochastic process of non-decreasing sample functions. The distribution function of sediment yield, frequently of interest in design problems, is

$$F_t(y) = P\{Y_2(t) \leq y\} \tag{2}$$

Let $T(y)$ be the minimum time required for the accumulated sediment yield to equal or exceed the amount y, or

$$T(y) = \inf\{t, Y_2(t) \geq y\} \tag{3}$$

which is often called the first-passage time. The distribution function for the first-passage time is

$$G_y(t) = P\{T(y) \leq t\} \tag{4}$$

Now
$$P\{T(y) > t\} = P\{Y_2(t) \leq y\}$$

Therefore,

$$G_y(t) = 1 - F_t(y) \tag{5}$$

Equations (1) through (5) are general, and can be applied to any situation. To illustrate how these expressions can be obtained, let us

Consider sediment yield from the standard fallow plot, used by Wischmeier and Smith (1960) in developing the Universal Soil Loss Equation (USLE). It is 22.1-m (72.6-ft) long, has a 9 percent slope, and is tilled in the direction of maximum slope. Two approaches can be used to develop the distribution function for sediment yield: (1) the empirical-analytic, and (2) the physically-based Monte Carlo simulation. The empirical-analytic approach requires the specification of a counting function, N(E), which is defined as the number of sediment yield events in the internal (o, t), where sediment yield events, rainfall events, and runoff events are defined as follows (See Fig. 1).

A rainfall event is defined as any continuous period of rainfall $\xi_1(t) > o$. A runoff event is any continuous period of runoff, and a sediment yield event is associated with each runoff event. Associated with the i^{th} rainfall event is the time of ending, R_i. Similarly, T_i

refers to the time of ending of the i^{th} runoff and sediment yield event.

The total sediment yield for the ν^{th} event can be written as

$$Y_\nu = \int_{T_{\nu-1}}^{T_\nu} \zeta_2(s)ds \quad (6)$$

and the sediment yield can be closely approximated as:

$$Y_2(t) = \sum_{\nu=0}^{N(t)} Y_\nu \; ; \quad Y_0 = 0. \quad (7)$$

If we assume that the individual sediment yield events are independent of N(t), we can write the distribution function as

$$F_t(Y) = P\{N(t) = 0\} + \sum_{\nu=1}^{\infty} P\{Z_\nu \leq Y\}\, P\{N(t) = \nu\} \quad (8)$$

where $Z_\nu = Y_1 + Y_2 + ... + Y_\nu$. Analytic expressions for the mean and variance for this case are well known, and if the expressions used for the density functions of N(t) and Z_ν are tractable, $F_t(Y)$ can be obtained analytically (Woolhiser and Todorovic, 1974; Woolhiser and Blinco, 1975).

A physically-based model of upland erosion, based on principles of hydrology, hydraulics, sediment transport, and erosion mechanics was presented by Foster and Meyer (1972). The model is based upon the continuity equation for mass transport under steady-state conditions.

$$\frac{\partial G}{\partial x} = D_r + D_i \quad (9)$$

where G = sediment load (weight/unit time/unit width), x = distance downslope, D_r = detachment (or deposition) rate of rill erosion (weight/unit time/unit area), and D_i = delivery rate of particles detached by interrill erosion to rill flow (weight/unit time/unit area).

They also included an approximate expression to account for the interaction of detachment (or deposition) and sediment load:

$$\frac{D_r}{D_{cr}} + \frac{G}{T_{cr}} = 1 \quad (10)$$

where D_{cr} = the detachment capacity of rill flow, and T_{cr} = the transport capacity of rill flow.

D_{cr}, T_{cr}, and D_i are functions of rainfall and runoff characteristics, soil properties, and vegetative cover.

Equations (9) and (10) can be combined by eliminating D_r.

$$\frac{\partial G}{\partial x} = \frac{D_{cr}}{T_{cr}} (T_{cr} - G) + D_i \quad (11)$$

The rill transport capacity during erosion ($T_{cr} > G$) is not the same as during deposition ($T_{cr} < G$) because of changes in the surface geometry. These equations can be combined with empirical expressions for D_i, D_{cr}, and T_{cr}, and the kinematic flow equations

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = R \quad (12)$$

and

$$q = Kh^m \tag{13}$$

where h = flow depth
t = time
q = runoff rate per unit width
x = distance
R = rainfall excess rate
K = slope-resistance coefficient and
m = dimensionless exponent

to obtain solutions for the general case of sediment concentration in overland flow for the rising, equilibrium, and recession hydrograph.

An empirical estimate of $F_t(Y)$ can be obtained by using several rainfall records (observed or simulated) of length t as input and ranking the amounts of simulated sediment output $Y_1(t)$, $Y_2(t)$, etc. A plotting position can be computed for each output, and $F_Z(Y)$ can be obtained graphically.

Space prevents a detailed discussion of techniques used to develop stochastic models of sediment yield for large watersheds. In general, the techniques have involved simulation with a concentration on the bed load equations which are, of course, more important for large streams.

The stochastic model developed by Murota and Hashino (1969) is an example of an approach that develops a deterministic relationship between sediment transport and daily rainfall, and then develops distribution functions for total sediment yield in an n-day period.

Renard and Lane (1975) and Renard and Laursen (1975) used simulation techniques to estimate distribution functions of sediment yield from semiarid watersheds. Basically, their model consisted of a stochastic runoff model and a deterministic equation to calculate sediment yield given a runoff hydrograph.

Smith, Fogel, and Duckstein (1974) used two empirical expressions relating runoff to rainfall characteristics, and sediment yield to both rainfall and runoff characteristics in developing a stochastic sediment yield model for a semiarid watershed.

APPLICATION OF STOCHASTIC SEDIMENT YIELD MODELS

There are very few instances in the literature in which stochastic sediment yield models have been applied to practical problems, even in the form of examples. Jacobi (1971) utilized an elementary sediment yield model in evaluating the economic worth of sediment yield data in a statistical decision framework.

As an example, Jacobi (1971) evaluated the worth of additional data in the design of sediment storage for Cochiti Dam in New Mexico. He found empirically that the expected opportunity loss was inversely proportional to record length, n, and that the expected marginal worth of 1 year's data was inversely proportional to n^2.

Duckstein, Szidarovszky, and Yakowitz (1977) demonstrated the conjunctive use of event-based simulation with a Bayesian approach to decision making to make maximum use of a very limited amount of data available to estimate sediment yield from a semiarid watershed. In an example, they demonstrated that although the estimated mean sediment yield for the Charleston Dam Site in Southern Arizona is 103.3 ac-ft, the optimal design value (in the sense that it minimizes the Bayes Risk) is 143.6 ac-ft.

CONCLUSIONS

From examples in the literature, it appears that stochastic models of sediment yield have important application in design of dams, debris-basins, and other hydraulic structures. The approaches used in developing stochastic sediment yield models range from detailed, physically-based models with distribution functions obtained by sampling from Monte Carlo simulations to empirical, event-based models for which distribution functions of sediment yield can often be obtained analytically.

The best approach for any particular application depends greatly on the available data as well as the size of the watershed.

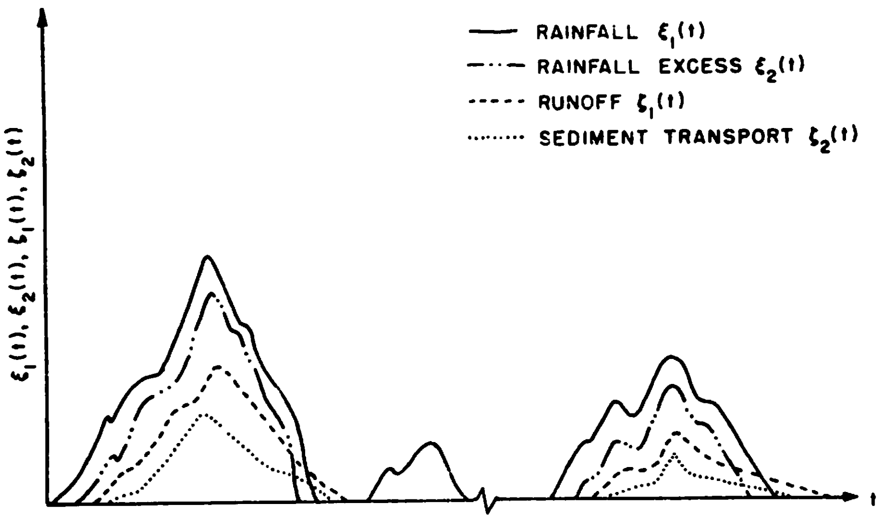

Figure 1. Sample functions of the processes $\xi_1(t)$, $\zeta_1(t)$, and $\zeta_2(t)$.

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