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# WATER INFILTRATION PROCESSES AND AIR-EARTH INTERFACE CONDITIONS

By

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## Abstract

The air-earth interface theory holds that *interfacial roughness and openness control the rates and routes of water infiltration by governing the flow of air and water in underlying macropore and micropore systems. Roughness* refers to the microrelief that produces depression storage, whereas *openness* refers to the macroporosity that is visible at the soil surface. Soil air and free surface water exchange freely across a *rough open* surface with consequent rapid water penetration via the relatively short broad straight paths of the macropore system. In contrast, surface exchange of air and water is greatly impeded by a *smooth closed* surface with consequent slow water penetration via the relatively long narrow tortuous paths of the micropore system. These relative differences in water penetration rates and routes are attributed to corresponding differences in phase continuity within the macropore system. Both air and water phases are maintained continuous by a rough open surface and discontinuous by a smooth closed surface. Discontinuity in the phases causes relatively high soil air back pressures and low soil water pressures, whereas phase continuity produces low air pressures and high water pressures.

The air-earth interface theory that surface roughness and openness control infiltration and the Darcy concept that hydraulic conductivity and gradient control infiltration are reconciled by introducing and defining a new hydraulic parameter, referred to as the *effective surface head*, which controls both the hydraulic conductivity and hydraulic gradient at the soil surface. Transmission characteristics of the soil profile are reflected in the magnitude of the effective surface head.

The air-earth interface concept appears to have considerable potential in the solution of land management problems wherein uncontrolled point infiltration, surface runoff and erosion are contributing factors. Such problems would be alleviated by designing land management systems to achieve a given level of surface roughness and openness or effective surface head.

The air-earth interface theory may be quantified by relating surface roughness and openness or effective surface head to the two parameters in Kostiakov's equation. These relationships were found to take the form of a power function for the coefficient of time in Kostiakov's equation and a linear function for the time exponent.

## Introduction

Uncontrolled infiltration often causes the inefficient use and irreversible loss of our vital soil and water resources. For instance, excessive tillage or overgrazing diminishes the soil's ability to absorb water, thereby increasing soil and water losses from the soil surface through the processes of evaporation, runoff, and erosion.

Many other problems are either directly or indirectly related to man's inability to control infiltration at appropriate levels. These include flash flooding of upland watersheds, excessive erosion of upland stream banks, sedimentation of waterways and reservoirs, pollution of surface and groundwaters, excessive evaporation from soil surfaces, inefficient leaching of soluble salts and excessive leaching of plant nutrients, inefficient on-site use of precipitation for vegetal production, inefficient water harvesting for off-site precipitation uses, slow recharge of groundwater and declining water tables, and inefficient irrigation of various land areas. Desertification of most semiarid and arid regions of the world is accelerated by excessive surface runoff and evaporation resulting from uncontrolled infiltration.

According to a new infiltration theory referred to as the *air-earth interface* (AEI) theory, interacting soil surface and water source conditions control water infiltration rates and water penetration routes (Dixon, 1972). In this paper the AEI theory is briefly reviewed and an approach to theory quantification for absolute infiltration control is presented.

## AEI Theory

The spatial domain of the AEI theory and its physical models is the *micro-interface* and its physical properties, *microroughness* and *macroporosity*. Micro-interfaces are defined as square or circular surfaces less than 1 $m^2$ in size; microroughnesses are soil surface irregularities having horizontal periodicities ranging from 1 to 100 cm; and macropores are soil voids assumed to be cylindrical tubes and plane cracks having diameters and widths ranging from 1 to 10 mm at the air-earth interface.

The AEI theory makes the general argument that *soil surface roughness and openness control infiltration of free surface water by governing the flow of air and water in underlying macropore and micropore systems,* wherein *roughness* refers to the microrelief that produces depression storage, and *openness* refers to the macroporosity that is visible at the soil surface. The macropore system includes the space immediately above the AEI and that space within macropores which fills and drains largely by gravity during and after soil surface exposure to free or ponded water (Fig. 1). Macropores include those voids produced by clay shrinkage, tillage,

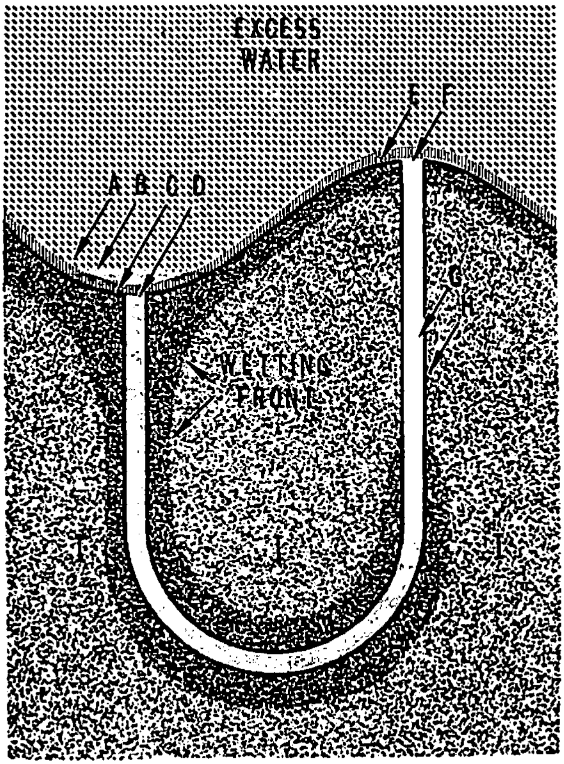


Fig. 1. Soil model containing a micropore system and a macropore system. The macropore system includes the space immediately above the air-earth interface and that within macropores, whereas the micropore system includes the space within and between individual soil individual soil aggregates. Symbol definitions are: A = plant residue cover on air-earth interface; B = free water surface; C = microdepression in air-earth interface; D = water intake port of macropore; E = micro-elevation in air-earth interface; F = soil air exhause port of macropore space; H = macropore wall; and I = micropore space. From Dixon and Peterson (1971).

earthworms, roots, internal erosion, ice lenses, pebble dissolution, and entrapped gas. In contrast, the micropore system includes the spaces within and between individual soil aggregates (textural and structural pores or simple and compound packing voids) that fill and drain largely by capillarity. Thus, during rapid wetting of an initially dry soil, the macropore and micropore systems contain water at pressures of near atmospheric and below atmospheric, respectively. The two systems of pores share common porous borders at the AEI and along macropore walls which allow intersystem flow of water and displaced soil air.

The AEI theory embodies six physical interfacial models (Fig. 2) representing two degrees of surface roughness and

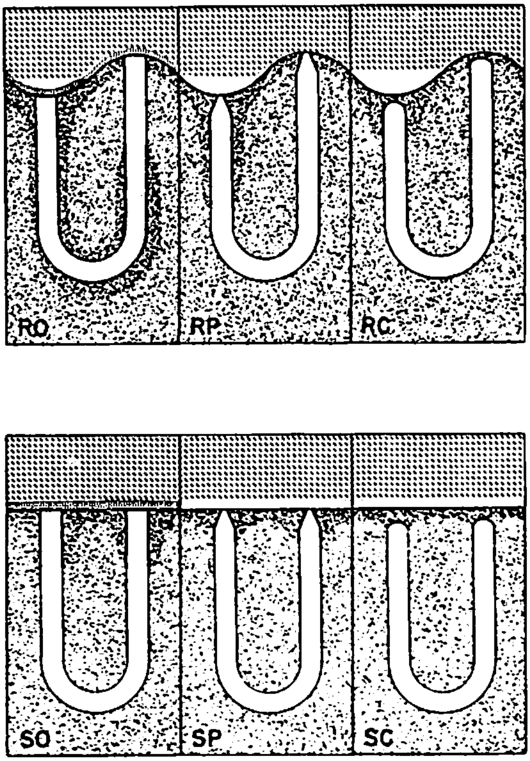


Fig. 2. Air-earth interface models and associated u-shaped macropore for water infiltration into soils. Models RO, RP and RC represent rough interfaces containing open, partly open (unstable) and closed macroports, respectively; whereas models SO, SP and SC represent smooth interfaces containing open, partly open (unstable) and closed macroports. From Dixon and Peterson (1971).

three degrees of surface openness (Dixon and Peterson, 1971). The subterranean part of the macropore system is depicted as a single U-shaped tube to graphically reflect its infiltration role as a water-intake air-exhaust circuit. Models RO, RP, and RC represent rough interfaces with open, partly open (unstable), and closed macropore interfacial openings or macroports, respectively. Models SO, SP, and SC represent plane (smooth) interfaces with open, partly open, and closed macroports, respectively. These models, which have been studied experimentally (Dixon, 1975b), are intended to guide relative infiltration control by serving as a reference framework within which needed modifications in existing surface conditions may be considered.

## AEI Theory Quantification

Although the AEI physical models help to explain the wide range in infiltration rates produced by varying surface conditions and provide physical principles upon which to base the design of surface management practices for relative infiltration control, they do not facilitate quantitative prediction for absolute control. Unfortunately, the physical system assumed in the development of the AEI theory is far too complex for detailed mathematical modeling. Even if the simultaneous flow rates and routes of two fluids in two interacting pore systems as affected by two dynamic AEI conditions could be successfully modeled, the large number of parameters required to do so would make the resulting mathematical model too cumbersome for practical infiltration control. Perhaps the most notable progress toward mathematical modeling of complex infiltration systems was recently reported by Morel-Seytoux (1976, and references therein). Natural complexities of delayed ponding and viscous flow of air were both considered in the derivation of equations for rainfall infiltration. However, the time of ponding is spatially highly variable under the upper boundary conditions assumed in the AEI theory. Micropores located in microdepressions saturate quickly under high intensity rainfall, but macropores located on microknolls may never saturate.

Progress toward quantification of the AEI theory involved three major steps: (1) identification, definition, and interpretation of important AEI theory parameters; (2) selection and interpretation of an appropriate two-parameter equation for modeling the AEI theory; and (3) determination of functional relationships between theory and equation parameters.

### Theory parameter identification.

Surface microroughness and surface macroporosity are the two principal physical parameters of the AEI theory. These two interrelated and interacting properties of the soil surface have yet to be characterized directly in a way that accurately reflects their infiltration roles. Such characterization presents a formidable task because of the great rapidity and intensity of physical and biotic structure-forming processes at the soil surface. A single hydraulic parameter has been chosen that integrates the effects of microroughness and macroporosity on the performance of the U-shaped water-intake air-exhaust circuits or the macropore systems (Dixon, 1975b). This parameter, referred to as the *effective surface head* $h_s$, is defined as the difference between surface water head $h_w$ and soil air pressure head $h_a$, or $h_s = h_w - h_a$. It usually has a narrow range of only a few centimeters of water surrounding the reference zero taken as ambient atmospheric pressure. The effective surface head is commonly less than zero where a large surface area becomes saturated, such as during intense rainfall and basin and border irrigation.

Studies of air pressure buildup under border-irrigated alfalfa (Dixon and Linden, 1972) led to the definition of $h_s$ by showing that soil air pressure affects infiltration by opposing the downward force of surface water within the macropore system. Whenever soil air pressure exceeded the sum of the hydrostatic pressure due to surface head and the soil bubbling pressure, macropores would exhaust soil air rather than infiltrate surface water, as evidenced by streams of bubbles emanating from surface openings of macropores. Thus the surface head, effective in driving water into open macropores, was the actual surface head minus the soil air pressure head.

Because of the limited area wetted, conventional infiltrometers and rainfall simulators cannot ordinarily produce measurable soil air back pressures and the resulting negative effective surface heads that are common during natural infiltration. Consequently, the actual surface head and effective surface head associated with these devices are essentially identical and always greater than zero. Several unique new infiltrometers, referred to as *closed-top infiltrometers* (Dixon, 1975a), were developed to simulate negative as well as positive $h_s$ in a narrow range surrounding zero. The design of these infiltrometers was based on the principle that a positive soil air pressure can be simulated by imposing an equivalent negative air pressure above the ponded-water surface.

Data from the closed-top infiltrometers indicated that infiltration is highly dependent on $h_s$ in a narrow range surrounding zero (Dixon, 1975a). Cumulative 30-minute infiltration increased 19% per cm of $h_s$ for one soil and 33% for another within an $h_s$ range of -3 to +1 cm. Such large effects are not consistent with some theoretical studies and some field studies that have been reported. For instance, Philip (1958) suggested about a 2% theoretical infiltration increase per cm of surface head at small times. In field studies, Horton (1940) and Lewis and Powers (1939) found no clear effect of ponded-water depth on infiltration. The observed large infiltration response to $h_s$ is attributed to the control that $h_s$ exerts over fluid flux in soil macropores; i.e., the rate and ultimate degree of macropore water saturation depends on $h_s$. Thus, $h_s$ determines not only the hydraulic gradients in the macropore system, but also the hydraulic conductivities.

Algebraic equation selection. The next step in quantifying the AEI theory was to select a suitable infiltration equation from those reported in the literature and then mathematically and physically interpret it relative to the AEI theory. The two-parameter time functions that were considered included:

| | |
|---|---|
| $I_v = AT^B$ | Kostiakov (1932) |
| $I_v = AT^{\frac{1}{2}} + BT$ | Philip (1957) |
| $I_v = AT^{\frac{1}{2}} + (B)$ | Ostashev (1936) |
| $I_v = AT + (B)$ | Darcy (1856) |

The equations of Ostashev and Darcy were modified slightly by adding a constant as shown in parentheses to improve their fitting ability and to make them more comparable with the other two equations.

The four equations were least-square fitted to data from (1) AEI, effective surface head and soil air pressure experiments; (2) border irrigation infiltrometers; (3) wet and dry infiltrometer runs; (4) sprinkled-water infiltrometers; and (5) ponded-water infiltrometers with both open and closed tops. Thus, a wide diversity of water source and infiltration system conditions were represented in this equation-fitting study, the results of which will be detailed in a subsequent paper. The conclusion was, however, that only Kostiakov's equation gives a consistently accurate fit regardless of the data source. Furthermore, it ranked equal to or better than each of the other equations for several other evaluation criteria. Consequently, Kostiakov's equation was selected for modeling the AEI theory of infiltration.

Infiltration rate $I_R$ and the rate of deceleration $I_D$ are given by the first and second derivative forms of Kostiakov's equation which are:

$$I_R = ABT^{B-1} \qquad I_D = AB(1-B)T^{B-2}$$

The integral and derivative forms of Kostiakov's equation indicate that where $0 < B < 1$:

(1) $I_V = 0$ and $I_R$ and $I_D$ are undefined for $T=0$;

(2) $I_V \to 0$, $I_R \to \infty$ and $I_D \to \infty$ as $T \to 0$; and

(3) $I_V \to \infty$, $I_R \to 0$ and $I_D \to 0$ as $T \to \infty$.

Thus, the infiltration volume increases at a decreasing rate monotonically with increasing time; and the infiltration rate and its deceleration decrease at a decreasing rate approaching zero asymptotically at large times.

The condition $0 < B < 1$ holds for most data sets from natural infiltration systems; however, infrequently the condition $B > 1$ prevails, indicating that the infiltration rate is increasing with time.

The mathematical interpretation of the parameters in the integral and derivative forms of Kostiakov's equation is readily apparent. If the unit for time is hours, then parameter A may be interpreted as either the first-hour infiltration volume $I_V$ or the mean first-hour infiltration rate $\overline{I}_R$; the parameter product AB is the instantaneous infiltration rate $I_R$ at the end of the first hour or at $T=1$, parameter B is first-hour end rate divided by the mean rate or $B = I_R/\overline{I}_R$ for $T=1$, and the time coefficient $[AB(1-B)]$ is the deceleration (negative acceleration) of the infiltration rate at $T=1$. Thus sets of infiltration data may be conveniently and meaningfully summarized in terms of the A and B parameters and the time period upon which they are based. Such summarizations give the first-hour infiltration and its abatement ratio and permit calculation of infiltration volume, rate, and deceleration for any selected time. Parameter A usually ranges from 0 to 20 and gives the integral curve its magnitude, whereas parameter B usually ranges from 0 to 1 and gives the integral curve its shape.

The A and B parameters may be quickly estimated from infiltration data since $A = I_V$ and $AB = I_R$ at $T=1$; however, better estimates are usually obtained by transforming the integral form to obtain the linear equation:

$$\ln I = \ln A + B \ln T ,$$

which can be least-square fitted to infiltration data. Such fits are easily performed with hand calculators programmed for simple linear regression analyses.

A physical interpretation of the Kostiakov equation and its parameters relative to the AEI theory is possible, although not as readily apparent as the preceding mathematical interpretation. The AEI theory assumes that all infiltrating surface water is subsequently stored in the soil profile. Thus, $I_V$ becomes the storage volume of infiltrated water, $I_R$ is the storage rate, $I_D$ is the deceleration in the storage rate, T is the elapsed time after incipient ponding during which storage has been occurring, parameter A is the storage during the first hour, AB is the storage rate at the end of the first hour, and B is a dimensionless ratio of AB and A which reflects the degree of storage rate abatement during the first hour.

Infiltration has long been recognized as a process reflecting the net effect of numerous concurrent decay or abatement factors (Horton, 1940) which cause the decreasing infiltration rates with increasing elapsed time after the onset of the process. In natural soils, under complex initial and boundary conditions, the abatement of capillary pressure gradient (the justification for the $T^{\frac{1}{2}}$ dependency) is often relatively unimportant compared with other infiltration abatement factors, some of which are infiltration-related abatement processes (Dixon, 1975b). These factors include (1) capillary pressure head reduction at the wetting front resulting from increasing moisture content with depth, (2) surface crusting or sealing, (3) soil subsidence or settling, (4) soil air pressure buildup and air entrapment, (5) clay mineral hydration, (6) eluviation and illuviation, (7) surface water head dissipation, (8) decreasing water phase continuity in the macropore system through air entrainment and entrapment, (9) macroporosity extent and continuity reduction with depth in the profile, and (10) anaerobic slime formation. Some other soil conditions, which will be referred to here as infiltration augmentation factors, tend to offset (and infrequently reverse) the normal abatement in infiltration rates. Such conditions include (1) increasing flow dimensionality with time, (2) increasing wettability with depth, (3) decreasing moisture content (or increasing air porosity) with depth, (4) decreasing water repellency with depth, (5) eluviation (micropiping) that increases surface macroporosity and subsurface macropore continuity, (6) increasing ponded water depth, (7) increasing surface area ponded, and (8) increasing water phase continuity in the macropore system through air displacement and absorption.

The magnitude of parameter B in Kostiakov's equation thus reflects the net interacting effect of the preceding abatement and augmentation factors on the time course of infiltration, with the magnitude being inversely related to the number and intensity of infiltration abatement factors and directly related to the number and intensity of augmentation factors that are active in a given infiltration system.

Values for B near zero, near one, and above one, indicate the dominance of abatement factors, little dominance of either abatement or augmentation factors, and dominance of augmentation factors, respectively. Since most of the abatement and augmentation factors are greatly affected by AEI conditions, parameter B may be regarded as a function of such conditions, especially where unfilled storage space is large enough to not dominate infiltration abatement. Parameter B is expected to be relatively large where effective surface head and surface microroughness and macroporosity are relatively large, and relatively small where these AEI conditions are relatively small.

Darcy-based flow theory for simple infiltration systems can also be useful in physical interpretation of parameter A in Kostiakov's equation. The coefficient in Darcy's equation is given by the product of the hydraulic conductivity and hydraulic gradient for a near-saturated stabile porous soil. For such soils, both the conductivity and gradient are relatively constant. However, for unsaturated soils, the conductivity and gradient are not constant, but are interdependent variables with the gradient decreasing and the conductivity increasing as the soil wets by infiltration. Thus, in accordance with Darcy's equation and the view of surface infiltration presented by Childs (1969), parameter A may be regarded as the product of the first-hour time-weighted means for hydraulic conductivity and hydraulic gradient at the soil surface. The surface hydraulic gradient and conductivity are greatly affected by surface microroughness and macroporosity and their hydraulic counterpart, effective surface head. Consequently, parameter A is also a function of these AEI conditions. Parameter A is expected to be relatively large where effective surface head and surface microroughness and macroporosity are relatively large and relatively small where these AEI conditions are relatively small.

In conclusion, the preceding mathematical and physical interpretations are in agreement that parameters A and B are interrelated. The physical interpretation indicates that both parameters depend on AEI conditions.

Theory versus equation parameters. The last step in quantifying the AEI theory was to relate its two parameters to the two parameters in Kostiakov's equation. The families of $I_V$ curves, shown in Figs. 3 and 4, were used for this purpose. Parameters A and B were determined by least-square fitting of Kostiakov's equation to the family of curves generated by varying surface roughness and openness at the AEI. Parameter means and the coefficients and exponents of the first and second derivative forms were then plotted as functions of the AEI condition (Fig. 5). The four AEI

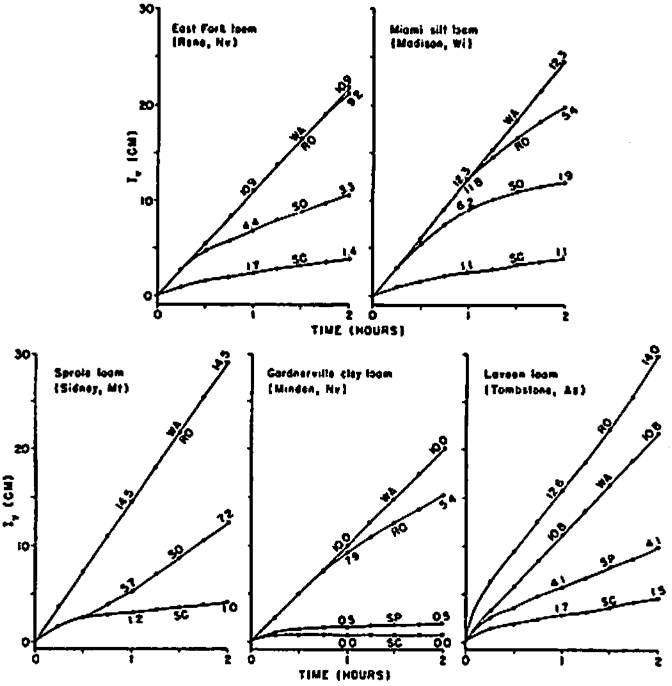


Fig. 3. Sprinkled-water infiltration under imposed air-earth interfaces RO and SC and naturally occurring interface either SO or SP. The curve labeled WA gives the total water applied by the infiltrometer spray nozzle. Numbers near curves at 1- and 2-hour times denote infiltration rates in cm $hr^{-1}$ for these times. From Dixon (1975).

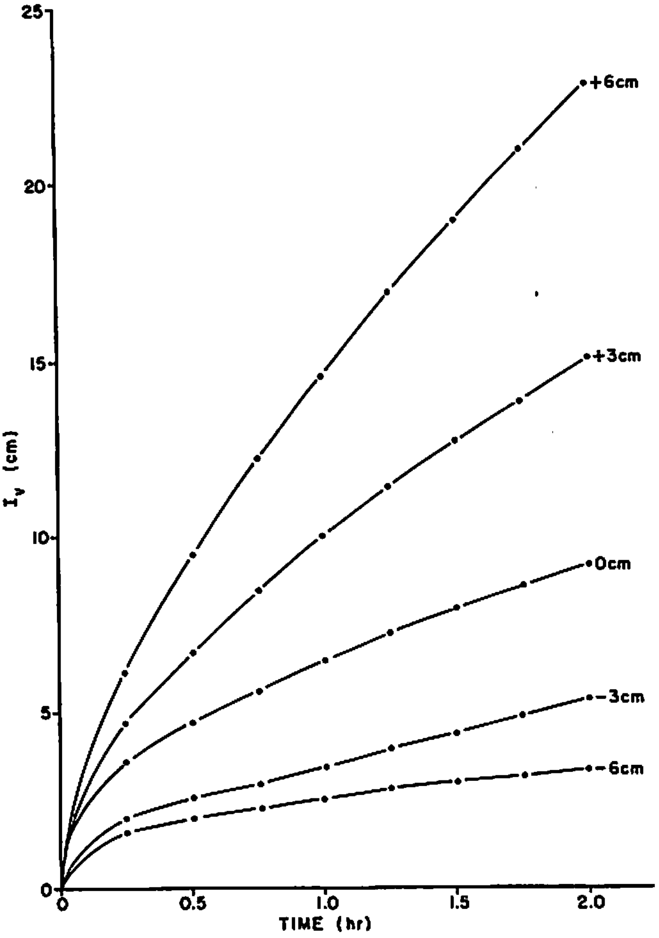


Fig. 4. Ponded-water infiltration $I_V$ as a function of time and effective surface heads ranging from a minus 6 to a plus 6 cm of water as produced by a closed-top infiltrometer.

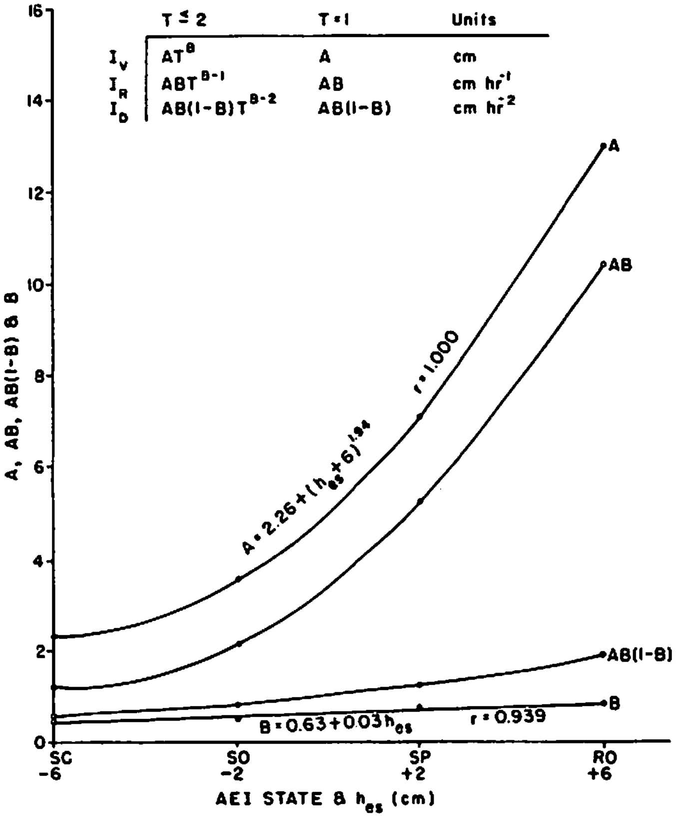
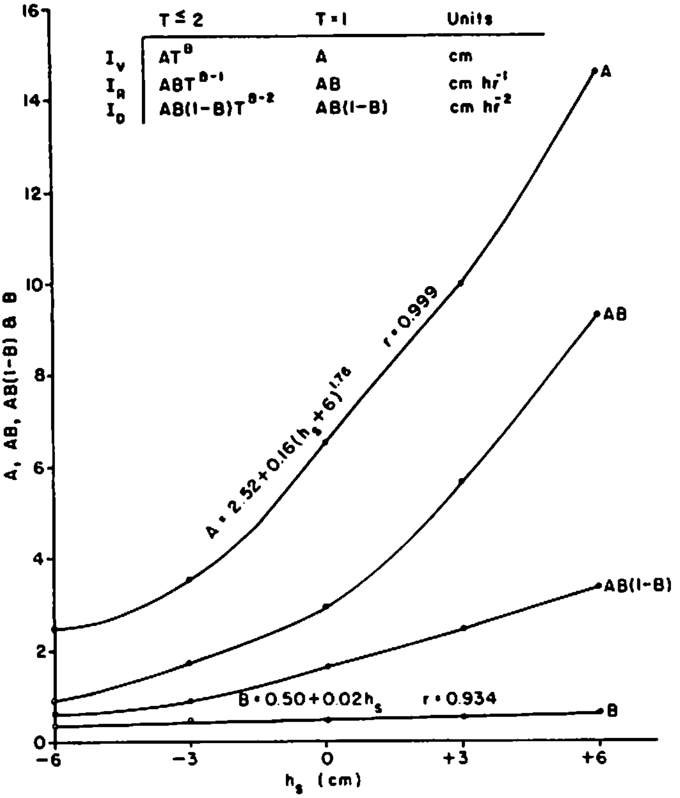


Fig. 5. Parameters for the integral and first and second derivative forms of Kostiakov's equation as functions of the AEI physical state and the estimated equivalent effective surface head $h_{es}$.

conditions, representing a broad range in surface roughness and openness, were assigned the effective surface head values that would be expected under intense rainfall over a large area. This assignment of approximate numerical values expedited subsequent linear regression analyses and facilitated comparison with the curves presented in Fig. 6.



|  | $T \leq 2$ | $T = 1$ | Units |
|---|---|---|---|
| $I_V$ | $AT^B$ | $A$ | cm |
| $I_R$ | $ABT^{B-1}$ | $AB$ | cm $hr^{-1}$ |
| $I_D$ | $AB(1-B)T^{B-2}$ | $AB(1-B)$ | cm $hr^{-2}$ |

Fig. 6. Parameters for the integral and first and second derivative forms of Kostiakov's equation as functions of effective surface head $h_s$.

The family of curves generated by varying the effective surface head (Fig. 4) were analyzed similarly to produce Fig. 6. The close correspondence of the shape and magnitude of the curves in Figs. 5 and 6 is consistent with the hypotheses that $h_s$ is the hydraulic manifestation of surface roughness and openness, and that the closed-top infiltrometer may be used on an uncrusted soil to determine the infiltration effects of these two interacting and interrelated physical conditions. Linear regression analyses indicated that parameter A is accurately described as a power function of the numerical estimates of the AEI condition and the effective surface head, whereas parameter B is linearly related to the AEI condition and $h_s$. Parameter A increases at an increasing rate with increasing surface roughness and openness and with increasing $h_s$ as indicated by power function exponents of 1.94 and 1.76. The coefficients AB and AB(1-B) corresponding to the instantaneous infiltration rate and its rate of deceleration at T=1, respectively, increase at an increasing rate with increasing time.

Although the curves in Figs. 5 and 6 exhibit surprisingly close correspondences, the small differences that do exist may be attributed to (1) error in estimating the numerical range for the AEI conditions, (2) differences in soil texture, and (3) differences in water source. The RO interface would probably have an effective surface head slightly below the estimated 6 cm. The soils represented by the curves shown in Fig. 5 have a mean texture slightly finer than that of the soil represented in Fig. 6. The curves of Fig. 5 are derived from sprinkled-water infiltration, whereas those of Fig. 6 are from ponded-water infiltration. The effect of soil texture would probably be relatively small compared to water source. Inherent to the sprayed-water source is the infiltration augmentation factor of increasing ponded area and depth with time. This factor may largely account for the differences in magnitude and shape of corresponding curves for the A and B parameters.

The functional relationships for the A and B parameters as given graphically and mathematically in Figs. 5 and 6 can provide a practical approach for quantifying the AEI theory. Further research is needed, however, before absolute infiltration for all soils can be predicted by this approach. This includes development of better methods for characterizing surface roughness and openness, evaluation of natural effective surface heads under diverse AEI and water-source conditions, and correlation of the measured effective surface head and corresponding surface roughness and openness. The curves in Figs. 5 and 6 are appropriate for medium-textured soils that are initially dry and well-structured. With the aid of closed-top infiltrometers, similar sets of curves need to be developed for coarse- and fine-textured soils. Methods that facilitate correcting for the infiltration effect of antecedent moisture and single-grain soil structure need to be developed.

Summation. The AEI theory provides a conceptual basis for relative infiltration control at the air-earth interface. Kostiakov's equation can be used in absolute infiltration control by interpreting the coefficient A as a function of effective surface head, with large A values being associated with rough open surfaces and positive effective surface heads and small A values with smooth closed surfaces and negative effective surface heads. Exponent B may be viewed as a function of infiltration abatement-augmentation factors with values near zero, near one, and above one, indicating the dominance of abatement factors, little dominance of either the abatement or augmentation factors, and dominance of augmentation factors, respectively. Since many of the abatement and augmentation factors affect the effective surface head and vice versa, parameters A and B are interdependent. Further theoretical and experimental research is needed to determine the independent effect of various infiltration abatement and augmentation processes on the parameters of Kostiakov's equation. The study of water infiltration as affected by dynamic surface boundary conditions is a fertile field for major experimental and theoretical advances.

## References

[1] Childs, E. C., 1969. "Surface Infiltration. An Introduction to the Physical Basis of Soil Water Phenomena," John Wiley and Sons, Ltd., New York, pp. 274-294.

[2] Darcy, H.P.G., 1856. "Les Fontaines Publiques de la Ville de Dijon," Dalmont, Paris.

[3] Dixon, R. M., 1972. "Controlling Infiltration in Bimodal Porous Soils: Air-Earth Interface Concept," Proc. 2nd Symp. Fundamentals of Transport in Porous Media, IAHR, ISSS, University of Guelph.

[4] Dixon, R. M., 1975a. "Design and Use of Closed-Top Infiltrometers," Soil Sci. Soc. Amer. Proc., 39: 755-763.

[5] Dixon, R. M., 1975b. "Infiltration Control through Soil Surface Management," Proc. Symp. on Watershed Management, Irrigation and Drainage Division, ASCE and Utah State University, pp. 543-567.

[6] Dixon, R. M. and Linden, D. R., 1972. "Soil Air Pressure and Water Infiltration under Border Irrigation," Soil Sci. Soc. Amer. Proc., 36:948-953.

[7] Dixon, R. M. and Peterson, A. E., 1971. "Water Infiltration Control: A Channel System Concept," Soil Sci. Soc. Amer. Proc., 35:968-973.

[8] Horton, R. E., 1940. "An Approach Toward a Physical Interpretation of Infiltration Capacity," Soil Sci. Soc. Amer. Proc., 5:399-417.

[9] Kostiakov, A. N., 1932. "On the Dynamics of the Coefficient of Water Percolation in Soils and the Necessity of Studying it from the Dynamic Point of View for the Purposes of Amelioration," Trans. Sixth Commission ISSS, Russian Pt. A., pp. 17-21.

[10] Lewis, M. R. and Powers, W. L., 1939. "A Study of Factors Affecting Infiltration," Soil Sci. Soc. Amer. Proc., 3:334-339.

[11] Morel-Seytoux, H. J., 1976. "Derivation of Equations for Rainfall Infiltration," Journal of Hydrology, 31: 203-219.

[12] Ostashev, N. A., 1936. "The Law of Distribution of Moisture in Soils and Methods for the Study of the Same," International Conference Soil Mechanics and Foundations Engineering Proceedings, Vol. 1 (Sect. K): 227-229.

[13] Philip, J. R., 1957. "The Theory of Infiltration: 4. Sorptivity and Algebraic Infiltration Equations," Soil Science, 84:257-264.

[14] Philip, J. R., 1958. "The Theory of Infiltration: 6. Effect of Water Depth Over Soil," Soil Science, 85: 278-286.