Testing a theoretical resistance law for overland flow on a stony hillslope

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Abstract
Overland flow, sediments, and nutrients transported in runoff are important processes involved in soil erosion and water pollution. Modelling transport of sediments and chemicals requires accurate estimates of hydraulic resistance, which is one of the key variables characterizing runoff water depth and velocity. In this paper, a new theoretical power–velocity profile, originally deduced neglecting the impact effect of rainfall, was initially modified for taking into account the effect of rainfall intensity. Then a theoretical flow resistance law was obtained by integration of the new flow velocity distribution. This flow resistance law was tested using field measurements by Nearing for the condition of overland flow under simulated rainfall. Measurements of the Darcy–Weisbach friction factor, corresponding to flow Reynolds number ranging from 48 to 194, were obtained for simulated rainfall with two different rainfall intensity values (59 and 178 mm hr\(^{-1}\)). The database, including measurements of flow velocity, water depth, cross-sectional area, wetted perimeter, and bed slope, allowed for calibration of the relationship between the velocity profile parameter \(\Gamma\), the slope steepness \(s\), and the flow Froude number \(F\), taking also into account the influence of rainfall intensity \(i\). Results yielded the following conclusions: (a) The proposed theoretical flow resistance equation accurately estimated the Darcy–Weisbach friction factor for overland flow under simulated rainfall, (b) the flow resistance increased with rainfall intensity for laminar overland flow, and (c) the mean flow velocity was quasi-independent of the slope gradient.

KEYWORDS
dimensional analysis, flow resistance, overland flow, rainfall, self-similarity, stony hillslopes, velocity profile

INTRODUCTION

Erosion by rainfall and runoff causes detachment, transport, and deposition of soil on hillslopes and is a major driver in landscape evolution. Water erosion processes are generally characterized as either interrill or rill erosion, which behave differently. On interrill areas, the main erosion process is due to raindrop impact, and the sediment is transported by flow from these areas to rills. Soil particle detachment in interrill areas is due to rainsplash, whereas the transport of detached sediments to rills is due to overland flow, characterized by shallow water depth and bed shear stress (Mutchler & Young, 1975; Toy, Foster, & Renard, 2002). Research on this topic aims to improve the understanding of overland flow and sediment transport under different conditions of morphology, soil type, vegetation cover, and rainfall regime (Dunkerley, 2018; Smith, Cox, & Bracken, 2007).
Flow resistance is fundamental to evaluate overland flow velocity, which can be also be used to estimate flow transport capacity (Ferro, 1998). Overland flow resistance is characterized by two main variables, which are the soil roughness and the rainfall intensity.

Katz, Watts, and Burroughs (1995) suggested that flow resistance over a smooth and sloping bed is the result of both the internal fluid resistance and the frictional resistance due to the channel boundaries. In the case of uniform laminar flow, if the flow shear stress varies linearly with the distance from the bed and the flow velocity is zero at the boundary, the local velocity increases parabolically upwards in the flow, reaching a maximum value of velocity at the free surface (Yalin, 1977). For laminar flow conditions, the Darcy–Weisbach friction factor $f$ assumes the following expression:

$$ f = \frac{K}{Re} $$

in which $K$ is a constant equal to 24 for uniform flow on a smooth bed. The overland flow under rainfall may be considered to be as a "disturbed laminar flow," because the impact of raindrops on a flow causes both an increase of flow turbulence and a retarding effect on mean flow velocity (Emmett, 1970).

Yoon and Wenzel (1971) suggested that, for $Re > 2,000$, the rainfall intensity effects on flow resistance are negligible, whereas for $Re < 2,000$ flow resistance always increases with rainfall intensity. The local flow velocity is influenced by rainfall with a retarding effect, which is higher near the free surface. This retarding effect is explainable by the concept of momentum transfer. The component of momentum carried by raindrops in the mean flow direction is negligible, if the direction of fall of raindrops is more or less normal to the water surface. For this reason, a part of mean flow momentum must be transferred to the added water in order to accelerate the raindrop mass. The transfer of mean flow momentum is not equally distributed, but it is greater near the surface, and so, the velocity retardation decreases from the surface downward. Taking into account that for given rainfall intensity and drop size, the rate of raindrop impact is constant, whereas the mean flow momentum increases with the flow Reynolds number; the ratio of momentum transfer diminishes with increasing Re values.

Unfortunately, after the experiments by Yoon and Wenzel (1971), few authors have carried out studies on flow resistance regarding overland flow with rainfall simulation.

Shen and Li (1973) investigated an overland flow in a 0.6-m-wide and 18.3-m-long flume, with plexiglass walls and a stainless steel bottom, using a rainfall simulator (with rainfall intensities ranging between 190 and 444 mm hr$^{-1}$). These authors confirmed that the Darcy–Weisbach friction factor depends on both the flow Reynolds number and the rainfall intensity for $Re$ values less than 2,000.

Savat (1977) carried out some experimental runs, characterized by a single rainfall intensity (60 mm hr$^{-1}$), for the determination of the average depth of a sheet flow using a simple weighing of the entire run and the water flowing in it. The experiments demonstrated that the effect of rainfall diminishes when the degree of flow turbulence or the slope angle increases.

Laboratory investigations were carried out by Katz et al. (1995) in a flume, 4.87 m long and 1 m wide, with a rough bed obtained by glued sand grains. Three slopes (4%, 6%, and 8%) and two simulated rainfall intensities (40.9 and 115.1 mm hr$^{-1}$) were used. These authors demonstrated that the roughness due to raindrop impact and the channel bed friction affect the $K$ coefficient of Equation (1).

Mügler et al. (2011) carried out an experiment, on a 10 m × 4 m rainfall simulation plot, to test four different roughness models with high-resolution velocity data. The analysis showed that the best results were obtained using a calibrated Manning coefficient $n$ model with a depth-dependent roughness formulation (Jain, Kothyari, & Ranga Raju, 2004).

Nearing et al. (2017) carried out experiments applying artificial rainfall (with rainfall intensities of 59 and 178 mm h$^{-1}$) to 2 m × 6 m plots at 5%, 12%, and 20% slope gradients. The experiments aimed to study the evolution of the surface roughness on a stony hillside and to test the slope–velocity equilibrium hypothesis. Nearing, Kimoto, Nichols, and Ritchie (2005) hypothesized that stony hillside in the semi-arid environment evolve to a state of “slope–velocity equilibrium.” The slope–velocity equilibrium is a state that evolves naturally over time due to the interactions between overland flow, erosion processes, and bed surface morphology, wherein steeper areas develop a relative increase in physical and hydraulic roughness such that flow velocity is a unique function of overland flow discharge independent of slope gradient.

Nearing et al. (2017) noticed that there was no relationship between the final rock cover of the investigated plots and either the slope gradient or the initial rock cover. The final random roughness (at the end of the experiment) varied with the slope gradient, with a rougher final surface resulting on steeper slopes. Measured runoff velocities supported the hypothesis that flow velocity is only dependent on unit flow discharge and, as a consequence, is independent of slope gradient. In other words, according to the experimental results by Nearing et al. (2017), the steeper slopes evolve to rougher surfaces compared with shallower slopes, and this increase of roughness with slope gradient. The hypothesis of slope–velocity equilibrium implies that the use of hydraulic equations, such as Manning and Darcy–Weisbach, in hillslope-scale runoff models can be problematic because the friction factors vary with both slope and rainfall intensity.

The slope–velocity equilibrium condition is a result similar to the feedback mechanism detected by Govers (1992); Di Stefano, Ferro, Palmeri, & Pampalone, 2018; Palmeri, Pampalone, Di Stefano, Nicolais, & Ferro, 2018) for rill flows and highlights the conclusion that if a constant roughness coefficient is assumed, then “velocity will be over-predicted on steep slopes and under-predicted on shallower slopes” (Govers, Takken, & Helming, 2000).

Despite the fact that the retarding effect of rainfall impact and the increase of flow turbulence are recognized effects, few studies have been undertaken on the effects of rainfall on hydraulic resistance (Ferro & Nicosia, 2020).

The aim of this paper was to modify a theoretical power–velocity profile, originally deduced neglecting the effect of rainfall impact,
taking into account the rainfall intensity effect. Then, a theoretical flow resistance law was obtained by integration of the new flow velocity distribution. The flow resistance law was tested using field measurements by Nearing et al. (2017) for an overland flow with simulated rainfall on stony hillslopes. The available measurements allowed (a) calibration of the relationship between the velocity profile and a dimensionless group representative of rainfall intensity; (b) assessment of the influence of rainfall on flow resistance; (c) assessment of the applicability of the theoretical flow resistance equation for an overland flow disturbed by rainfall impact; and (d) verification of the hypothesis of quasi-independence between the flow velocity and the slope gradient.

2 | THE THEORETICAL FLOW RESISTANCE EQUATION

One of the main tasks of open-channel flow hydraulics is deducing a theoretical flow resistance law by integration of a known flow velocity distribution in the cross section (Ferro, 1997; Powell, 2014).

For overland flow, subjected to rainfall, on a stony hillslope, the local flow velocity profile \( v(y) \) along a given vertical can be expressed by the following functional relationship (Barenblatt, 1987, 1993; Ferro, 1997, 2017, 2018):

\[
\phi \left( \frac{dv}{dy}, y, h, d, u_*, s, i, \rho, \mu, g \right) = 0, \tag{2}
\]

in which \( \phi \) is a functional symbol, \( y \) is the distance from the bottom, \( h \) is water depth, \( d \) is a characteristic diameter of the soil particles representative of the grain roughness, \( u_* = \sqrt{\frac{gR}{s}} \) is the shear velocity, \( R \) is the hydraulic radius, \( s \) is the slope, \( i \) is the rainfall intensity, \( \rho \) is the water density, \( \mu \) is the water viscosity, and \( g \) is acceleration due to gravity.

According to the \( \Pi \)-theorem (Barenblatt, 1987), Equation (2) can be expressed in a dimensionless form as follows:

\[
\Pi_1 = \phi_1(\Pi_2, \Pi_3, \Pi_4, \Pi_5, \Pi_6, \Pi_7), \quad \Pi_2 = \frac{h}{y}, \tag{5}
\]

\[
\Pi_3 = \frac{d}{y}, \tag{6}
\]

\[
\Pi_4 = s, \tag{7}
\]

\[
\Pi_5 = \frac{i}{u_*}, \tag{8}
\]

\[
\Pi_6 = \frac{u_*y}{v_k}, \tag{9}
\]

\[
\Pi_7 = \frac{h}{y}, \tag{10}
\]

in which \( v_k = \mu/\rho \) is the kinematic viscosity.

According to Barenblatt (1987), “In some cases, it turns out to be convenient to choose new similarity parameters—products of powers of the similarity parameters obtained in the previous step.” In other words, Barenblatt (1987) suggested combining the original dimensionless groups to obtain new similarity parameters.

From Equation (5) to (6) it follows:

\[
\Pi_{2,3} = \frac{\Pi_2}{\Pi_3} = \frac{h}{y} = \frac{h}{y} \tag{11}
\]

Coupling Equations (8), (5), and (9), the following dimensionless group is obtained:

\[
\Pi_{5,2,6} = \Pi_5\Pi_2\Pi_6 = \frac{i}{u_*} \frac{y}{y} \frac{h}{v_k} = \frac{h}{v_k} = \frac{Re}{\Pi_1}, \tag{12}
\]

which can be referred to as rain Reynolds number.

Coupling Equations (9), (6), and (11), the following equation is obtained:

\[
\Pi_{6,3,2} = \Pi_6\Pi_3\Pi_{2,3} = \frac{u_*y}{v_k} \frac{d}{y} = \frac{u_*h}{v_k}, \tag{13}
\]

whereas from Equations (10) and (5), the following dimensionless group is obtained:

\[
\Pi_{7,2} = \sqrt{\frac{8}{\Pi_1^2}} \frac{1}{\Pi_2} = \sqrt{\frac{8}{\Pi_1^2}} \frac{u_*}{g} \frac{y^{1/3}}{h^{1/3}} = \sqrt{\frac{8}{g}} \frac{s^{1/2}}{h^{1/2}} = \sqrt{\frac{8}{g}} h^{1/2} = \frac{V}{\sqrt{g} h} = F, \tag{14}
\]

in which \( V \) is the mean flow velocity and \( F \) is the flow Froude number.

The functional relationship (3) can be rewritten in the following form:
\[ \Pi_1 = \phi_2(\Pi_{23}, \Pi_4, \Pi_{526}, \Pi_{632}, \Pi_{72}). \]  

(15)

where \( \phi_2 \) is a functional symbol.

Introducing into Equation (15) the expression of each dimensionless group, this functional relationship can be rewritten as

\[ \gamma \frac{dv}{dy} = \phi_2 \left( \frac{h}{d} \frac{i h}{n_k} \frac{u}{n_k} \frac{u}{n_k} \right). \]  

(16)

Assuming Incomplete Self-Similarity in \( u, y/n_k \) (Barenblatt & Monin, 1979; Barenblatt & Prostokishin, 1993; Butera, Ridolfi, & Sordo, 1993; Ferro, 2017; Ferro & Pecoraro, 2000), Equation (16) yields

\[ \frac{1}{u_c} \frac{dv}{dy} = \frac{u}{n_k} \left( \frac{u}{n_k} \right) \phi_3 \left( \frac{u}{n_k} \frac{h}{d} \frac{s.Re_i}{F} \right). \]  

(17)

in which \( \phi_3 \) is a functional symbol.

Rearranging Equation (17) results in

\[ \frac{1}{u_c} \frac{dv}{dy} = \left( \frac{u}{n_k} \right) \left( \frac{u}{n_k} \right) \frac{d}{\delta} \phi_1 \left( \frac{u}{n_k} \frac{h}{d} \frac{s.Re_i}{F} \right). \]  

(18)

Integrating Equation (18), and assuming equal to zero the integration constant (Barenblatt & Prostokishin, 1993; Butera et al., 1993; Ferro & Pecoraro, 2000), the following power velocity distribution is obtained:

\[ \frac{v}{u_c} = \Gamma \left( \frac{u}{n_k} \frac{h}{d} \frac{s.Re_i}{F} \right) \left( \frac{u}{n_k} \right)^{\delta}. \]  

(19)

in which \( \delta \) is an exponent which can be calculated by the following theoretical equation (Barenblatt, 1991; Castaing, Gagne, & Hopfinger, 1990):

\[ \delta = \frac{1.5}{\ln(Re)}. \]  

(20)

in which \( Re = V h/n_k \) is the flow Reynolds number.

Equation (19) can be rewritten as follows:

\[ \frac{v}{u_c} = \Gamma \left( \frac{u}{n_k} \frac{h}{d} \frac{s.Re_i}{F} \right) \left( \frac{u}{n_k} \right)^{\delta}. \]  

(21)

Taking into account that the shear Reynolds number \( Re_i \) can be rewritten as

\[ Re_i = \frac{u_i h}{n_k f}. \]  

(22)

Equation (22) demonstrates that \( Re_i \) implicitly considers the variability of the depth to sediment ratio \( h/d \).

Ferro (2018) also deduced the following relationship, which states that \( F \) is related to Shields parameter, which is dependent on \( Re^* \) (Buffington & Montgomery, 1997; Di Stefano & Ferro, 2005; Shields, 1936):

\[ F \frac{h}{d} \frac{\gamma}{\gamma} = \frac{V^2}{g} \frac{\gamma}{\gamma} \frac{gRe_s}{8} \frac{\gamma}{\gamma} = \frac{\gamma}{\gamma} \frac{gRe_s}{8} \frac{\gamma}{\gamma} = \frac{\gamma}{\gamma} \frac{8}{8} \frac{8}{8} \frac{\phi_4(Re_i)\phi_4(Re_i)}. \]  

(23)

in which \( \phi_4 \) is a functional symbol, \( \gamma_s \) is the soil particle specific weight, and \( \gamma \) is the water specific weight. As according to Equation (23) \( F \) takes into account both the depth sediment ratio \( h/d \) and the shear Reynolds number \( Re^* \). The velocity profile can be rewritten as follows:

\[ \frac{v}{u_c} = \Gamma(s.Re_i) \left( \frac{u}{n_k} \right)^{\delta}. \]  

(24)

Integrating the power velocity distribution (Equation 24), the following expression of the Darcy–Weisbach friction factor \( f \) is obtained (Barenblatt, 1993; Ferro, 2017; Ferro & Porto, 2018):

\[ f = 8 \left[ \frac{(\delta + 1)(\delta + 2)}{21-\delta} \right]^{-\frac{1}{\delta}}. \]  

(25)

Starting from Equation (24) and establishing that \( y = \alpha h \), the distance from the bottom at which the local velocity is equal to the cross section average velocity \( V \), the following value \( \Gamma_y \) of \( \Gamma \) can be obtained (Ferro, 2017; Ferro & Porto, 2018):

\[ \Gamma_y = \frac{V}{u_c} \left( \frac{h}{n_k} \alpha \right)^{\alpha}. \]  

(26)

in which \( \alpha \) is a coefficient, smaller than unity, that allows consideration both that the average velocity \( V \) is located below the water surface and that the mean velocity profile, whose integration gives \( V \), is calculated by averaging, for each distance \( y \), the velocity values \( v \) measured in different verticals.

Ferro (2017) deduced the following theoretical equation for calculating \( \alpha \):

\[ \alpha = \left[ \frac{(\delta + 1)(\delta + 2)}{21-\delta} \right]^{1/\delta}. \]  

(27)

For flow Reynolds number less than 2,000, the calculated \( \alpha \) values vary in a very narrow range, and the mean value equal to 0.132 is applicable.

The applicability of the flow resistance Equation (25) was tested in previous studies for different hydraulic conditions without the rainfall effect (Di Stefano, Ferro, Palmeri, & Pampalone, 2017; Di Stefano,
Ferro, Palmeri, Pampalone, & Agnello, 2017; Ferro, 2017, 2018; Ferro & Porto, 2018; Palmeri et al., 2018), and the following theoretical expression of $\Gamma_v$ function (Ferro, 2018) was proposed:

$$\Gamma_v = \frac{aFb}{sc}, \quad (28)$$

in which $a$, $b$, and $c$ are coefficients to be estimated by the measurements.

When a flow is subjected to the rainfall impact, the $\Gamma$ parameter of the velocity profile also depends, according to Equation (24), on $Re_i$ and the following expression of $\Gamma_v$ can be used:

$$\Gamma_v = \frac{aFb}{scRe_i^e}, \quad (29)$$

in which $a$, $b$, $c$, and $e$ are coefficients to be estimated by the measurements. When the rainfall effect is neglected, Equation (29) reduces to Equation (28) for $e = 0$.

3 | MEASUREMENTS OF OVERLAND FLOW RESISTANCE UNDER RAINFALL USED IN THIS INVESTIGATION

Nearing et al. (2017) carried out experiments applying simulated rainfall to plots at three different values of slope gradient (5%, 12%, and 20%). The experiments were conducted on 6-m-long, 2-m-wide, and 0.3-m-deep pivoted steel boxes with an adjustable slope (Figure 1). The soil used in the experiment (Luckyhills–McNeal) was gravely sandy loam, containing approximately 52% sand, 26% silt, and 22% clay.

The Walnut Gulch Rainfall Simulator was equipped with a single oscillating boom with four V-jet nozzles that could produce rainfall rates ranging between 13 and 190 mm hr$^{-1}$. Wind shields were used to avoid wind effects on raindrops. Before the beginning of each experiment, the simulator was positioned over the soil box and calibrated, and then the soil layer, approximately 20 cm deep, was wetted with a rainfall having a constant intensity (35 mm hr$^{-1}$) to obtain equal moisture conditions.

The experiments were carried out using two different values of rainfall intensity, namely, 59 and 178 mm hr$^{-1}$, and runoff rate was measured using a V-shaped flume equipped with an electronic depth gauge, which was calibrated periodically during the experiment with timed runoff weight samples. Overland flow velocity was measured using a salt solution (three different point of application located at 1.65, 3.5, and 5.8 m from the outlet) and by monitoring the electrical resistivity by electrical sensors embedded at the end of the plot. Two litres of the solution were uniformly applied on the surface using a perforated PVC pipe placed across the plot. The data were collected at 0.37 s intervals with real-time graphical output using LoggerNet3 software and a CR10X3 data-logger by Campbell Scientific. The salt curve started at salt application time, and the measurement was stopped when the residual resistivity corresponding to the base level was reached. Peak values from the salt curve were used to measure flow velocity. The average flow depth was calculated by the runoff rate and the mean flow velocity assuming a rectangular cross section 2 m wide.

Surface rock cover was measured at 300 points on a 20 cm × 20 cm grid using a hand-held, transparent, size guide arranged over the surface. A single-point laser sliding along a notched rail placed across the plot and pointed down on the plot was used to objectively identify the sample point locations. The technique ensured that surface rock was measured at the same points every time during the experiment. Rock cover percentage was considered to be the percentage of points with rocks greater than 0.5 cm.

Surface elevations were measured along three 2-m-long transects oriented across the plot located at 0.9, 2.9, and 4.9 m from the lower edge of the plot. Elevation points along these transects were measured at 5-mm intervals with a 0.2-mm vertical resolution using a Leica3 E7500i laser distance metre mounted on an automated linear motion system.

Two replications for each treatment were made, with measurements of runoff rate, flow velocity, rock cover, and surface roughness. These experiments were characterized by Reynolds numbers varying between 48 and 194 (which suggest laminar flow in the absence of raindrop impacts) and average Froude numbers between 0.08 and 1.05 (99.4% were subcritical).

The experimental data used in this paper are available online at https://doi.org/10.5194/hess-21-3221-2017-supplement.

4 | RESULTS

The variable $Re_i$, accounting for rainfall intensity, describes this rainfall effect and is not correlated with $Re$ (Figure 2). Therefore, the Reynolds number $Re$ is not able to explain the rainfall effect, and both $Re$ and

![Figure 1](https://example.com/figure1.jpg)

**Figure 1** View of the soil box used by Nearing et al. (2017) for the experiments.
Re are considered to be predictive variables of the Darcy–Weisbach friction factor $f$.

The effect of rainfall on the flow resistance law was tested calibrating the theoretical relationship (Equation 29) among the velocity profile parameter $\Gamma$, the channel slope, the flow Froude number, and the rain Reynolds number $Re$.

Using the whole database, constituted by 168 data points, the following equation to calculate the parameter $\Gamma_v$ was found:

$$\Gamma_v = 0.323 \frac{F^{1.2833}}{s^{0.6698} Re^{0.0787}}. \quad (30)$$

Equation (30) is characterized by a coefficient of determination equal to 0.999, and in Figure 3, the comparison between the $\Gamma_v$ values calculated by Equation (26), with $\alpha = 0.132$, and those calculated by Equation (30) is shown.

Introducing Equation (30) into Equation (25), taking into account that $Re^\delta$ is always equal to 4.4817 and using the mean value of $\delta$ (0.327), being as its range is narrow, the following equation to estimate the Darcy–Weisbach friction factor was obtained:

$$f = 12.408\frac{\Gamma_v^{0.1187}}{F^{1.9342}}. \quad (31)$$

Figure 4 shows the comparison between the measured values of the Darcy–Weisbach friction factor and those calculated by Equation (31); this agreement is characterized by a root mean square error (RMSE) equal to 1.4474. The friction factor values calculated by Equation (31) were characterized by errors that were less than or equal to ±10% for 99.4% of cases and less than or equal to ±5% for 88.1% of cases.

The analysis was also developed neglecting the influence of the rain Reynolds number and calibrating Equation (28) using the entire database, the following equation to calculate the parameter $\Gamma_v$ was obtained:

$$\Gamma_v = 0.4054 \frac{F^{1.2794}}{s^{0.6633}}. \quad (32)$$

Equation (32) is characterized by a coefficient of determination equal to 0.990. Figure 5 shows the comparison between $\Gamma_v$ values calculated by Equation (26), with $\alpha = 0.132$, and those calculated by Equation (32). Figure 5 clearly shows that Equation (32) underestimated $\Gamma_v$ values corresponding to a rainfall intensity $i$ of 59 mm hr$^{-1}$, whereas it overestimated those corresponding to $i = 178$ mm hr$^{-1}$.

Introducing Equation (32) into Equation (25) and substituting $Re^\delta = 4.4817$ and $\delta = 0.327$, the following flow resistance equation was obtained:
In Figure 6, the agreement between the measured friction factor values and those calculated by Equation (33) are shown. This agreement is characterized by RMSE equal to 1.4926, and the friction factor values calculated by Equation (33) are characterized by errors that are less than or equal to ±10% for 71.4% of cases and less than or equal to ±5% for 22.6% of cases.

Notwithstanding Equations (31) and (33) are characterized by similar values of RMSE (1.4474 and 1.4926, respectively), the difference in terms of errors in calculating the Darcy–Weisbach friction factor is substantial. Figure 7, which shows the frequency distribution of the errors corresponding to the two examined cases, demonstrates that errors in the estimate of the Darcy–Weisbach friction factor by Equation (31) were appreciably lower than those obtained by Equation (33).

\[ f = 8.8088 \left( \frac{\alpha}{\sqrt{\frac{gR}{s/f}}} \right)^{1.008} \]  

Finally, for both Equations (31) and (33), the Darcy–Weisbach friction factor \( f \) increased with a power of slope gradient having an exponent approximately equal to 1.

Taking into account that \( V = \sqrt{\frac{8gR}{s/f}} \), this result confirms that flow velocity is approximately independent of slope.

5 | DISCUSSION

Figure 2 confirms that raindrop impact affects the \( f \)-Re relationship, and the \( K \) coefficient of Equation (1) is dependent on both channel grain resistance and rainfall intensity. Taking into account that the measurements used in this investigation are characterized by Reynolds numbers that suggest laminar flow in the absence of raindrop impacts, rain resistance can appreciably contribute to total resistance (Smith et al., 2007).

Equation (31) takes into account two different effects on flow resistance: the friction factor due to bottom roughness, represented by \( F \), and to rainfall impact represented by \( R_{ei} \). Equation (31) also shows that the flow resistance increased for increasing rain Reynolds number values. This result can be explained because the \( R_{ei} \) increases with rainfall intensity, and for Re values less than 2,000, in agreement with previous studies (Yoon & Wenzel, 1971), flow resistance increases with rainfall intensity.

The underestimation and overestimation of Equation (32) for \( i = 59 \) and 178 mm hr\(^{-1}\), respectively, shown in Figure 5, produces an opposite bias on the calculation of the Darcy–Weisbach friction factor (Figure 6), which establishes that the effect of rainfall impact cannot be neglected for the laminar flow regime.

These influences of the rainfall intensity on flow resistance are also highlighted by the frequency distribution of the errors shown in Figure 7. The errors associated to Equation (31) vary in a very narrow range (−6%, +10%). In other words, the frequency distribution of the errors associated with Equation (33), which denotes a bias in the friction factor estimate (the errors vary from −12% to +30%),
demonstrates that the roughness model is incomplete. Other variables must be used to explain the friction factor variability, including the effect of rainfall impact on flow resistance.

Finally, the exponent of slope quasi-equal to 1 for Equations (31) and (33) demonstrates that, for a laminar overland flow, the increase of velocity due to slope can be balanced by the retarding effect due to the rainfall impact.

Further experiments should be carried out for testing the applicability of the theoretical flow resistance equation for turbulent flow subjected to the impact of rainfall over a wider range of intensities.

6 | CONCLUSIONS

The approach using the theoretical flow resistance law, deduced by the II-Theorem of dimensional analysis and self-similarity theory, was modified for taking into account the effect of rainfall intensity and was applied to the overland flow data by Nearing et al. (2017) obtained by rainfall simulator experiments.

The available measurements, corresponding to two different rainfall intensities and three different values of slope, were used to calibrate the theoretical relationship between the velocity profile parameter \( \Gamma \), the channel slope, the flow Froude number and the rain Reynolds number, which is a dimensionless group representing the effect of rainfall intensity on flow resistance.

The results confirmed that, for flow Reynolds number less than 2,000, flow resistance always increases for greater rainfall intensity values.

This investigation also showed that the Darcy–Weisbach friction factor \( f \) increased with a power of slope gradient, having an exponent approximately equal to 1 and, as a consequence, showed that the overland flow mean velocity was quasi-independent of slope.

In conclusion, the lack of knowledge in the field of overland flow subjected to rainfall should stimulate experimental activity for studying overland flow in different rainfall and surface roughness conditions.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are openly available in https://doi.org/10.5194/hess-21-3221-2017-supplement.

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