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Runoff Curve Numbers with Varying Site Moisture

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Introduction

Calculating direct storm runoff volume by using runoff curve numbers is an international technique. It was originated by, and is still strongly identified with, the United State Department of Agriculture Soil Conservation Service. The ultimate reference document on the methodology is their National Engineering Handbook, Section 4, Hydrology (1), hereinafter simply referred to as "NEH-4." In it, the algebraic and hydrologic relations between storm rainfall, soil-site storage, and storm runoff are given as follows:

$$Q = \frac{(P - 0.2S)^2}{P + 0.8S}, P \geqq 0.2S \quad \text{(1)}$$

$$\text{CN} = \frac{1{,}000}{10 + S} \quad \text{(2)}$$

in which P = storm rainfall; Q = direct runoff; CN = a dimensionless expression of S called "Curve Number"; and S = a watershed storage parameter. The CN may vary from 0 (Q = 0 for all P) to 100 (Q = P for all P). Several features should be noted: (1) Eq. 1 is valid only for $P \geqq 0.2S$; Q = 0 otherwise; (2) the system as stated is in inches, thus the 10 and 1,000 in Eq. 2 must carry inch dimensions, although conversions can be made to the metric system; (3) CN has no intrinsic meaning, it is only a convenient transformation of S to establish a 0 to 100 scale; and (4) steps in applying the procedure are well documented and standardized so that its usage is straightforward and reproducible.

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Tables and charts of CN for different vegetative types and hydrologic soil groups are presented in NEH-4 and elsewhere, although background calibration information on these is rare.

A naturally occurring problem in applying this method is the effect of watershed wetness on CN. As a runoff coefficient, CN must reasonably be expected to vary with soil and site moisture. This is handled in NEH-4 by the introduction

TABLE 1.—Definition of Antecedent Moisture Conditions

Class (1)	5-day Total Antecedent Rainfall, in inches	
	Dormant season (2)	Growing season (3)
I	<0.5	<1.4
II	0.5 – 1.1	1.4 – 2.1
III	>1.1	>2.1

Note: Source—NEH-4 (1), Table 4.2.

TABLE 2.—Relationships between CN and AMC Classes

Antecedent Moisture Class		
I (1)	II (2)	III (3)
100	100	100
87	95	98
78	90	96
70	85	94
63	80	91
57	75	88
51	70	85
45	65	82
40	60	78
35	55	74
31	50	70
22	40	60
15	30	50
9	20	37
4	10	22
0	0	0

Note: Source—NEH-4 (1), Table 10.1 (condensed).

of: (1) Three soil moisture classes, I, II, and III, which modify CN and are defined on the antecedent 5-day rainfall; and (2) a table of equivalent curve numbers, indicating appropriate alternate CN to be selected. Tables 1 and 2 present this information and Fig. 1 is a graphical presentation of Table 2 (from Table 10.1 in NEH-4 (1) which contains no analysis of the source of this information).

Some real and conceptual difficulties arise in applying this portion of NEH-4 procedures. First, the relationships are shown as discrete, and not continuous, thus implying sudden shifts in CN, with corresponding quantum jumps possible in calculated runoff. Secondly, NEH-4 contains no background development or statement of assumptions, leaving only appeals to agency authority as a foundation for professional beliefs, and not faith based on physical reasoning or reconciliation with reality. Both of these are grevious shortcomings, and thus some adjustment procedure is necessary.

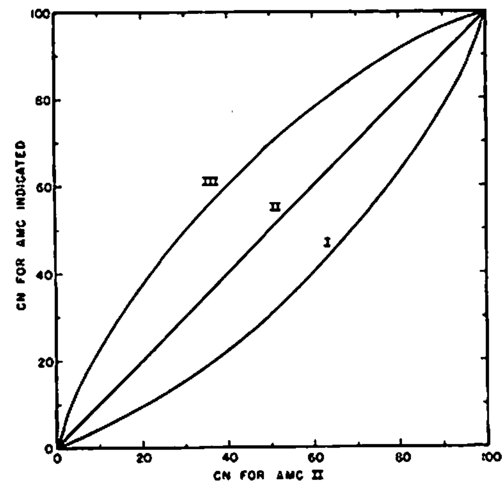

FIG. 1.—Effect of AMC on CN as Referenced on AMC II [Smoothed Curve of Points Adapted from Table 10.1 in NEH-4 (1)]

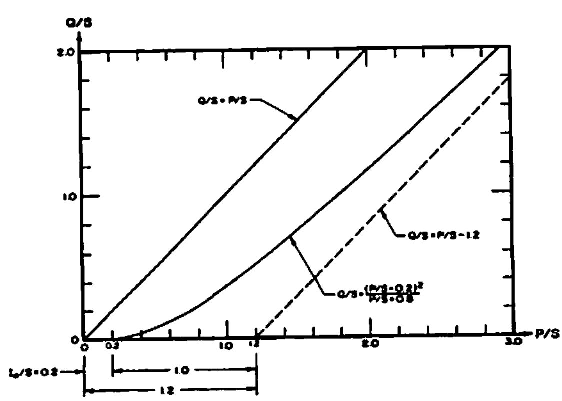

FIG. 2.—Graphical Representation of Eq. 1 or Eq. 6, or both, Standardized on S

Other than the tables in NEH-4, the literature on the topic is sparse. Sobhani (3) has developed algebraic expressions of the relationships between CN's under different moisture classes. These are

$$CN_{I} = \frac{CN_{II}}{2.334 - 0.01334\, CN_{II}} \quad (3)$$

$$\mathrm{CN}_{\mathrm{III}} = \frac{\mathrm{CN}_{\mathrm{II}}}{0.4036 + 0.0059\,\mathrm{CN}_{\mathrm{II}}} \quad \ldots (4)$$

These equations are empirical, and are curves fit statistically to Table 10.1 in NEH-4. They are based on relationships between S values under different moisture classes and are accurate within about ± 1 CN. Also, although not presented here, the NEH-4 points, as shown in Fig. 1, plot as three equally spaced and parallel straight lines on a double normal probability plot, with CN taken as a percent probability. Simanton, Renard, and Sutter (2) partitioned Antecedent Moisture Condition (AMC) Class I into four subdivisions to analyze data solely within that class. Their analysis, based on data from the semiarid Walnut Gulch watershed, suggested that CN increased as antecedent moisture increases. Williams and LaSeur (4) varied CN with soil moisture in a basin rainfall-runoff model by varying S with losses to runoff in a moisture accounting system, according to the expression

$$S = 20 - V \quad \ldots (5)$$

in which V = the site moisture storage, and all dimensions are in inches.

Eq. 5 assumes S ranges from 0 in. to 20 in. This concept of moisture status will be further developed on a conservation-of-mass basis to provide a logic-based alternate to the NEH-4 approach, and to avoid both of the objections previously described. It is also offered as an approximation of what may have been the original reasoning leading to the NEH-4 relationships, now apparently lost.

Development

Taking Eq. 1, expanding the numerator, and applying polynomial division yields

$$Q = P - S\left(1.2 - \frac{S}{P + 0.8S}\right), P \geqq 0.2S \quad \ldots (6)$$

A geometric representation of this is given in Fig. 2. It can be easily seen that the ultimate possible difference (as $P \rightarrow \infty$) between rainfall P and direct runoff Q is not S, but $1.2S$, denoted $\mathcal{V}$ here so that

$$\mathcal{V} = 1.2S \quad \ldots (7)$$

This may be envisioned as the total water storage available on the site, for a given condition of soil, vegetation, and moisture status. This makes no statement concerning the total soil water storage under such as an "oven dry" condition, but only as defined by the current state of soil moisture. The NEH-4 also makes no such distinction. As shown in explanatory Fig. 3, the total absolute capacity of the site is an unknown.

At time 1, given a curve number, the storage available is

$$\mathcal{V}_1 = 1.2S = 1.2\left(\frac{1{,}000}{\mathrm{CN}_1} - 10\right) \quad \ldots (8)$$

Any change in $\mathcal{V}$ will be the sum of evapotranspiration losses (ET), and interim rainfall inputs (P) less any runoff (Q), so that at time 2

$$\not V_2 = \not V_1 + ET - (P - Q) = 1.2S_2 \qquad (9)$$

Therefore, from Eqs. 8 and 9

$$\not V_2 = 1.2\left(\frac{1{,}000}{CN_1} - 10\right) + [ET - (P - Q)] = 1.2S_2 \qquad (10)$$

and by definition

$$CN_2 = \frac{1{,}000}{10 + S_2} \qquad (11)$$

Substituting and simplifying leads to

$$CN_2 = \frac{1{,}200}{\dfrac{1{,}200}{CN_1} + [ET - (P - Q)]} \qquad (12)$$

While Q is certainly a function of P and CN (through Eqs. 1 and 2). Eq. 12 is nonetheless the most practical simplification of the relationships. Attempts to further simplify lead to burdensome algebraic expressions. A solution in nomographic form is given in Fig. 4.

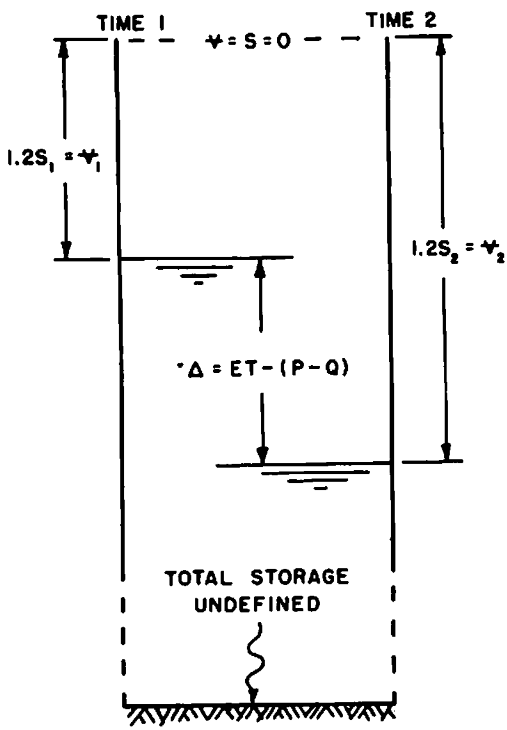

FIG. 3.—Diagrammatic Analogy of Site Moisture Defining $\not V$, S, and Δ

The terms ET, P, and Q are the interim or antecedent values that act on an initial condition of a given CN_1 to effect a change to CN_2. For clarity in understanding, they might be subscripted a to signify their antecedent nature.

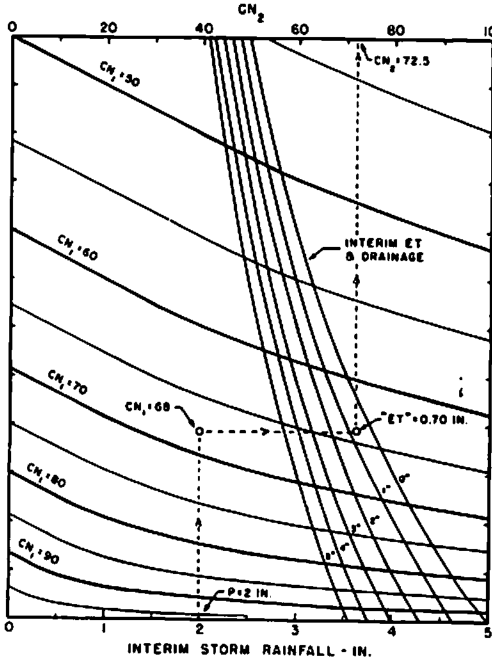

FIG. 4.—Nomographic Solution to Eq. 12, Solution for Example 1 is Shown

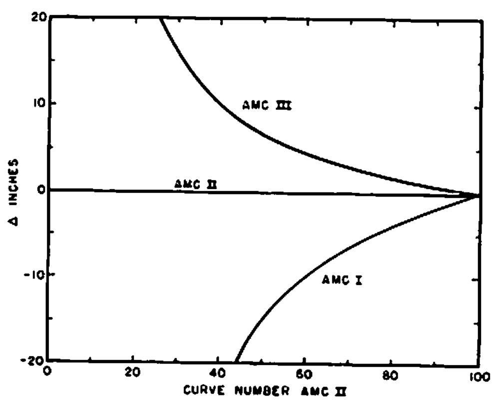

FIG. 5.—Site Moisture Storage Changes Required to Achieve NEH-4 Antecedent Moisture Criteria

The ET term should be understood to incorporate interim drainage losses as well as evapotranspiration.

Application

The method outlined might be used in place of the NEH-4 routines, i.e., as a substitute for Tables 1 and 2, herein. However, an estimate or assumption of the interval evapotranspiration losses must be supplied, which should incorporate allowances for any drainage. Thus, the following examples:

1. A forested watershed in midsummer is at CN = 68. Over 7 days a 2-in. storm occurs. What is the resulting CN?

Estimating ET (including drainage) at 0.10 in./day, or a total of 0.70 in., Eq. 12 gives CN_2 = 72.5. Alternate values of ET of 0.05 in./day and 0.15 in./day yield CN_2 = 74.1 and 71.0 respectively. Using NEH-4 methods, CN would not change. A graphical solution is given in Fig. 4.

2. In a dormant condition, a pasture has a CN = 82. Over 5 days, no rainfall occurs. What is the resulting CN?

Under dormant conditions, ET may be neglected. Using a nominal drainage value of 0.01 in./day and Eq. 12, CN_2 = 81.7. Using NEH-4 methods, CN_2 = 66.

3. How large a storm must fall on a watershed of CN = 80 to raise the CN to 85?

Assuming that ET = 0 during the storm, solving Eq. 12 with the information inserted yields $P - Q = 0.882$ in. From Eq. 6, this may be equated to

$$S\left(1.2 - \frac{S}{P + 0.8S}\right)$$

For CN_1 = 80, S = 2.5 in. Substituting and solving yields P = 0.951 in.

Analysis

Application.—This technique requires estimates of both evapotranspiration and drainage, neither of which are routinely available to the extent of information on regional design rainfall frequency-duration. Local guidelines or design estimates are certainly needed, and might be drawn from available field plot, lysimeter, climatologic, and watershed data. For perspective, evapotranspiration estimates might be limited to an upper level of about 0.25 in./day under optimum conditions, and/or proportioned annual or seasonal P-Q rates where local runoff data is available. Drainage data on a watershed basis is almost nonexistent, and it can only be estimated with knowledge of local soils, geology, and baseflow. However, unmanageable as this may seem, the currently used NEH-4 procedure implies both ET and drainage in its AMC classification and CN transfer routine (Tables 1 and 2, and Fig. 1, herein) without conscious recognition of their roles. Definition of the problem and analysis of its components, as undertaken here, should direct attention to the underlying processes, and hopefully lead to more credible estimates of both curve number and direct runoff.

Comparisons.—The preceding exercises invite comparisons with NEH-4 procedures. Unfortunately, there are differences in assumptions and resolutions that make such attempts both confusing and not entirely appropriate. For example, there is no clear definition herein of "AMC-II," the standard reference moisture condition. Curve number is taken only as a site water storage deficit, and NEH-4 provides no quantitative descriptions of these terms. The ET term is omitted from the NEH-4 methodology, although it undoubtedly plays a role and is included in a masked form. An additional constraint is the adherence to a 5-day time interval.

However, some enlightenment can be gained by simply calculating, via Eq. 12, the interim change of site storage [ET − (*P* − *Q*)] necessary to achieve the curve number changes dictated in NEH-4 and shown in Table 2. Table 3 gives this calculation, and a graphical presentation is given in Fig. 5. It indicates, for example, that to convert from CN = 75 at AMC II to CN = 57 at AMC I, 5.05 in. of moisture storage must be evacuated in 5 days. Similarly, to convert

TABLE 3.—Site Moisture Storage Changes Required to Achieve NEH-4 Antecedent Moisture Criteria

CN AMC II (1)	CN AMC I (2)	Δ, in inches (3)	CN AMC III (4)	Δ, in inches (5)
100	100	0	100	0
95	87	−1.16	98	0.39
90	78	−2.05	96	0.83
85	70	−3.03	94	1.35
80	63	−4.05	91	1.81
75	57	−5.05	88	2.36
70	51	−6.39	85	3.03
65	45	−8.21	82	3.83
60	40	−10.00	78	4.62
55	35	−12.47	74	5.60
50	31	−14.71	70	6.86
40	22	−24.55	60	10.00
30	15	−40.00	50	16.00
20	9	−73.33	37	27.57
10	4	−180.00	22	65.45
0	0	—	0	—

from CN = 75 at AMC II to CN = 88 at AMC III, a net gain of 2.36 in. must be achieved in 5 days. These figures may be contrasted with reasonable expectations based on Tables 1 and 2, site considerations of soil and vegetative characteristics, and drainage and evaportranspiration. Clearly, reasonable comparisons are strained in the lower curve number ranges, say below about CN = 70. These contrasts are clearer for AMC III when one accounts for evapotranspiration, drainage, and runoff. Thus, for CN = 70 a rainfall of 3.17 in. and no evapotranspiration or drainage is required to achieve the Δ of 2.36 in. Similarly for AMC I and CN = 70, evapotranspiration and drainage alone, with no interim rainfall, must account for 6.39 in. of moisture evacuation.

Limitations.—Moisture losses to evapotranspiration and drainage should be limited by site conditions. Although no elaboration is given, NEH-4 gives a rudimentary definition of AMC I as ". . . the lower limit of moisture.", suggesting a boundary. Thus, e.g., drawing from Table 3, no more than 3.03 in. of net site losses may be possible on a site where CN-II is 85. Similar reasoning might also imply an upper limit as specified by AMC III, which NEH-4 defines as ". . . the upper limit of moisture." However, adhering to this could deny the possibility of a CN approaching 100 on any site by virtue of complete saturation.

A conceptual limitation of interest is the basic definition of CN solely in terms of site moisture status (albeit with the limitations described previously), and not in infiltration characteristics. Ideally, the capacity of a site to withhold rainfall from runoff is not only a function of the site storage capacity, but the loss rate thresholds as well. Relationships between the two must either be assumed implicitly or taken to be unimportant. Changes in land condition or practice would influence both of these properties.

Future Directions.—The conservation of mass approach to curve moisture relationships described offers several opportunities of reasoned manipulation, modification, and improvement in applying the runoff curve number method. Local or regional modification of the moisture relationships might be established based on the analysis of rainfall records and site properties. For greater ease of comprehension and more meaningful algebraic expression, the runoff Eq. 1 and the curve number Eq. 2 might be recast in terms of $\mathcal{V}$ rather than S, leading to

$$Q = \frac{\left(P - \frac{\mathcal{V}}{6}\right)^2}{P + \frac{2}{3}\mathcal{V}}, \; P \geqq \frac{\mathcal{V}}{6} \quad \ldots\ldots (13)$$

$$\mathrm{CN} = \frac{1{,}200}{12 + \mathcal{V}} \quad \ldots\ldots (14)$$

Also, hopefully, the site moisture capacities implied by Eq. 1 and further developed here could be used in defining soil hydrology and curve numbers in future design and environmental impact work. The role of infiltration capacity and its relationship to curve number/site moisture properties needs to be explored, explained, and applied to this technology.

Summary and Conclusions

Runoff curve numbers may be defined in terms of site moisture levels, and this interpretation was exploited to explain and enlarge upon presently used techniques. The relationships derived may, with judgment, be used as an alternate to handbook methods. The procedure provides insights and raises issues, and may be used as a point of departure for future work.

Acknowledgments

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Appendix I.—References

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Appendix II.—Notation

The following symbols are used in this paper:

CN = runoff curve number, defined as $1{,}000/(10 + S)$;
ET = evapotranspiration and drainage losses;
P = interim storm rainfall;
Q = direct storm runoff, as defined by $Q = (P - 0.2S)^2/(P + 0.8S)$;
S = an index of potential site retention;
$\mathcal{V}$ = maximum potential site retention = $1.2S$; and
Δ = net site moisture change = ET + $(P - Q)$.

Subscripts

1, 2 = beginning and ending times of study interval; and
I, II, III = antecedent moisture class (AMC) as defined by NEH-4.

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KEY WORDS: **Antecedent precipitation**; Curves; Rainfall; **Runoff; Soil moisture**; Watershed management; Watersheds

ABSTRACT: Runoff curve numbers (CN) as used to estimate runoff volume (Q) from storm rainfall (P), are usually modified by handbook methods to account for the influence of watershed wetness. An alternate technique, based on the SCS rainfall-runoff formula and conservation of mass, is developed, illustrated, and discussed. The expression developed is: $CN_2 = 1{,}200/(1{,}200/CN_1) + ET\text{-}(P\text{-}Q)$. The methodology requires inputs of the interim evapotranspiration and drainage (ET), but is not unreasonably sensitive to estimates of them.

REFERENCE: Hawkins, Richard H., "Runoff Curve Numbers with Varying Site Moisture," *Journal of the Irrigation and Drainage Division*, ASCE, Vol. 104, No. IR4, **Proc. Paper 14254**, December, 1978, pp. 389-398