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SIMPLE TIME-POWER FUNCTIONS FOR RAINWATER INFILTRATION AND RUNOFF

by

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ABSTRACT

The equations of Darcy, Kostiakov, Ostashev, Philip, and four modified Philip equations were evaluated for use in predicting and controlling rainwater infiltration and rainfall excess in crop and rangelands. These eight equations were least-square fitted to data from ring, border-irrigation, closed-top, and sprinkling infiltrometers. Kostiakov's equation satisfied the evaluation criteria better than the other seven equations. The parameters of Kostiakov's equation were physically interpreted by relating their magnitudes to some physical, biological, and hydraulic characteristics of the infiltration system. These characteristics included several infiltration abatement and augmentation processes and factors that are controlled at the soil surface by land management practices. The eight equations were also fitted to rainfall data to permit calculating runoff from small surface areas about the size of a typical crop plant. Comparison of the regression curves for infiltration and rainfall suggested that land management practices that appropriately alter the soil surface will permit wide-range control of infiltration, runoff, and erosion; and thereby achieve conservation and more efficient use of soil and water resources for crop production. The most important soil surface conditions affecting infiltration were microroughness, macroporosity, plant litter, and effective surface head.

INTRODUCTION

Rainwater infiltration and runoff are hydrologic processes of vital importance to plants and people, and thus deserve considerable modeling effort. Models which are simple, yet physically sound, are needed by land managers to implement better use and protection of land resources. Such models can advance the understanding of basic hydrologic processes; and this understanding, in turn, can lead to the prediction and control of such processes. Control of rainwater infiltration and runoff can help alleviate land management problems such as excessive runoff and erosion; flash flooding of upland watersheds; sedimentation of waterways and reservoirs; non-point source pollution of surface waters; inadequate soil water for seed germination, seedling establishment, and optimal plant growth; excessive leaching of soluble salts and plant nutrients; pollution of ground waters; slow aquifer recharge and declining water tables; excessive loss of water by surface evaporation; and accelerated land deterioration and desertification.

The cost of such land management problems to society is of gigantic magnitude. Worldwide desertification alone is estimated (Dregne, 1978) to be costing 15.6 billion dollars a year in lost agricultural production, 3.2 billion due to waterlogging and salinization, 6.8 billion to rangeland deterioration, and 5.6 billion to deterioration of rain-fed cropland. Such land deterioration is usually only partially reversible by even the best land management practices.

Much effort has already been expended on the development of point or small area infiltration models (Parr and Bertrand, 1960). In a series of papers, Dixon and coworkers (Dixon, 1977) have evolved a descriptive concept for controlling rainwater infiltration, referred to as the air-earth interface *(AEI)* concept. The main purpose of the study reported herein was to evaluate several simple infiltration equations for use in quantifying the *AEI* concept. This concept indicates that surface microroughness and macroporosity (or their hydraulic counterpart - - effective surface head) control rainwater infiltration. Quantification involved the selection of a simple infiltration equation having parameters sensitive to these *AEI* conditions. Such an infiltration equation will be useful in predicting maximum cumulative infiltration for a given land management practice. However, prediction of excess rainfall, surface runoff, and non-point source pollution requires use of a reference rainstorm. Dixon (1966) showed that a maximum-intensity storm could be generated for reference purposes by plotting maximum rainfall depths (50-year frequency) against their duration times. This yields a cumulative rainwater curve similar in shape to that for cumulative infiltration. This paper investigates the possibility that infiltration equations can also be fit to the maximum-depth rainfall data that are available (Hershfield, 1961) for numerous locations throughout the United States. The area between corresponding infiltration and rainfall curves

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would then provide a quantitative indication of either the rainwater or infiltration capacity excess.

THEORETICAL CONSIDERATIONS

All infiltration equations can be interpreted in terms of the general transport law:

$$\text{FLUX VOLUME} = \text{TRANSMISSION COEFFICIENT} \times \text{DRIVING FORCE GRADIENT} \times \text{ELAPSED TIME}$$

The flux volume is conveniently expressed as unit depth of surface water infiltrating I_v; the driving force gradient, as a drop in hydraulic head H per unit soil depth L or as a dimensionless hydraulic gradient $(i = H/L)$; and the transmission coefficient as a proportionality (permeability) constant or hydraulic conductivity K given numerically by the flux volume when both the gradient and time are unity. Thus, for infiltration volume in centimeters (cm) and time in hours (hr), the general transport equation becomes:

$$I_v\ (cm) = K\ (cm/hr) \times i\ (cm/cm) \times T\ (hr)$$

The equations of Kostiakov (1932), Philip (1957), Ostashev (1936), and Darcy (1856) were considered for study because they all (1) express infiltration volume I_v (or depth of surface water) as an explicit function of time; (2) contain two parameters $(A\ \&\ B)$ after adding constant terms to Ostashev's and Darcy's equations; (3) transform easily to linear forms for least-square regression analyses; and (4) differentiate readily to infiltration rate I_R and infiltration deceleration I_D forms (Table 1).

Table 1. Four Historic Infiltration Equations, Their Linear Transforms, and Their First and Second Derivatives.

INFILTRATION EQUATION	LINEAR TRANSFORM	1ST DERIVATIVE (RATE)	(-)2ND DERIVATIVE (DECELERATION)*
(1) Kostiakov $I_v = AT^B$	$\ln I_v = \ln A + B \ln T$	ABT^{B-1}	$AB(1-B)T^{B-2}$
(2) Philip $I_v = AT^{1/2} + BT$	$I_v/T^{1/2} = A + BT^{1/2}$	$1/2\ AT^{-1/2} + B$	$1/4\ AT^{-3/2}$
(3) Ostashev $I_v = AT^{1/2}$	$I_v = AT^{1/2} + B$	$1/2\ AT^{-1/2}$	$1/4\ AT^{-3/2}$
(4) Darcy $I_v = AT$	$I_v = AT + B$	A	0

*Deceleration is the negative of the 2nd derivative.

Darcy's equation was derived empirically to describe the volume of water absorbed by a saturated stable sand bed having water ponded at the top and free drainage at the bottom. For viscous flow in a stable saturated porous media, the absorption coefficient A is given by the product of the hydraulic conductivity K and the hydraulic gradient i or $A = Ki$. For the simple infiltration system that Darcy used, both K and i could be maintained time invariant. Even in wet infiltrating soils, neither K nor i are constant because of incomplete water saturation, soil instabilities (particularly near the surface), and changing water potentials at irregular upper and lower soil boundaries. Natural soil infiltration systems are never open to atmospheric pressure along their lower boundaries. Instead, they range from partially open to completely closed. Darcy's equation applies best to infiltration in wet stable soils, wherein i is approximately unity and infiltration is driven almost entirely by the gravitational force. In a dry-soil infiltration system, i and K are interrelated variables, both of which are functions of the soil water content with K increasing and i decreasing with increasing water content. Since i usually decreases more rapidly than K increases, the rate of infiltration tends to decrease with time.

Ostashev's equation was derived to describe the volume of water absorbed horizontally (gravitational gradient = 0) by a dry stable homogeneous porous media. The infiltrated volume decreases with $T^{1/2}$ owing to the abatement in capillary pressure gradient as the wetting front advances. Similar to Darcy's equation, the absorption coefficient A may be interpreted as a product of mean time-weighted K and i during the first time unit. However, in this case K is the unsaturated hydraulic conductivity which is several orders of magnitude less than the saturated hydraulic conductivity and i is the hydraulic gradient produced by capillarity. Vertical infiltration into a dry stable system, where gravity as well as capillarity is driving the process, will abate more slowly than permitted by $T^{1/2}$. The time exponent would thus be somewhat greater than 1/2. The wetter the soil initially, the greater would be the relative gravitational contribution, and the greater would be the exponent. Thus, in Darcy's equation where absorption is driven only by the force of gravity, infiltration is proportional to T; whereas, in Ostashev's equation where capillarity is the sole driving force, infiltration is proportional to $T^{1/2}$.

Philip's equation was derived analytically for the downward absorption of water into an initially dry stable porous medium. The first and second terms give the infiltration contributions of the capillary and gravitational driving forces, respectively. Thus, Philip's equation accounts for the infiltration effects of both forces, essentially by combining (adding) Ostashev's and Darcy's equations. Parameters A and B may be regarded as capillary and gravitational absorption coefficients, respectively.

Kostiakov's equation was empirically derived to describe the time-course of infiltration as an initially dry soil absorbs irrigation water - - usually at a decreasing rate until soil saturation is closely approached. The absorption coefficient A may be interpreted in much the same way as the corresponding coefficients in Darcy's and Ostashev's equations; i.e., as the product of the mean K and i for the first unit of time. Large A values are associated with soil surfaces that are microrough and macroporous or with conditions favoring a relatively large contribution of the gravitational force to infiltration (Dixon, 1977 and Dixon and Simanton, 1977). In contrast, small A values are associated with a smooth microporous surface where capillarity is the major force driving infiltration. Parameter A gives the I_v-time curve its magnitude, whereas parameter B gives this curve its shape. For $0<B<1$, the infiltration rate is abating wih time (the usual case) and for $B>1$ infiltration is increasing or augmenting with time (the exceptional case).

The magnitude of parameter B in Kostiakov's equation reflects the net effect of numerous interrelated and interacting infiltration abatement and augmentation processes and conditions (Dixon, 1975b and 1976). The abatement processes and conditions include (1) decreasing capillary pressure gradient due to deepening wetting front; (2) surface sealing under raindrop impact; (3) decreasing capillary pressure gradient due to increasing moisture content with depth; (4) soil settling causing macropores to collapse; (5) decreasing soil wettability with depth; (6) increasing water repellency with depth; (7) decreasing available storage space with time; (8) decreasing storage space with depth because of increasing moisture content, rock, etc.; (9) decreasing macroporosity both in number and continuity with depth; (10) swelling of clay colloids with corresponding shrinkage of macropores; (11) anaerobic slime formation; (12) rising soil air pressure and the consequent entrapment of soil air in macropores; and (13) freezing of the infiltrated water with consequent blockage of fluid flow routes. The augmentation processes and conditions include (1) increasing flow dimensionality with time; (2) eluviation and illuviation leading to micropipe formation; (3) increasing soil wettability with depth; (4) decreasing water repellency with depth; (5) increasing ponded-water depth with time; (6) soil water absorption of entrapped air; (7) macropore formation through solution of soluble salts; (8) increasing ponded surface area with time; and (9) melting of soil ice by the infiltrated water.

Parameter B values ranging from 0.0 to 0.5, 0.5 to 1.0, and 1.0 to 1.5 indicate the dominance of abatement factors, little dominance of either abatement or augmentation factors, and dominance of augmentation factors, respectively. Since the abatement and augmentation processes and factors interact with each other in different combinations and intensities to control the time-weighted means for hydraulic conductivity and hydraulic gradient, parameters A and B in Kostiakov's equation are interrelated. Kostiakov's equation is a general equation in a relative sense, since parameter B can assume values appropriate for almost any combination of the abatement and augmentation processes. In contrast, the equations of Darcy and Ostashev represent special cases of Kostiakov's equation (Dixon, 1976). Philip's equation, as indicated previously, is essentially a combination of these two special cases. The form of Darcy's equation accounts for no infiltration decay or augmentation, and those of Ostashev's and Philip's account for only one of the infiltration abatement and augmentation factors; i.e., the decrease in capillary pressure gradient resulting from the increasing distance of the wetting front from the ponded-water source.

The equations of Darcy, Ostashev, and Philip are often said to be physically based -- meaning that the parameters have physical significance. Such physical significance, however, is restricted to the simple ideal infiltration systems for which these equations were derived. All infiltration equations are, or become, empirical when applied to the complex soil and water-source conditions found in crop, forest, and rangelands. The magnitude of a parameter determined by fitting an infiltration equation to data from such land areas, usually reflects conditions not present in the simple ideal system. Consequently, an adequate physical interpretation of the parameter must account for the major factors affecting this parameter in the natural infiltration system being studied. To assume that the theoretical physical significance still holds can be extremely misleading, thereby leading to much confusion.

PROCEDURE

EVALUATION CRITERIA

Evaluation criteria were developed to (1) facilitate initial screening of the many infiltration equations for selecting several for subsequent fitting accuracy tests and (2) guide final selection of the best equation for modeling the *AEI* concept. These criteria included:

1. Parameter number restricted to two.
2. Infiltration expressed explicitly as a function of time.
3. Equation parameters sensitive to *AEI* conditions.
4. Infiltration volume approaches zero as time approaches zero.
5. Infiltration rate approaches zero as time approaches infinity or as the unfilled profile storage space approaches zero.
6. Infiltration rate approaches infinity as time approaches zero.
7. Equation accounts for net infiltration effect of diverse interacting infiltration abatement and augmentation processes or factors that are affected by *AEI* conditions.
8. Equation gives consistently accurate fit of data collected under widely varying conditions of surface microroughness, surface macroporosity, and effective surface head.
9. Fitting of equation to infiltration data always yields positive-valued parameters.
10. Equation, with positive-valued parameters, yields rates that can either decelerate, or remain constant, or accelerate with increasing time.
11. Mathematical and physical interpretation of parameters valid for widely varying *AEI* conditions.
12. Equation form aids data interpretation, summarization, extrapolation, and interpolation.
13. Simple first and second derivative forms of the infiltration volume equation for calculating infiltration rates and rate changes.
14. Equation easily least-square fitted to infiltration data to obtain parameter estimates.
15. Infiltration easily calculated using equation and parameter estimates.
16. Equation has simplest form possible, yet satisfies preceding criteria.

EQUATIONS TESTED

The first two evaluation criteria limited to four the possible choices of published infiltration equations available for subsequent testing (Table 1). Initial testing suggested that the fitting accuracy of Philip's equation could be improved somewhat by selecting powers of time that would be more appropriate for complex infiltration systems than the 1/2 and 1 that pertain strictly to the simple system for which the equation was derived. Accordingly, the four modified Philip equations (given in Table 2) were tested in addition to those given in Table 1. These modified equations are of the form,

$$I = AT^x + BT^y$$

where the values of x in the four equations range from 0.1 to 0.3 and y from 0.8 to 1.1. Parameters A and B can be interpreted in a similar manner to that given previously when discussing the theoretical basis for Philip's equation. However these parameters probably have more physical significance for a natural infiltration system than the corresponding parameters in Philip's equation, but less significance for the ideal system assumed by Philip.

Table 2. Modified 2-Parameter Philip Equations, Their Linear Transforms, and Their First and Second Derivatives.

MODIFIED INFILTRATION EQUATION	LINEAR TRANSFORM	1ST DERIVATIVE (RATE)	(-)2ND DERIVATIVE (DECELERATION)
(5) $I_v = AT^{0.1} + BT^{1.1}$	$I_v/T^{0.1} = A + BT$	$0.1AT^{-0.9} + 1.1BT^{0.1}$	$0.09AT^{-1.9} - 0.11BT^{-0.9}$
(6) $I_v = AT^{0.2} + BT^{1.2}$	$I_v/T^{0.2} = A + BT$	$0.2AT^{-0.8} + 1.2BT^{0.2}$	$0.16AT^{-1.8} - 0.24BT^{-0.8}$
(7) $I_v = AT^{0.3} + BT^{1.3}$	$I_v/T^{0.3} = A + BT$	$0.3AT^{-0.7} + 1.3BT^{0.3}$	$0.21AT^{-1.7} - 0.39BT^{-0.7}$
(8) $I_v + AT^{0.3} + BT^{0.8}$	$I_v/T^{0.3} = A + BT^{0.5}$	$0.3AT^{-0.7} + 0.8BT^{-0.2}$	$0.21AT^{-1.7} - 0.16BT^{-1.2}$

EQUATION FITTING

The equations of Kostiakov, Philip, Ostashev, Darcy, and the four modified Philip equations (Tables 1 and 2) were evaluated by fitting them to field infiltrometer data. This evaluation included the following steps:

1. Transform infiltration equations to their linear form as given in Tables 1 and 2.
2. Perform least-square linear regression analyses to obtain parameter values.
3. Use parameter values in the untransformed equations (except in Darcy's and Ostashev's equation) to obtain calculated infiltration values.
4. Evaluate differences between calculated and observed infiltration by statistically testing for fitting accuracy.
5. Determine data group means and standard deviation of these means for the fitting-accuracy statistics.

6. Select infiltration equation giving the most accurate data fit by ranking the data group means and their standard deviations.

As reflected in steps 3 through 6, the untransformed equations (with the two exceptions) were used to determine data fitting accuracy. The magnitude of correlation coefficients for the transformed equations of Philip and the modified Philip equations were not useful in evaluating fitting accuracy. This type of linear transformation often causes the regression line slope to fluctuate around zero, with consequent low correlation coefficients, even though the equation fit might be quite accurate, as indicated by closeness of observed and calculated infiltration values. Nevertheless, these transforms provided a reliable (and simple) method for estimating parameter values as verified by a iterative computer method for accurately determining parameter values. The statistical tests of fitting accuracy referred to in step 4 included the (1) relative mean absolute deviation *(RMAD)* of calculated infiltration from observed infiltration, (2) slope of the regression line *(SRL)* for calculated infiltration versus observed infiltration, (3) intercept of the vertical axis *(IVA)* by the regression line for calculated infiltration versus observed infiltration, and (4) coefficient of determination *(CD)* for the linear regression of calculated infiltration versus observed infiltration. The accuracy of equation fit approaches perfection as the means for *RMAD*, *SLR*, *IVA*, and *CD* approach zero, one, zero, and one, respectively; and as the standard deviations for *RMAD*, *SLD*, *IVA*, and *CD* all approach zero.

DATA SOURCES

Infiltration and rainfall data sets used in evaluating the eight infiltration equations are summarized in Tables 3 and 4. Published rainfall data (Table 5) were selected from locations near the field infiltration sites of the authors and their coworkers in order to generate reference rainstorms for use in interpreting the infiltration data.

Table 3. Cumulative Infiltration Data Used to Test Fitting Accuracy of Eight Infiltration Equations.

LOCATION(S) AND SOURCE	SOILS AND VEGETATION	INFILTROMETER DESCRIPTION	NUMBER OF TESTS AND REPS.	DATA POINTS PER TEST	TEST TIME (HOURS)
U.S. Survey, 68 soil sites (Free et al., 1940)	Wide range in both	Constant-head, single ring, 23 cm O.D., 61 cm long	124 24	5	3
Wisconsin, Montana, Nevada, and Arizona soil sites (Dixon, 1977)	Wide range in both	Modified Purdue sprinkling type, 10 cm/hr full cone nozzle, 1-meter-square plot frame (Dixon & Peterson, 1964 & 1968)	15 2	8	2
Site near Fallon, NV (Dixon & Linden, 1972)	Loamy Border-irrigated alfalfa	Border-irrigation type, 1-meter-square plot frame, ponded-water depth same as variable irrigation head	12 2	10	3
Site near Reno, NV	Loamy Border-irrigated alfalfa	Double-square closed-top type (Dixon, 1975a)	59 5	8	2
Santa Rita Experimental Range, Continental, AZ (Authors, unpublished)	Loamy to sandy Partial grass cover	Modified Purdue sprinkling type, 10 cm/hr full-cone nozzle, 1-meter-square plot frame (Dixon & Peterson, 1964 & 1968)	2 12	8	1

Table 4. Rainfall Maximum Depth-Duration Data Used to Test Fitting Accuracy of Eight Infiltration Equations.

LOCATION(S) AND SOURCE	STORM FREQUENCY TESTED (YEARS)	NUMBER OF TESTS	DATA POINTS PER TEST	TEST MAXIMUM DURATION (HOURS)
1. Madison, WI; Sidney, MT; Reno, NV; Tombstone, AZ; & Continental, AZ (Hershfield, 1961)	1, 10, 100	30	6	2
2. Madison, WI; Williston, ND; Miles City, MT; Reno, NV; Tucson, AZ (Shands and Ammerman, 1963)	Maximum Recorded	5	6	2
3. Walnut Gulch Experimental Watershed, Tombstone, AZ (Authors, unpublished)	1 & 100	20	6	1

RESULTS AND DISCUSSION

EQUATION RANKING

The last step in the fitting procedure described previously was to select the infiltration equation giving the most accurate data fit by ranking data group means and their standard deviations. In the ranking of each data group, the means and standard deviations for the four fitting-accuracy statistics (*RMAD*, *SLR*, *IVA*, and *CD*) were all given the same weight. Then each data group was weighted equally to determine the grand ranks given in Table 5. The overall grand rank may be determined from the "rank total" column in this table.

In general, the rank of an equation reflects its ability to accurately predict the nature of the time dependency of cumulative infiltration or maximum depth rainfall relative to the other equations. In turn, the equation's ability to correctly assess this time dependency is a function of the exponents of time appearing in the equation (Table 6). As a group, the four modified equations (No. 5 to 8) ranked better than the four historic equations since the time exponents of these equations were especially chosen to be appropriate for complex natural infiltration systems. Darcy's equation (No. 4) ranked surprisingly well relative to the other historic equations, apparently because of some strong infiltration augmentation processes operative under sprinkled-water infiltration in semiarid regions. Increasing surface ponded area (and depth of ponding) and increasing capillarity with soil depth interact to produce S-shaped cumulative infiltration curves (Dixon, 1977). The straight line of Darcy's equation fits such data better than the curves of the other equations. However, Kostiakov's equation fits the S-shaped infiltration curves almost as well as Darcy's equation since the fitted-parameter *B* will approximate unity in such cases. Because of its constant term, Darcy's equation still has a slight advantage over Kostiakov's for fitting this kind of data.

Table 5.

Summary Ranking of Eight Infiltration Equations Relative to Their Ability to Accurately Fit Data from the Sources Given in Tables 3 and 4.

EQUATION NUMBER	INFILTRATION DATA	RAINFALL DATA	RANK TOTAL
(1)	5	2	7
(2)	6	3	9
(3)	7	5	12
(4)	4	8	12
(5)	1	7	8
(6)	2	6	8
(7)	3	4	7
(8)	4	1	5

Table 6.

Approximate Proportionality of Cumulative Infiltration and Powers of Time for the Eight Infiltration Equations Evaluated.

EQUATION NUMBER	POWERS AT: SMALL TIMES	LARGE TIMES
(1)	>0.0	>0.0
(2)	0.5	1.0
(3)	0.5	0.5
(4)	1.0	1.0
(5)	0.1	1.1
(6)	0.2	1.2
(7)	0.3	1.3
(8)	0.5	0.8

The poorest fits of infiltration data were obtained with Philip's and Ostashev's equations (No. 2 and 3) since they generally tend to overestimate infiltration at small times, and sometimes underestimate it at large times. Where early infiltration abatement processes are strong, the overestimation of infiltration by the first term in Philip's equation is compensated for by the second term which becomes negative in the least-square fitting procedure. Consequently, the negative-valued parameter B for such a natural infiltration system is a coefficient that corrects for the wrong assumption made in the first term, rather than a coefficient related to saturated hydraulic conductivity and the gravitational contribution as assumed in the equation's derivation (Taylor and Ashcroft, 1972).

The equations containing two time terms (No. 2, 5, 6, 7, and 8) have an inherent data fitting advantage. However, this advantage is diminished somewhat if time exponents are inappropriate. The exponent in the first term should be relatively small to reflect the rapid rate of infiltration abatement at small times, whereas the exponent in the second term should be relatively large to reflect the slow rate of infiltration abatement (sometimes actually augmentation) at large times. Our results indicate that an excellent fit for a given set of data can be obtained by letting x in equation No. 9 equal one standard deviation less than the mean parameter B in Kostiakov's equation, and by letting y equal one standard deviation greater than this parameter's mean value, where parameter B is calculated by the least-squares linear regression method or estimated by simply dividing the 60-minute infiltration rate by the 60-minute infiltration volume.

Whenever the net effect of interacting (and often compensating) infiltration abatement and augmentation processes caused infiltration to proceed approximately at the power(s) of time of one of the infiltration equations, then that particular equation would fit the data quite well. However, such circumstantial and fortuitous equation fits should not be construed as verifying theory or the physical soundness of the equation. Adequate validation of theory requires that an equation accurately fit the data for the reasons assumed in the derivation of this theory.

The results of this fitting study indicate that Darcy's equation fits infiltration data accurately when infiltration approaches linearity as caused by weak abatement processes or strong augmentation processes, or a combination of the two. Therefore, accurate fitting of Darcy's equation was favored by a (1) microrough macroporous soil surface, (2) sprinkled-water source, (3) initially wet soils, and (4) semiarid or arid climate. Darcy's equation fitted data poorly when infiltration abatement processes were relatively intense, such as in the case of a rapidly sealing soil surface under raindrop impact. Ostashev's and Philip's equations fitted infiltration data accurately when infiltration abatement and augmentation processes were at moderate levels of intensity. Thus, accurate fits were obtained for soils that were (1) initially dry and fully wettable; (2) stable, smooth and microporous at the surface; (3) completely covered with vegetation and (4) relatively deep.

Of the four historic equations, only Kostiakov's equation satisfied evaluation criteria No. 7 and 8 by consistently fitting infiltration data accurately regardless of the intensities and combinations of the various infiltration abatement and augmentation factors. Kostiakov's equation also satisfied each of the remaining 14 evaluation criteria as well as, or better than, the other equations (Table 7). Although the four modified equations fit the data slightly better than Kostiakov's equation (Table 5), they are more complex and, thus, more difficult to interpret both mathematically and physically. The results of this study indicate that Kostiakov's equation is a general infiltration equation possessing sufficient flexibility to account for a wide range of natural conditions affecting infiltration.

Table 7. Evaluation Criteria Satisfied by Each of Eight Infiltration Equations.

EQUATION NUMBER	EVALUATION CRITERION SATISFIED (+) LIST NUMBER*																
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL
(1)	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	16
(2)	+	+	+	+		+							+	+	+		8
(3)	+	+	+		+	+							+	+	+		8
(4)	+	+	+										+	+	+		6
(5)	+	+	+	+		+	+	+	+				+	+	+		11
(6)	+	+	+	+		+	+	+	+				+	+	+		11
(7)	+	+	+	+		+	+	+	+				+	+	+		11
(8)	+	+	+	+	+	+							+	+	+		9

*Numerical listing is given in section entitled "Evaluation Criteria."

The accuracy of equation fit to the rainfall data (Table 5) may also be interpreted with the aid of the time exponents given in Table 6. A graphical plot of maximum rainfall depths versus their durations reveals a marked abatement (convex curvature upward) both at small and large times. Therefore, the relatively accurate fit of equations No. 1 and 8 and the inaccurate fit of equation No. 4 to the rainfall data would be anticipated from the relative magnitude of the time exponents in these equations. Thus, Kostiakov's equation appears well suited to modeling a reference rainstorm of this type in addition to cumulative rainwater infiltration. This hypothetical storm will tend to overestimate cumulative rainfall for the specific return frequency, especially during the first 30 minutes. The physical signifiance of modeling reference rainstorms with Kostiakov's equation is being studied, and will be discussed in greater detail elsewhere.

EQUATION SIGNIFICANCE

The equation of Kostiakov has both mathematical and physical significance for the natural infiltration systems it attempts to model. The general physical significance of Kostiakov's equation relative to the other equations was briefly discussed in an earlier section.

Mathematically, Kostiakov's equation is extremely simple, with infiltration volume I_v being expressed as a one-term power function of time. Infiltration rate I_R and the deceleration I_D in this rate are given by the first and second derivative forms of Kostiakov's equation (Table 1). The integral and derivative forms of Kostiakov's equation indicate that where $0 < B < 1$:

(1) $I_v = 0$ and I_R and I_D are undefined for $T = 0$;

(2) $I_v \rightarrow 0$, $I_R \rightarrow \infty$ and $I_D \rightarrow \infty$ as $T \rightarrow 0$; and

(3) $I_v \rightarrow \infty$, $I_R \rightarrow 0$ and $I_D \rightarrow 0$ as $T \rightarrow \infty$.

Thus, the infiltration volume increases at a decreasing rate monotonically with increasing time; and the infiltration rate and its deceleration decrease at a decreasing rate approaching zero asymptotically at large times. The condition $0 < B < 1$ holds for most data sets from natural infiltration systems; however, infrequently the condition $B > 1$ prevails, indicating that the infiltration rate is increasing with time.

The mathematical interpretation of the parameters in the integral and derivative forms of Kostiakov's equation is readily apparent. If the unit for time is hours, then parameter A may be interpreted as either the first-hour infiltration volume I_v or the mean first-hour infiltration rate $\bar{I}_R$; the parameter product AB is the instantaneous infiltration rate I_R at the end of the first hour or at $T = 1$, parameter B is first-hour end rate divided by the mean rate or $B = I_R/\bar{I}_R$ for $T = 1$, and the time coefficient $[AB(1-B)]$ is the deceleration (defined as the negative of acceleration) of the infiltration rate at $T = 1$. Thus, sets of infiltration data may be conveniently and meaningfully summarized in terms of the A and B parameters and the time period upon which they are based. Such summarizations give the first-hour infiltration and its abatement ratio and permit calculation of infiltration volume, rate, and deceleration for any selected time. Parameter A usually ranges from 0 to 20 (assuming I_v is in cm) and gives the integral curve its magnitude, whereas parameter B usually ranges from 0 to 1, and gives the integral curve its shape.

The A and B parameters may be quickly estimated from infiltration data since $A = I_v$ and $AB = I_R$ at $T = 1$; however, better estimates are usually obtained by transforming the integral form to obtain the linear equation:

$$\ln I_v = \ln A + B \ln T,$$

which can be least-square fitted to infiltration data. Such fits are easily performed with hand calculators programmed for simple linear regression analysis.

A physical interpretation of Kostiakov's equation and its parameters relative to the *AEI* concept is possible, although not as readily apparent as the preceding mathematical interpretation. In general, the *AEI* concept assumes that all infiltrating surface water is subsequently stored in the soil profile. Thus, I_v becomes the storage volume of infiltrated water, I_R is the storage rate, I_D is the deceleration in storage rate, T is the elapsed time after incipient ponding during which storage has been occurring, parameter A is the storage during the first hour, AB is the storage rate at the end of the first hour, and B is a dimensionless ratio of AB and A which reflects the degree of storage rate abatement during the first hour.

Specifically, the *AEI* concept assumes that the two interacting and interrelated soil surface physical properties - - microroughness and macroporosity - - control free-water infiltration into soils. Surface microroughness and soil macroporosity interconnect and interact with each other to form ponded-water intake and soil-air exhaust circuits that govern the entry of water into soils. Dixon (1977) has shown that the hydraulic equivalent of these two surface conditions is the effective surface head; whereas the biological equivalent appears to be plant litter. Thus, the *AEI* concept as presently formulated indicates that infiltration is controlled at the soil surface - - physically by interconnected microroughness and macroporosity; hydraulically, by effective surface head; and biologically, by plant litter.

Physical interpretation of Kostiakov's equation in terms of the *AEI* concept involves relating the parameters of this concept to those of Kostiakov's equation. Dixon (1977) found that parameters A and B were sensitive to standard microroughness-macroporosity treatments. Assigning equivalent effective surface heads to these treatments, facilitated expressing A and B parameters as functions of surface treatments. Relationships between A and B and effective surface head were also determined. A closed-top infiltrometer was used to obtain infiltration data (Table 3) under effective surface heads ranging from -6 to +6 cm of water head. Parameters *A* and *B* were then determined by least-square fitting of Kostiakov's equation to the data points as shown in Fig. 1. These parameters were then related to effective surface head - - again by the least-square linear regression method (Fig. 2.). Graphs like this can serve to quantify the air-earth interface concept of infiltration and thereby facilitate absolute control over infiltration through soil surface management. The microroughness-macroporosity and plant litter equivalents of effective surface head can also be included on the horizontal axis. In practice then, soil surface management would be directed to achieving levels of the *AEI* concept parameters that would give the desired control over rainwater infiltration.

Rainwater excess (or runoff) can be approached similarly, except that a reference rainstorm is required for calculating the runoff data. This approach is illustrated in Fig. 3, 4, and 5 for the plant litter parameter. For simplicity, only extreme levels of the plant litter parameter are shown. These figures indicate that (1) litter provides a factor-of-ten control over infiltration (Fig. 3), (2) runoff from the litter-covered surface under the 100-year storm is negligible (Fig. 4), and (3) runoff from the bare surface is 90% of the total cumulative rainfall (Fig. 5). Dixon (1977) has also reported an order-of-magnitude control of infiltration for the other two *AEI* concept parameters.

In the example shown in Fig. 5, potential runoff or precipitation excess is determined by subtracting accumulative infiltration from accumulative rainfall, using either the actual data or data calculated with the fitted Kostiakov equations. This provides a set of calculated runoff data to which Kostiakov's equation can again be fitted. The resulting *A* and *B* parameter for runoff can then be analyzed in a manner similar to that shown in Fig. 2 for infiltration. This approach should facilitate prediction and control of runoff not only from individual plant-sized land areas, but from larger land areas as well. The physical significance of modeling precipitation excess or rainwater runoff with Kostiakov's equation will be discussed in a subsequent paper.

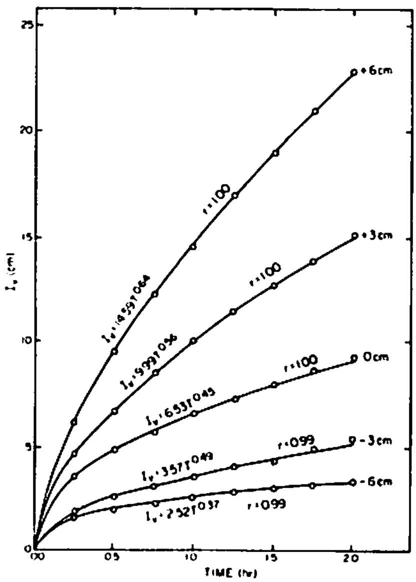

Fig. 1. Infiltration volume I_v as a function of time for effective surface heads ranging from -6 *cm* to +6 *cm* of water with least-square determined Kostiakov equations and correlation coefficients for the linear transforms of these equations.

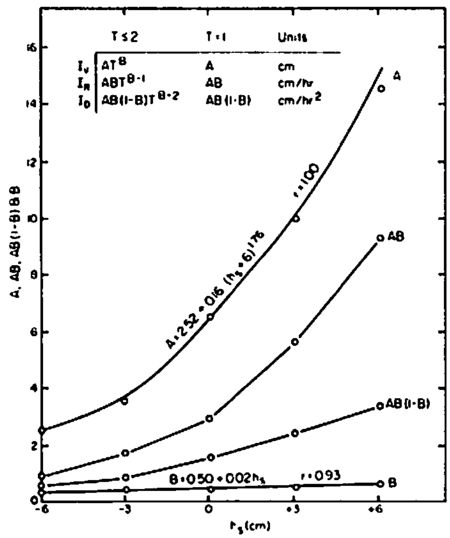

Fig. 2. Parameters for the infiltration volume I_v, infiltration rate I_R, and infiltration deceleration I_D forms of Kostiakov's equation as functions of effective surface head h_s with least-square determined functions for parameters *A* and *B*.

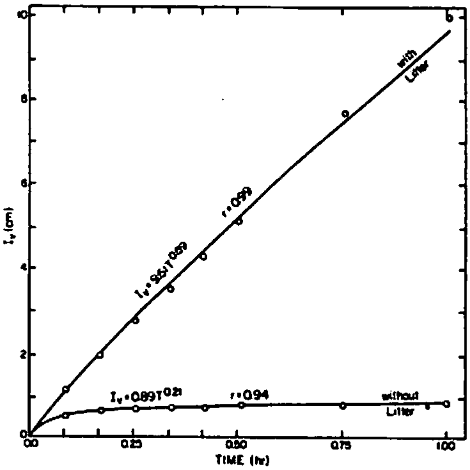

Fig. 3. Infiltration volume I_v as a function of time for litter-covered and bare soil surfaces with least-square determined Kostiakov equations and correlation coefficients for the linear transforms of these equations.

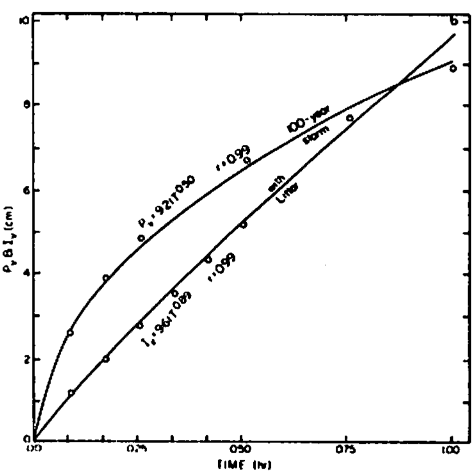

Fig. 4. Precipitation P_v and infiltration volume I_v for a litter-covered surface with least-square determined Kostiakov equations and correlation coefficients for the linear transforms of these equations.

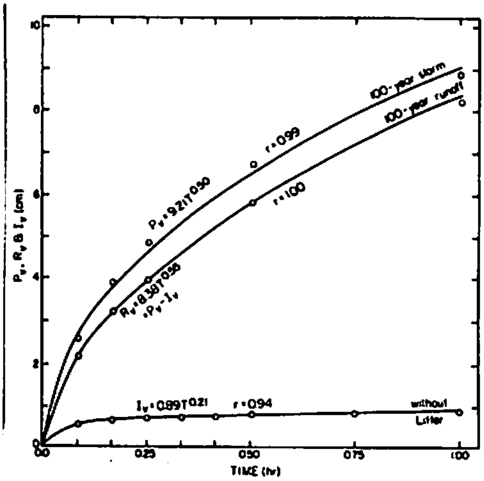

Fig. 5. Precipitation P_v, runoff R_v, and infiltration volume I_v for a bare soil surface with least-square determined Kostiakov equations and correlation coefficients for the linear transforms of these equations.

SUMMARY AND CONCLUSIONS

Kostiakov's equation was selected for modeling the *AEI* concept of infiltration because of its simple mathematical form, its ability to accurately and consistently fit data from diverse sources, and its meaningful parameters which provide a convenient method for summarizing infiltration data and predicting and controlling infiltration and runoff. Use of this equation for quantifying the *AEI* concept involves the determination of functional relationships between equation parameters and concept parameters.

Kostiakov's equation also accurately fits maximum-depth rainfall-durationn data that is widely available. This provides a reference rainfall curve for comparing with infiltration curves, and thus, the opportunity for calculating rainfall excess or potential runoff. Kostiakov's equation will also fit such calculated data and the resulting parameters can then be related to the *AEI* concept parameters. Infiltration and runoff control would then be achieved by directing land management practices to effecting appropriate levels of the *AEI* concept parameters. The *AEI* concept has a biological, a physical, and a hydraulic parameter, each of which appear to exert an equivalent and controlling influence on the infiltration process. These parameters are plant litter, surface microroughness-macroporosity, and effective surface head.

Additional research is needed to (1) develop better methods for field evaluating the *AEI* concept parameters, (2) evaluate the concept parameters under diverse field conditions, (3) relate measured parameters to measured infiltration, and (4) develop economic methods for imposing and maintaining infiltration and runoff control treatments on large land areas. Since the green plant is the best land management tool available for holding soil and water resources in place and for increasing the soil resource, the sample size for this research should be approximately equal to the space occupied by several crop plants in a monoculture, and several plant communities in a multiculture. Control of key hydrologic processes at this spatial scale will help keep the vital land resources - - soil and water - - within easy reach of plant roots. Consequently, such control can lead to improved land management practices for increasing and stabilizing land productivity.

REFERENCES CITED

Darcy, H. P. G. 1856. Les Fontaines Publiques de la Ville de Dijon. Dalmont, Paris.

Dixon, R. M. 1966. Water infiltration responses to soil management practices. PhD Thesis, Univ. Wisconsin, 175 p, Univ. Microfilms, Ann Arbor, Mich. (Order No. 66-5903, Diss. Abstr. XXVII:4).

Dixon, R. M. 1975a. Design and use of closed-top infiltrometers. Soil Sci. Soc. Amer. Proc. 39:755-763.

Dixon, R. M. 1975b. Infiltration control through soil surface management. Proceedings Symposium on Watershed Management. Irrigation and Drainage Division, ASCE and Utah State University. pp. 543-567.

Dixon, R. M. 1976. Comments on "Derivation of an equation of infiltration" by H. J. Morel-Seytoux and J. Khanja. Water Resources Research, AGU 12(1):116-118.

Dixon, R. M. 1977. Air-earth interface concept for wide-range control of infiltration. Annual Meeting of American Society of Agricultural Engineers, Raleigh, North Carolina. Paper No. 70-2062, 33pp.

Dixon, R. M., and D. R. Linden. 1972. Soil air pressure and water infiltration under border irrigation. Soil Sci. Soc. Amer. Proc. 36:948-953.

Dixon, R. M., and A. E. Peterson. 1964. Construction and operation of a modified spray infiltrometer and a flood infiltrometer. Wisc. Agr. Exp. Sta. Res. Rpt. 15.

Dixon, R. M., and A. E. Peterson. 1968. A vacuum system for accumulating runoff from infiltrometers. Soil Sci. Soc. Amer. Proc. 32:123-125.

Dixon, R. M., and J. R. Simanton. 1977. Water infiltration processes and air-earth interface conditions. Proc. Third International Hydrology Symposium, June 27-29, Fort Collins, Colorado.

Dregne, H. E. 1978. Desertification: Man's abuse of the land. Journal of Soil and Water Conservation 33(1):11-14.

Free, G. R., G. M. Browning, and G. W. Musgrave. 1940. Relative infiltration and related principle characteristics of certain soils. USDA Technical Bulletin 729, 51pp.

Hershfield, D. M. 1961. Rainfall frequency atlas of the United States. Technical Paper No. 40. Weather Bureau, U.S. Dept. Commerce.

Kostiakov, A. N. 1932. On the dynamics of the coefficient of water percolation in soils and the necessity of studying it from the dynamic point of view for the purposes of amelioration. Trans. Sixth Commission ISSS. Russian Pt. A, pp. 17-21.

Ostashev, N. A. 1936. The law of distribution of moisture in soils and methods for the study of the same. International Conference Soil Mechanics and Foundations Engineering Proceedings, Vol. 1 (Sect. K):227-229.

Parr, J. F., and A. R. Bertrand. 1960. Water infiltration into soils. Advances in agronomy 12:211-363.

Philip, J. R. 1957. The theory of infiltration: 4. Sorptivity and algebraic infiltration equations Soil Science, 84:257-264.

Shands, A. L., and D. Ammerman. 1963. Maximum recorded United States point rainfall for 5 minutes to 24 hours at 207 first order stations. U.S. Weather Bureau Technical Paper 2.

Taylor, S. A., and G. L. Ashcroft. 1972. Movement of soil water. In: Physical Edaphology. W. H. Freeman and Company, San Francisco. pp. 185-247.