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EFFECTS OF RAINFALL INTENSITY ON RUNOFF CURVE NUMBERS

by

Richard H. Hawkins

ABSTRACT

The runoff curve number rainfall-runoff relationships may be defined in two ways: (1) by formula, which uses total storm rainfall and a curve number, but not intensity or duration descriptors; and (2) rainfall loss accounting using a  $\phi$  rate and a specific intensity duration distribution of the function  $i(t) = 1.5P(5(1+24t/T)^{-1/2}-1)/T$ , where  $i(t)$  is the intensity at time  $t$  for a storm of duration  $T$ . Thus, the curve number method is found to be a special case of  $\phi$  index loss accounting. The two methods are reconciled through the identity  $1.2S = \phi T$ , leading to the relationship  $CN = 1200/(12+\phi T)$ . The effects of rainfall intensity on curve number are felt through deviations from the necessary causative intensity-duration curve. Some sample alternate distributions are explored and the effects on curve number shown. Limitations are discussed.

INTRODUCTION

BACKGROUND

Runoff curve numbers are coefficients used in the calculation of "direct storm runoff" by a methodology first pioneered by and still strongly identified with the U.S. Soil Conservation Service (SCS). The primary reference on the subject matter is their National Engineering Handbook, Section 4, Hydrology (10), herein abbreviated "NEH-4." Direct storm runoff is taken to be a function of antecedent rainfall, storm rainfall depth  $P$ , and a storage index  $S$  (variable with antecedent rainfall and land condition), in accordance with the equation

$$Q = (P - 0.2S)/(P + 0.8S) \quad (1)$$

where  $P$ ,  $Q$ , and  $S$  are in inches depth, and  $P > 0.2S$ . The general algebraic and geometric relationships in Eq. (1) are shown in Figure 1. Note that as  $P$  increases,  $P-Q$  approaches 1.2S.

The storage index  $S$  is an indicator of land condition, and may vary from 0 (an impervious watershed;  $Q = P$ ) to infinity (a completely absorbent watershed;  $Q = 0$ ), with virtually all realistic values falling in between. To create a more intuitively pleasing expression of  $S$ , and to "linearize" the relationships (10),  $S$  was transformed by the equation:

$$CN = 1000/(10+S) \quad (2)$$

to define "curve number,"  $CN$ . It is dimensionless and varies from 100 (completely impervious;  $Q = P$ ) to 0 (completely absorbent;  $Q = 0$ ).

The technique is normally applied in a design mode, using a handbook estimate of  $CN$  in accordance with soil, vegetation type, land condition and moisture status, and a design frequency storm. Runoff hydrographs may be subsequently calculated via a standard unit hydrograph procedure. It is, of course, also possible to determine  $S$ , and thus  $CN$ , from any real  $P$  and  $Q$  data pair by solution of Eq. 1 for  $S$  to

$$S = 5(P + 2Q \sqrt{4Q^2 + 5PQ}) \quad (3)$$

PROBLEMS IN APPLICATION

The very simplicity of the methodology which makes it universally appealing becomes the point of contention when the technique is taken beyond the faithful application suggested in NEH-4. Real world

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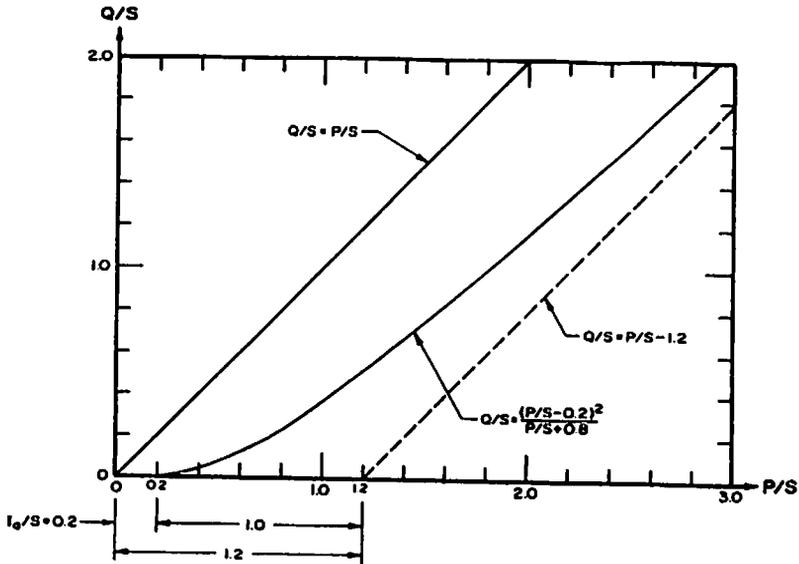


Figure 1. The SCS rainfall-runoff relationship standardized on the storage index  $S$ .

exceptions to the watershed "model" in Eq. 1 are not difficult to find (6, 8). Also, the complaint is often aired that Eq. 1 makes no recognition of rainfall intensity, although explanation diagrams in NEH-4 suggest an infiltration process. Smith (9) has elaborated in detail on the topic:

1. The CN methodology cannot respond to differences in storm intensity...It cannot distinguish between the effect of 4 inches of precipitation in 1 hour, and 4 inches in 12 hours, although both the infiltration amounts and runoff rates will be considerably different.
2. Closely related to the above, the SCS methodology does not properly predict initial abstractions ( $I_a$ ) for shorter, more intense storms, since it assumes ( $I_a$ ) to be constant.
3. The CN method cannot be extended to properly predict infiltration patterns within a storm. ...Attempts to use the CN Method within a storm have highlighted its physical invalidity - the resulting infiltration decay curve ( $P > I_a$ ) is forced to rise and fall with rainfall rate, rather than be controlled by soil conditions as in nature.
4. The CN method postulates a maximum depth of infiltration ( $S$ ), after which all rainfall becomes runoff. Selection of an  $S$  to approximate response to short storms can produce poor results for extended storms. Existence of such an  $S$  is not physically supported."

Smith's specific comments reinforce the intuitive notion that a runoff coefficient should incorporate infiltration characteristics, which then must bring rainfall intensity into consideration as well.

Associating intensity and infiltration to curve number could lead to greater utilization of existing field measurements of infiltration. For example, Gifford and Hawkins have summarized grazing influences in infiltration (3). While not on a watershed-wide reference basis, these measurements do represent a sample of the real world, and may be used, with appropriate caution, in decision making and process models (4). Unfortunately, there is no documented relationship between infiltration rates and curve number, and, thus, the body of infiltration data is not fully usable.

#### ANALYSIS

The problems described above arise from the lack of a time dimension in the basic mechanics of the method. Without time considerations, neither infiltration nor intensity can be incorporated. The time concept must be introduced through new relationships or assumptions. A brief foray into such follows.

Through expansion of the numerator and polynomial division, Eq. 1 may be expressed as

$$Q = P - S \left[ 1.2 - \frac{S}{P + 0.8S} \right] \quad (4)$$

This may be seen as a form of hydrologic accounting

$$Q = P - \text{Losses} \quad (4a)$$

so that

$$\text{Losses} = S \left[ 1.2 - \frac{S}{P + 0.8S} \right] \quad (5)$$

Denoting Losses as "L", acknowledging S to be a constant, and differentiation with respect to time yields:

$$\frac{dL}{dt} = \frac{dP}{dt} \left[ \frac{S}{P + 0.8S} \right]^2 \quad (6)$$

The time rate of loss is  $dL/dt$  or infiltration rate  $f$ , while  $dP/dt$  is simply rainfall intensity,  $i$ . Thus,

$$f = i \left[ \frac{S}{P + 0.8S} \right]^2 \quad (7)$$

where P is the cumulative rainfall. A dimensionless expression, standardized on  $f/i$  and  $P/S$  is shown in Figure 2. Eq. 7 supports Smith's contention that "...infiltration... is forced to rise and fall with rainfall rate..." which was also observed by Aron et al. (1) in a digital model using the SCS method. Also, Simanton, Renard, and Sutter (8) found that observed curve numbers varied with storm intensity. Eq. 7 will give a reasonable and credible infiltration relationship with accumulated rainfall if the intensity is constant; otherwise, it produces erratic results. This matter will not be further pursued in this paper.

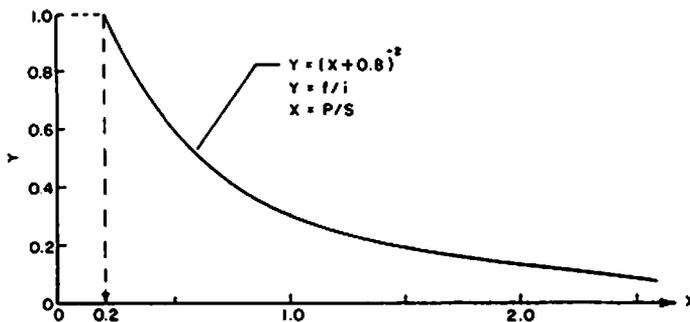


Figure 2. Dimensionless expression of infiltration rate from the SCS equation.

#### SYNTHESIS

As suggested above, rainfall intensity considerations are meaningful only when infiltration is also introduced. Horton's equation is perhaps the most widely used and easily visualized expression of infiltration, although it is pragmatic rather than analytic, and dependent on time only. Despite its empirical nature, it is a good illustration and reference model:

$$f(t) = f_0 + (f_c - f_0)e^{-kt} \quad (8)$$

where  $f_0$ ,  $f_c$ , and  $k$  are coefficients representing respectively the initial infiltration rate (at  $t = 0$ ), the long-term residual rate (as  $t \rightarrow \infty$ ), and a recession constant describing the rate that  $f(t)$  approaches  $f_c$  from  $f_0$ . The  $f_c$  descriptor is used here as a reference concept: a residual long-term or constant rate of water intake. In fact, a constant rate is often approached in field situations of  $i > f$  in very modest time frames such as 20 - 40 minutes, although this is certainly not a universal statement.

#### PHI INDEX

An average storm infiltration rate, called the  $\phi$  (phi) index has been commonly used to determine rainfall excess for generating composite unit hydrographs. Although it represents the combined effects of infiltration, interception and depression storage, it is functionally identical to average infiltration rate as taken from most plot studies. It requires the further simplifying assumptions of a constant rate process, and uniform distribution across the watershed area. As justification, it may be envisioned as either (1) a storm long average of an admittedly more complex phenomenon; (2) a real expression of  $f_c$  with the early low intensity storm bursts satisfying initial moisture demands and drawing the rate down to near  $f_c$ , or (3) a pragmatic assumption necessary to proceed without undue fuss.

$\Phi$  indexes have also been used in investigative situations. Arteaga and Rantz (2), and Lane and Wallace (7) have analyzed rainfall-runoff data for small semiarid watersheds, and found  $\phi$  to vary positively with storm intensity. This was interpreted as an expression of a variable source area, with accompanying variable loss rates.

### INTENSITY-DURATION

Rainstorms are of naturally varying characteristics, differing in their total depth  $P$ , duration  $T$ , the maximum and minimum intensities, as well as the distribution and sequence of rates within a storm. If the internal sequence is ignored, the individual storm intensities or bursts may be rearranged in descending order in as small a time increment as desired. The result is an intensity-duration curve directly analogous to the flow-duration curves so popular in conventional engineering hydrology. An explanatory example is seen in Figure 3.

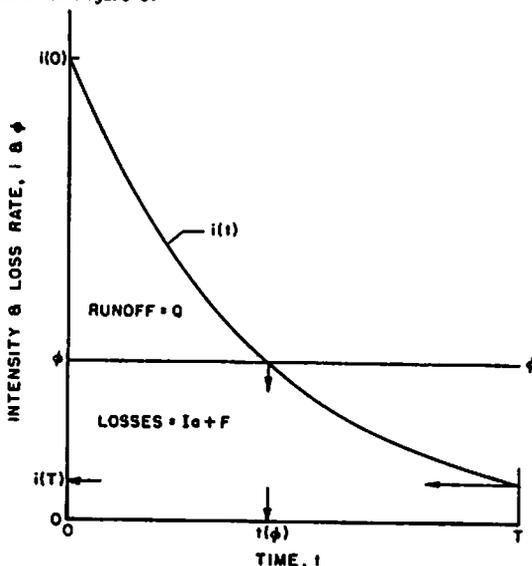


Figure 3. An idealized storm intensity-duration function, with  $\phi$ -index loss rate superimposed. Note that  $i(0)$  may be infinite, or that  $i(T)$  may be zero. Discrete or discontinuous functions are also possible.

For descriptive purposes, the distributions may be cast as algebraic functions such as frequency distributions are in statistics. Indeed, duration curves are forms of frequency or probability distributions, although this will not be pursued here. They may also be portrayed as discrete functions of finite time intervals. Both approaches will be illustrated later.

This tactic allows orderly classification of rainstorms without concern for the temporal sequence of bursts, and, thus, superimposition of the constant loss rate  $\phi$ . Two storms of different cumulative rainfall appearance may be identical when described by their intensity-duration characteristics.

### STORM RUNOFF: RAINFALL PARTITIONING

The superimposition of  $\phi$  on the intensity duration function produces (1) total storm rainfall excess (or runoff), and (2) the net time duration of the runoff,  $t(\phi)$ . The runoff volume is, by integration

$$Q = \int_0^{t(\phi)} i(t) dt - \phi t(\phi) \quad i(0) > \phi \quad (9)$$

where  $t(\phi)$  is merely the inverse of  $i(t)$ , at  $i = \phi$ . The total storm rainfall  $P$  is

$$P = \int_0^T i(t) dt \quad (10)$$

and the average storm intensity is

$$\bar{i} = \frac{P}{T} = \frac{1}{T} \int_0^T i(t) dt \quad (11)$$

A simple example will illustrate this. Suppose storm intensities occur triangularly distributed, from a maximum of  $i(0)$  (at  $t = 0$ ) to 0 (at  $t = T$ ). Then

$$i(t) = i(0) \left(1 - \frac{t}{T}\right) \quad (12)$$

It is then easily shown that

$$t(\phi) = T(1 - \phi/i(0)) \quad (12a)$$

$$P = i(0) T/2, \text{ and, not surprisingly,} \quad (12b)$$

$$\bar{T} = i(0)/2 \quad (12c)$$

Q is then calculated from (9) as

$$Q = \int_0^{t(\phi)} i(0) \left(1 - t/T\right) dt - \phi t(\phi) \quad (12d)$$

With appropriate integration and substitution, this yields

$$Q = (P - \phi T/2)^2 / P \quad (13)$$

This gives storm "runoff" as a function of storm rainfall ( $P$ ), loss rate ( $\phi$ ), and storm duration ( $T$ ) for a rainstorm of the assumed triangular intensity distribution. It is displayed in Figure 4. Similar calculations made for several other assumed continuous functions, and for a well known discrete storm distribution found in NEH-4, are summarized in Table 1. This approach will be called "rainfall partitioning," although it is merely a formalized  $\phi$  index rainfall accounting using the notion of storms with known intensity-duration characteristics.

Table 1. Characteristics of selected intensity-duration distributions and runoff.

Identification	$i(t)$	$i(0)$	$t(\phi)$	$I_a$	Q
Uniform <sup>1/</sup>	$i(0) = \text{constant}$	$P/T$	undefined	$\phi T$	$P - \phi T$
Triangular <sup>1/</sup>	$i(0)(1-t/T)$	$2P/T$	$T(1-\phi)$	$\phi T/2$	$(P - \phi T/2)^2 / P$
Quadratic <sup>1/</sup> , <sup>2/</sup>	$i(0)(1-t/T)^2$	$3P/T$	$T(1-\phi^2)$	$\phi T/3$	$P(1-3\phi + 2\phi^3/2)$
Exponential <sup>1/</sup> , <sup>2/</sup>	$i(0)(1-t/T)^n$	$(n+1)P/T$	$T(1-\phi^{1/n})$	$\phi T/(n+1)$	$P(1-\phi)[n(1-\phi^2)+1]$
Logarithmic	$abt^{b-1}$	infinite	$[\phi/(ab)]^{1/b-1}$	0	$\frac{1}{P^{1-b}(\phi T b)^{1-b}} \frac{b}{5} (1-b)^{4/5}$ $P - \phi T$
"CN" <sup>1/</sup> , <sup>2/</sup> , <sup>3/</sup>	$i(0)[5(1+24/T)^{-1/5}-1]/4$	$6P/T$	$T[(5/4\phi+1)^2-1]/24$	$\phi T/6$	$(P-0.25)^2/(P+0.85)$
NEH-4 Design (6 hour)	N/A	$4.44P/T$	N/A	$\phi T/4.44$	$\frac{\Sigma(1-\phi)\Delta t}{P-\phi T} \frac{(1-\phi)}{(\phi < i_{min})}$

Notes: 1. for all  $P > I_a$ ; 2.  $P = \phi/i(0)$ ; 3.  $5 = \phi T/1.2$ ; 4.  $P < \phi T/b$ ; 5.  $P > \phi T/b$ .

#### SCS EQUIVALENTS

By comparison of Figures 1 and 3 and their associated algebra, some identities between the two systems may be written.

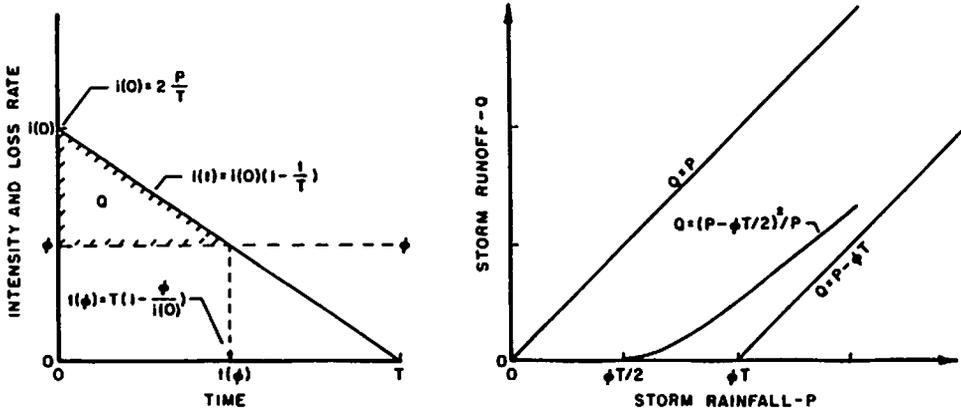


Figure 4. Illustrative example of intensity-duration accounting and resulting rainfall-runoff relationship.

Maximum losses: The maximum possible difference between rainfall and runoff in the SCS method is 1.25. With rainfall partitioning, and for any intensity distribution, this would be  $\phi T$ , or continuous losses over the storm duration  $T$  at rate  $\phi$ . Thus,

$$\phi T = 1.25 \quad (14)$$

or

$$\phi = \frac{1.25}{T}; S = \frac{\phi T}{1.2} \quad (14a)$$

Thus, loss rate  $\phi$  (an infiltration rate?) may be described in terms of  $S$ , and, thus, CN and storm duration. The inclusion of storm duration reaffirms the earlier held principle of the necessity of a time dimension. By substituting the above into (2)

$$CN = \frac{1200}{12 + \phi T} \quad (15)$$

and

$$\phi = \frac{12}{T} \left( \frac{100}{CN} - 1 \right) \quad (15a)$$

The storm duration (i.e., time) consideration cannot be escaped. CN is not a function of  $\phi$  alone.

#### "Initial abstraction."

The minimum rainfall necessary to initiate runoff in the SCS method is taken as 0.25, a relationship only poorly documented in NEH-4. For  $P < 0.25$ ,  $Q = 0$ . With rainfall partitioning, this threshold is attained at  $\phi = i(0)$ . For the triangular distribution example previously described, this can be shown to be  $\phi T/2$ . Equating this to 0.25 is inconsonant with Eqs. 14 and 15: equivalences may be stated between either the initial abstractions or the ultimate losses, but not necessarily both. The latter was used as a standard for comparison because of its key role in SCS hydrology conceptualization.

#### SYNTHESIS OF SCS EQUATION

At this point, the challenge may be issued to discover the intensity-duration function which, with  $\phi$  index loss accounting, will produce the standard SCS runoff equation (Eq. 1). As shown in Table 1, this is found to be

$$i(t) = \frac{i(0)}{4} \left[ \frac{5}{\sqrt{1 + 24t/T}} - 1 \right] \quad (16)$$

This will be called the "CN" distribution. A derivation for it is found in Appendix II. The runoff expression from the rainfall partitioning approach reduces to Eq. 1 with the introduction of  $S = \phi T/1.2$  and  $i(0) = 6P/T$ . Figure 5 shows Eq. 16 graphically.

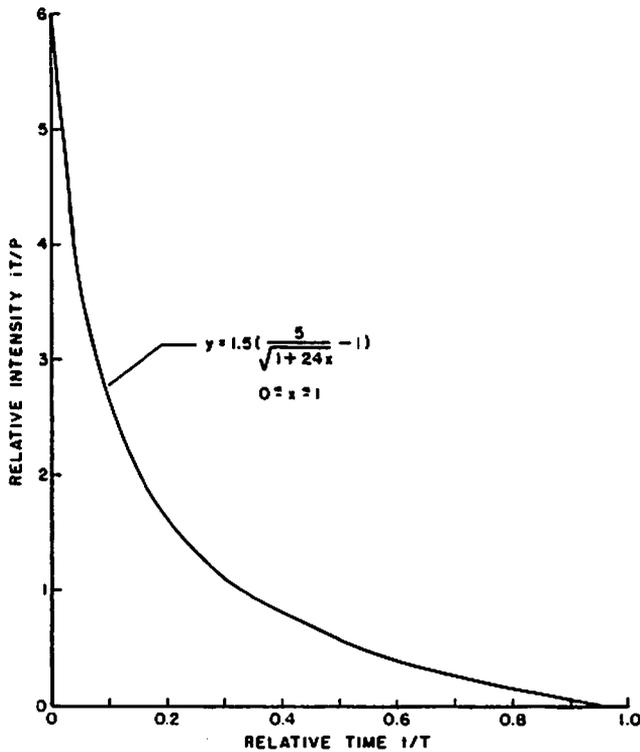


Figure 5. Intensity-duration distribution necessary to generate SCS rainfall-runoff relationship with  $\phi$ -index loss accounting.

#### EFFECTS OF INTENSITY

##### STRATEGY

There are now two means of generating storm runoff from storm rainfall to match the conventional SCS relationships (1) the formula itself, i.e., Eq. 1., which inputs only total storm rainfall and a curve number; and (2) the described rainfall partitioning approach, which draws on storm volume and duration, a constant loss rate, and a fixed distribution of rainstorm intensities. To the extent that the partitioning approach is valid, the intensity characteristics of other storm distributions may be examined for the effect on runoff, and, thus, (by equivalence with Eq. 1) their effects on curve number (CN). The question is further reduced to the difference between the intensity-duration distributions, or more specifically, deviation from Eq. 16, the so-called "CN" distribution.

This places importance on rainstorm characteristics. There is little literature dealing directly with rainstorm intensity-duration as used herein. However, the distributions may be inferred from other standard forms of hydrometeorological presentations, as discussed in limited detail in Appendix IV. For the purposes of discussion here, either theoretical or institutionalized design distributions may be used. Figure 6 shows a 6-hour design distribution found in NEH-4. Differentiation and reordering, as shown in Figure 6, leads to an intensity duration plot. It, along with the theoretical triangular distribution, will be used for example comparisons in the following section.

##### COMPARISONS

The effects of intensity may now be examined by comparing runoff calculated by rainfall partitioning against that from Eq. 1. This presumes that rainfall partitioning is the more physically justifiable or process documented approach, or is at least the less incredible option.

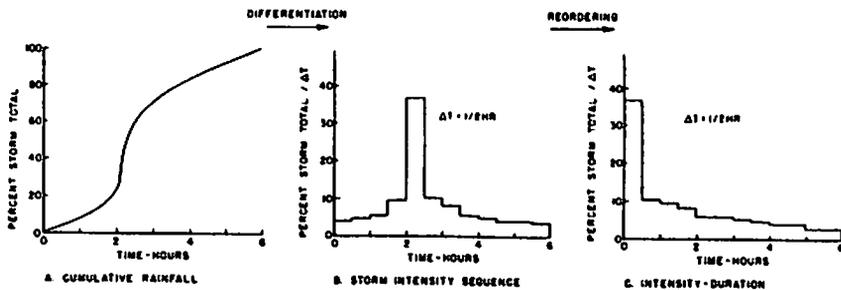


Figure 6. Illustrative description of rainstorm intensity-duration from design cumulative rainfall.

Three approaches will be compared: 1) the SCS formula or direct handbook method as a base; and rainfall partitioning with both 2) the NEH-4 6-hour design storm distribution and 3) the triangular distribution previously set forth. As fixed by the NEH-4 design storm, a 6-hour duration is used, as is an arbitrary example choice of  $CN = 80$ . Thus, from Eq. 2,  $S = 2.50$  in, and from Eq. 15a,  $\phi = 0.50$  in/hr.

For a storm of  $P = 3$  in, the following results are obtained (See Table 1 for calculating equations).

1. SCS formula (Eq. 1)  $Q = 1.25$  in
2. NEH-4 design storm  $Q = 0.95$  in
3. Triangular distribution  $Q = 0.75$  in

The 3.0 rainfall and the above runoffs define apparent curve numbers via Eq. 3 as follows:

1. SCS formula  $CN = 80.0$
2. NEH-4 design storm  $CN = 74.7$
3. Triangular distribution  $CN = 70.8$

These calculations may be made for a spectrum of rainfalls. Results for rainfalls up to 5 in are shown in Figure 7. For these cases, the results indicate that to achieve the same runoffs from rainfall partitioning by the SCS formula, curve numbers should be reduced accordingly, implying that the NEH-4 formula (Eq. 1) overestimates runoff.

Other distributions might also be used, drawn on specific local meteorological data or on inferred local design intensity duration functions, as outlined in Appendix IV.

## DISCUSSION

### USAGE

The rationale and methodology developed in this paper may be applied in two ways. First, as a guide in the selection of curve number in a design application; the choice should depend on the intensity distribution of the storm at hand as compared to the  $CN$  distribution. No rules-of-thumb are offered, but some guidelines may be implied from the in-text experience with the NEH-4 6-hour design storm.

Second, where conditions merit, the rainfall partitioning approach might be used as an alternate to the curve number method. As with all untested methods, it should be applied with full user awareness of the assumptions, and insofar as possible, within the limitations thus imposed and outlined below.

### LIMITATIONS

The contrasts made and inferences for curve number influences presume that the  $\phi$  index - rainfall partitioning method is at least comparable (perhaps superior) to the runoff curve number method. Unfortunately, there is scant documentation to support either opinion. Also, rainfall partitioning requires inputs of intensity-duration information and a  $\phi$  index. The former may be approximated from reduced meteorologic presentations of depth-duration-frequency. The latter is intended to correspond to a constant basin-wide loss rate, or (approximately) residual infiltration rate,  $f_c$ , for which a large body of knowledge already exists. Application should be limited to situations of homogeneous small watersheds

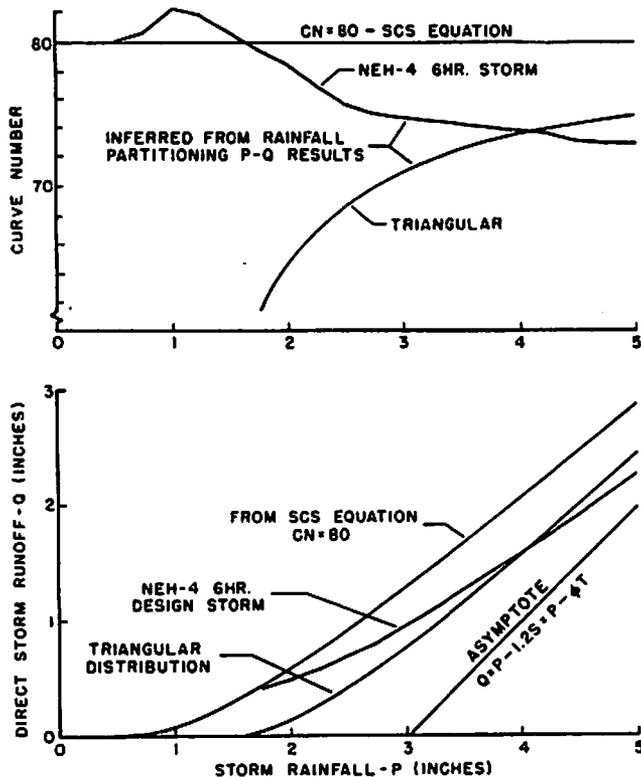


Figure 7. Comparisons of runoff and apparent curve number for a six-hour storm with CN = 80, by NEH-4 method and by rainfall partitioning for NEH-4 six-hour design storm and a triangular distribution of storm intensities. Although differences in runoff do exist for the upper two curves below about P = 1.75 in, the plotting scales do not permit showing them. The upper plot shows the apparent CN, or the CN indicated by the P and the Q.

where rainfall intensities are areally constant at any instant. However, distributed forms of the method could also be developed.

#### FUTURE WORK

The questions raised here may merit more attention. The basic item to be serviced is the appropriate usage of the runoff curve number method, and possible adjustments, augmentations, or replacement. Thus, the rainfall partitioning approach developed here should also be examined and compared to the curve number method. This rapidly directs attention to intensity-duration descriptions of rainstorms, and to loss rates for various land types. Interpretation to curve number forms will demand transfer information for soil groups, vegetative types, and land treatment effects.

#### ACKNOWLEDGEMENTS

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#### APPENDIX 1 - Notation

The following symbols are used

- a - one hour rainfall for a specified frequency and location or one hour intercept in  $P(t) = at^b$
- b - characteristic exponent describing rainfall volume duration in  $P(t) = at^b$
- CN - runoff curve number  $1000/(10 + S)$  inches
- f - infiltration rate (general)
- $f_c$  - residual constant infiltration rate
- $f_0$  - initial infiltration rate
- F - total infiltrated depth following satisfaction of  $I_a$
- $I_a$  - initial abstraction, or rainfall depth necessary to initiate runoff in SCS equation. Equal to 0.2S
- $i(0)$  - maximum rainfall intensity
- $i(t)$  - minimum rainfall intensity
- n - exponent describing intensity duration function
- P - storm rainfall depth
- Q - storm runoff depth
- T - storm duration
- t - time

- $t(\phi)$  - time when  $\phi = i(t)$ , total net duration of rainfall excess  
 $S$  - watershed storage index in SCS equation  
 $\rho$  -  $\phi/i(0)$   
 $\phi$  - net constant loss rate

#### APPENDIX II

##### Derivation of "CN" Intensity Distributions

Referring to Figure III-1, a generalized rainfall partitioning diagram with a loss rate  $\phi$ , the following may be written:

First, from NEH-4,

$$Q = P - (I_a + F) \quad (III-1)$$

in which  $I_a + F \rightarrow I_a + S$  as  $P \rightarrow \infty$ , and which leads to

$$I_a + F = 1.2S - \frac{S^2}{(P + 0.8S)} \quad (III-2)$$

a result obtained by expansion and polynomial division in Eq. 1.

Second, the rainfall volume under the  $\phi$  rate in Figure III-1, in differential terms is

$$td\phi = d(I_a + F) \quad (III-3a)$$

$$t = \frac{d}{d\phi} (I_a + F) \quad (III-3b)$$

Substituting Eq. III-2 and  $S = \phi T/1.2$  (i.e., Eq. 13a) in III-3b leads to

$$I_a + F = \phi T - \frac{\phi^2 T^2}{1.2^2(P + 0.8\phi T/1.2)} \quad (III-4)$$

Differentiating with respect to  $\phi$  as per Eq. III-3, and simplifying produces

$$t = T \left[ 1 - \frac{25\phi T}{6} \frac{3P + \phi T}{(3P + 2\phi T)^2} \right] \quad (III-5)$$

This may be simplified by letting  $x = t/T$  and  $y = \phi \frac{T}{P}$ . Solving via the quadratic equation gives

$$y = \phi \frac{T}{P} = 1.5 \left[ \frac{5}{\sqrt{1 + 24x}} - 1 \right] \quad (III-6)$$

We are concerned with the situation of  $\phi = 1$ ; thus, resubstituting  $x$  and simplifying yields

$$i = i(t) = 1.5 \frac{P}{T} \left[ \frac{5}{\sqrt{1 + 24t/T}} - 1 \right] \quad (III-7)$$

It can be seen that  $P/T$  is the mean storm intensity  $\bar{T}$ , and that when  $t = 0$ ,  $i(0) = 6P/T$ , or that the maximum intensity is 6 times the mean.  $t(\phi)$  is defined by the Eq. III-5.

#### APPENDIX III

##### Intensity duration Relationships with Storm Size and Average Intensities

The term "intensity-duration" as used in this paper is intended as analogous to flow duration in surface water hydrology, except that the time base is storm duration, and not an average annual sampling period. It is a reordering or rearrangement of storm rainfall intensities by descending values. Thus, two attributes result: (1) The slope is non positive, and (2) the area under the curve is the total storm rainfall. In functional representation, it will be called  $i(t)$ .

Another form of rainfall, "intensity-duration," also exists in widespread publications. For example, U.S. Weather Bureau Technical Paper 25 (11) gives curves of average intensities for durations to 24 hours. These should thus more properly be called "average intensity-duration" curves, to avoid confusion. In functional representation here, these will be called  $\bar{T}(t)$ .

Also, storm characteristics are sometimes described by total cumulative storm rainfall as a function of duration. A common form (5) is of the structure

$$P(t) = at^b \quad 0 < b < 1 \quad (IV-1)$$

Naturally, a probability or return period identification is also made. In Eq. IV-1, above, this is felt through the magnitude of the "a" coefficient. The three forms of storm intensity discussed ( $P(t)$ ,  $T(t)$ , and  $i(t)$ ) are mathematically related, as shown in the following.

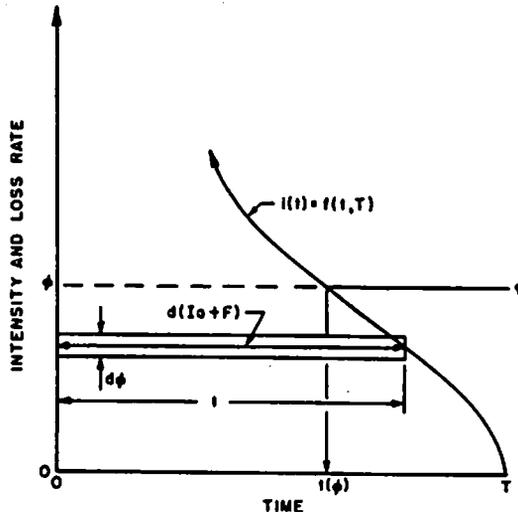


Figure III-1. Reference diagram for derivation of the intensity distribution function necessary to generate the SCS rainfall-runoff relations.

The average intensity at time  $t$  is achieved from Eq. IV-1 by division by  $t$ , or

$$\bar{T}(t) = \frac{P(t)}{t} = at^{b-1} \quad (IV-2)$$

The intensity-duration function is obtained by differentiation of Eq. IV-1 with respect to time, so that

$$i(t) = \frac{dP(t)}{dt} = abt^{b-1} \quad (IV-3)$$

As is easily seen, the three are related through a series of factors involving  $t$  and  $b$ . Thus, given any of the three forms, the other two may be obtained. Adherence to the exponential structure in Eq. IV-1 is necessary for these pleasing relationships. However, using the above logic, segmented or discontinuous functions can also be treated. Such literature is widespread, although the information is in various forms.

As an example, consider the 100-year rainfall for Los Angeles, as given in U.S. Weather Bureau TP 25. The following are scaled from the average intensity-duration plot: for 24 hours,  $\bar{T} = 0.095$  in/hr; for 1 hour,  $\bar{T} = .61$  in/hr. From this, substituting into Eq. IV-2, and solving two simultaneous equations, it can be found that  $a = .61$ , and  $b = .413$ . Inserting these into IV-3 produces

$$i(t) = .252t^{-0.587} \quad (IV-4)$$

Note that in this form  $i(0)$  is infinite, and thus, any rainfall  $> 0$  will have intensities greater than any non-infinite  $\phi$ , and thus some runoff, however small. Also note that  $i(T)$  will always be positive.