HYDROLOGY AND WATER RESOURCES IN ARIZONA AND THE SOUTHWEST, VOLUME 7, p. 153-162

Proceedings of the 1977 meetings of the Arizona Section of the American Water Resources Association and the Hydrology Section of the Arizona Academy of Science, held in Las Vegas, Nevada, April 15-16.

# SIMULATION OF SUMMER RAINFALL OCCURRENCE IN ARIZONA AND NEW MEXICO

by

Herbert B. Osborn and Donald Ross Davis

## INTRODUCTION

Thunderstorms produce most of the annual rainfall and almost all runoff from arid and semiarid rangelands in the Southwest. Thunderstorms also produce major flood peaks from small (100-square miles) watersheds in the Southwest. Therefore, developing models that can be used for predicting runoff in river basins, for flood plane zonings, and for estimating flood damage, is important to engineering design, particularly in regions where thunderstorms are a significant portion of the rainfall and runoff. Such models also provide basis for estimating erosion and sediment transport, as well as estimating precipitation available for forage growth.

Osborn, Lane and Kagan (1974) used records from 95 recording rain gages on the 58-square-mile U. S. Department of Agriculture Walnut Gulch Experimental Watershed in southeastern Arizona to develop a simplified stochastic model for air-mass thunderstorm rainfall. Osborn, Mills, and Lane (1972) used the thunderstorm rainfall model and a previously developed rainfall-runoff relationship (Osborn and Laursen, 1973) to predict runoff from Walnut Gulch, and reported the resulting accuracy and certainty of the output.

A regional model based on Walnut Gulch and Alamogordo Creek air-mass thunderstorm rainfall models, Agricultural Research Service (ARS) and National Weather Service (NWS) 24-hour rain gage records in Arizona and New Mexico, and the NWS climatological data for the Southwest is being developed. The regional model includes a prediction model for thunderstorm rainfall at a point which is based on daily point rainfall occurrence probabilities (> 0.01 inch) from 15 years of records from 15 NWS 24-hour recording and 7 standard rain gages in Arizona and New Mexico (Figure 1).

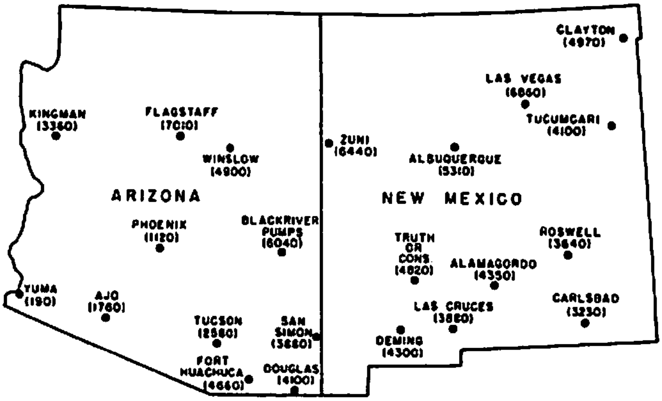


Fig. 1. Location and elevation of 22 selected NWS rain gages in Arizona and New Mexico.

---

The authors are Research Hydraulic Engineer, Southwest Watershed Research Center, Agricultural Research Service, Tucson, Arizona, and Asst. Professor, Department of Hydrology and Water Resources, University of Arizona, Tucson, Arizona, respectively.

The 15 stations with recording rain gages were chosen primarily for continuity of record and thunderstorm identification. Using 15 years of record (1958-1972) for evaluation seemed to be a good balance between fewer stations with longer records, and more stations with shorter records. The 7 stations with standard gages were chosen primarily to fill gaps in the 15-gage network.

## PREDICTION OF RAINFALL OCCURRENCE

There are significant differences in thunderstorm rainfall in different regions of Arizona and New Mexico which complicate such a model (Petterson, 1969, pp. 130-131; Osborn, 1971). In southeastern Arizona, for example, most thunderstorms can be classified as air-mass. Thus, the Walnut Gulch air-mass thunderstorm model is based on this simplifying assumption, whereas in eastern New Mexico, for example, frontal activity is an important consideration in estimating rainfall from summer thunderstorms. In the higher mountains of northern and central Arizona and New Mexico, low intensity winter rain and snow are a more important source of precipitation than are summer thunderstorms, although thunderstorms still produce a significant amount of rainfall.

The proposed rainfall occurrence model has three parameters: elevation, latitude, and longitude. We used these parameters because they could be identified at any location, as opposed to trying to fit known rainfall distributions at certain locations with one, or a combination of mathematical distributions that are assumed to represent subregions as well as the specific point.

## RAINFALL OCCURRENCE

In developing the model, the 22 stations were considered representative of their geographic and topographic locations. However, most of the stations are located in or near cities, and not for geographic or climatological considerations. The stations ranged from near sea level (Yuma) to over 6,000 feet (Flagstaff and Las Vegas), from northern Arizona and New Mexico (Flagstaff, Winslow, Albuquerque, and Las Vegas) to southern Arizona and New Mexico (Yuma, Douglas, Las Cruces and Carlsbad). For example, smoothed curves for average daily point rainfall probability at Douglas, Flagstaff, Tucson, and Phoenix, in Arizona, and Albuquerque, Las Cruces, Roswell, and Tucumcari, in New Mexico, illustrate both the similarities and differences in summer rainfall in the Southwest (Figures 2 and 3). The curves are the accumulation of events which may result from one or more of several atmospheric conditions. The conditions are extremely simplified in the model to represent moisture flows into Arizona from the Southwest (SW), the "monsoon" season when moisture flows into the Southeast from the Gulf of Mexico (SE), and frontal (continental) storms pushing into Arizona and New Mexico from the north and west.

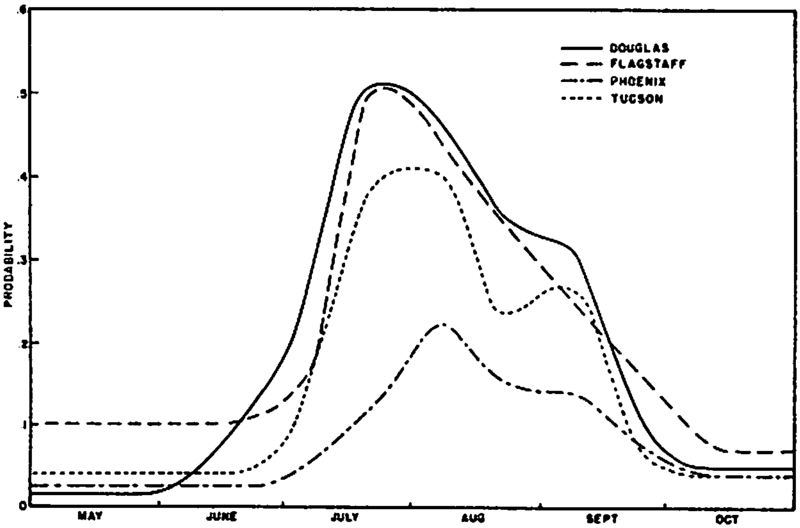


Fig. 2. Average seasonal rainfall probabilities for selected Arizona NWS rain gages.

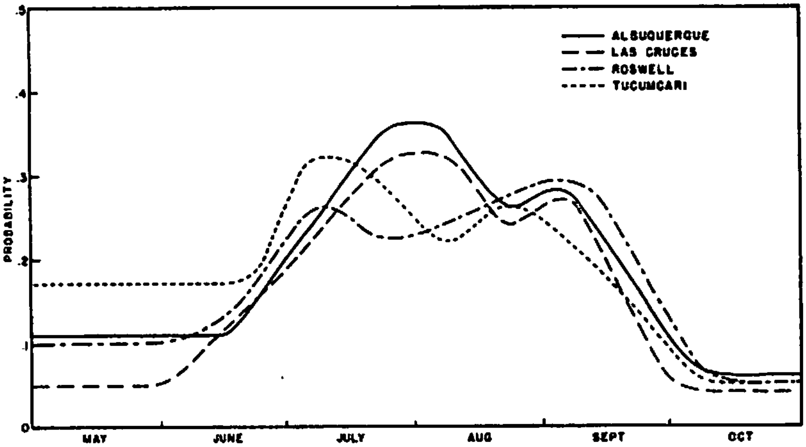
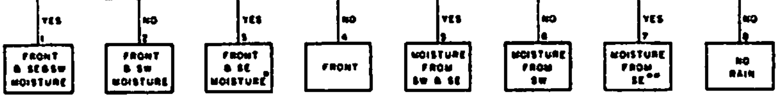


Fig. 3. Average seasonal rainfall probabilities for selected New Mexico NWS rain gages.

The model is an effort to follow, with simplifying assumptions, what actually happens, physically, to produce rainfall in Arizona and New Mexico. A flow diagram (Figure 4) follows through a logical sequence in determining if rainfall occurs. As already mentioned, the magnitude and areal extent of predicted events are based on stochastic models of thunderstorm rainfall developed from records from the Walnut Gulch and Alamogordo Creek Watersheds.



* AREAL RAINFALL GENERATED WITH ALAMAGORDO CREEK THUNDERSTORM RAINFALL MODEL.

** AREAL RAINFALL GENERATED WITH WALNUT GULCH AIR-MASS THUNDERSTORM RAINFALL MODEL.

Fig. 4. Simplified schematic diagram of summer rainfall occurrence in Arizona and New Mexico.

All probabilities for each of the three systems are determined independently, as indicated in Figure 4, and the "combination" events are assumed to represent the less frequent, exceptional storms that occur in the Southwest.

## FRONTAL RAINFALL OCCURRENCE

Frontal rainfall frequency from May through September was assumed constant over time at any specific location. Based on trial and error, the probability of occurrence of frontal rainfall on day n, $P_F(n)$, assuming no frontal rainfall on the previous day, was expressed by the equation:

$$P_F(n) = .12 + .008\ (103-\ell_o),- 0.012\ (37-\ell_a),\ P_F(n) \geq 0, \tag{1}$$

where $\ell_o$ = longitude in degrees, and

$\ell_a$ = latitude in degrees.

The approximate limits in the equation are:

$103^o < \ell_o < 114^o$

$31^o < \ell_a < 37^o$

Once frontal rain occurs, the system tends to persist. Continued rainfall from the system seems highly correlated with elevation, whereas the initial occurrence of frontal rainfall is most highly correlated with latitude, as well as significantly correlated with longitude.

If frontal rainfall was predicted on day n, the chance of rainfall on day n+1, $P_F(n)$, was given by the equation:

$$P_F(n+1) = P_F(n)\ \frac{h}{1000}$$

where $P_F(n+1) \leq 0.75$, and h = elevation in feet ($1000^{ft} < h < 8000^{ft}$).

$$\text{Also, } P_F(n+2) = P_F(\underline{n+1});\ P_F(n+3) = P_F(n+2), \text{ etc.} \tag{2}$$

## SW RAINFALL OCCURRENCE

From May through September, the average probability of SW rainfall at any location was assumed constant over time. SW rainfall occurrence decreases with latitude and increases with longitude and elevation. The probability of occurrence of SW rainfall on day n, $P_{sw}(n)$, assuming no SW rainfall on the previous day, was given by:

$$P_{sw}(n) = .08 + .00001\ h + .01(31-\ell_a) - .01\ (114-\ell_o), \tag{3}$$

$$P_{sw}(n) \geq 0.$$

Once SW rainfall occurs, there is a much greater chance of rainfall the next day. This persistence is highly correlated with elevation, suggesting that the system, although present over a wide region, may be too weak or lack the moisture to produce rainfall at lower elevations. If rain was predicted on day n, the chance of rain on day n+1, $P_{sw}(n+1)$ was given as:

$$P_{sw}(n+1) = P_{sw}(n)\ \frac{h}{1000}$$

where

$$P_{sw}(n+1) \leq 0.65.$$

Also,

$$P_{sw}(n+2) = P_{sw}(n+1);\ P_{sw}(n+3) = P_{sw}(n+2), \text{ etc.} \tag{4}$$

Once no rain is predicted, the program returns to $P_{sw}(n)$.

## COMBINED FRONTAL AND SW RAINFALL

If both frontal and SW rainfall were predicted on the same day, a much greater chance of rainfall occurring was assumed on the following day. Assuming F and SW are independent, both frontal and SW rain have been predicted on day n, $P_{F+SW}(n+1)$ was given by:

$$P_{F+SW}(N+1) = P_F(n+1) + P_{SW}(n+1) - P_F(n+1) \times P_{SW}(n+1) \tag{5}$$

where $P_{F+SW}(n+1) \leq 0.85$

also $P_{F+SW}(n+2) = P_{F+SW}(n+1)$, etc.

As before, once no rain is predicted, the program returns to $P_F(n)$ and $P_{SW}(n)$.

## SE EVENTS

Approximate values for the occurrence of air-mass thunderstorm rainfall were developed by subtracting estimates of frontal and SE events from all summer rains. The resulting curves indicated three distinct subregions within Arizona and New Mexico with different air-mass thunderstorm frequency characteristics (Figures 5 and 6).

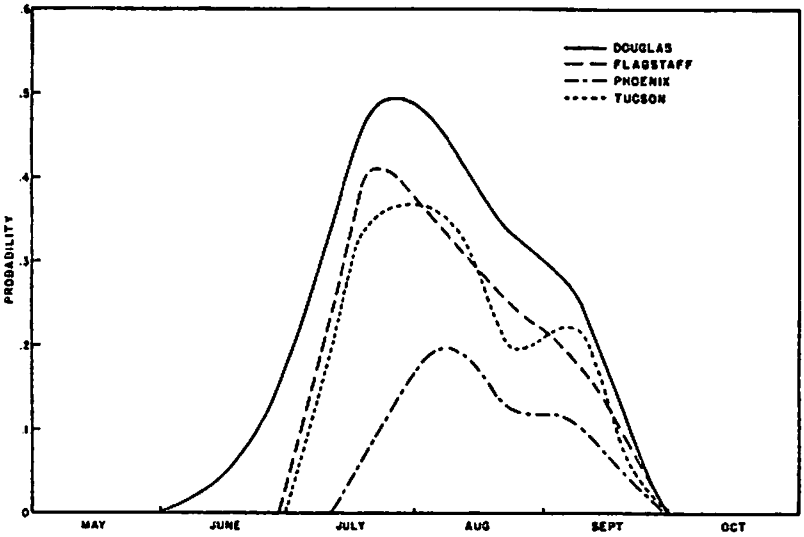


Fig. 5. Average seasonal air-mass thunderstorm rainfall probabilities for selected Arizona NWS rain gages.

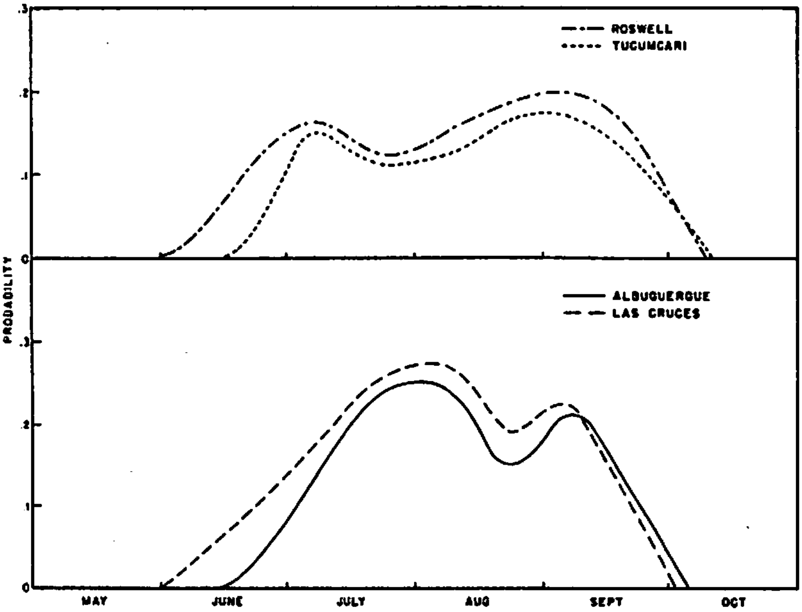


Fig. 6. Average seasonal air-mass thunderstorm rainfall probabilities for selected New Mexico NWS rain gages.

Region I - Eastern New Mexico

Region II - The Rio Grande Valley, western New Mexico, and the upper Gila and Little Colorado River Basins

Region III - The remainder of Arizona

Roswell, Las Cruces, and Douglas were chosen as the base stations for Regions I, II, and III, respectively, to predict air-mass thunderstorm rains primarily because of their location and their good records.

Although a Bernoulli random variable adequately described the occurrence of air-mass thunderstorm rainfall during the peak of the season, a second variable was needed to predict the beginning of the season. The beginning date for the "monsoon" season (SE) was generated using a random variable normally distributed around June 22 for Roswell, June 15 for Las Cruces, and July 3 for Douglas (dates were estimated from NWS data). For the first two regions, 4 days are added to the mean of the normal distribution for every added degree of latitude. For Region III, 2 days were added for each degree of longitude, as well as 4 days for each degree of latitude, and 3 days were subtracted for each 1,000 feet elevation. These values were estimated by trial and error based on NWS data.

Occurrence of air-mass thunderstorm rainfall was determined using frequency curves from the 8 stations (Figures 5 and 6), and adjusting the curves according to the latitude and elevation at the desired location. Occurrences increase with elevation and decrease with latitude in all three regions. The basic equation was

$$P_{SE}(n) \text{ at location} = P_{SE}(n) \text{ at base station} \times R, \tag{6}$$

where $P_{SE}(n)$ = the probability of air-mass thunderstorm rainfall occurring on a given day, and

$R$ = the ratio between probabilities at the given location and the base station.

The multiplier, R, for a given location was determined from the following set of equations. The equations varied only in the base latitudes and elevations for each region (which are for the base stations at Roswell, Las Cruces, and Douglas).

$$\text{Region I} \quad - R_1 = 1 + .14\ (33.4 - \ell_\alpha) - .10\ (\frac{3640-h}{1000}) \tag{7}$$

$$\text{Region II} \quad - R_2 = 1 + .14\ (32.4 - \ell_\alpha \ - .10\ (\frac{3880-h}{1000}) \tag{8}$$

$$\text{Region III} \quad - R_3 = 1 + .14\ (31.5 - \ell_\alpha) - .10\ (\frac{4100-h}{1000}) \tag{9}$$

where $1000^{ft} < h < 8000^{ft}$. The equations were determined primarily from estimates of July-August air-mass thunderstorm rainfall. Since the estimates were inexact, statistical correlation between the estimated and predicted values would be misleading.

## EVALUATION

The model was based on location parameters with an effort to explain the storm systems, but we made no attempt to rigorously define these systems. Such terms as "frontal occurrence," "southwest moisture," "southeast moisture" are used as general support for a three-component prediction model, based on one topographic and two geographic parameters. Rigorous definitions of these terms and the rainfall associated with them would be too complex to use in a regional model. The model was developed to predict rainfall occurrence, with an effort to relate the equations logically to the meteorology of the Southwest. The model has a Markovian feature, since rainfall occurrence on any day depends on whether or not it rained on the previous day.

However, the principal assumptions that were made and the rules that were possibly violated with these assumptions should be discussed. The equations purportedly relate to frontal systems, flow of moist air into Arizona and New Mexico from the southeast and southwest, the coexistence of these systems, and their persistence. Among the principal assumptions are:

(1) Frontal rains (or rains from frontal systems) can be assumed random from May through September.

(2) SW rains (rainfall occurring from moisture pushed into Arizona and New Mexico from tropical storms in the Pacific) can be assumed random from May through September.

(3) Persistence of either frontal systems or SW moisture is highly dependent on elevation.

(4) SE rainfall can be predicted by a seasonal Bernoulli random variable based on probabilities from a base station in each of three designated subregions.

(5) Any two, or all three, systems can occur simultaneously to produce rainfall events.

In the western United States, frontal systems tend to move further south in the winter. However, based on NWS weather maps, frontal systems are still fairly frequent in the Southwest in the summer, particularly in the northern regions of the southwest. Because of the low probabilities, it is difficult to determine a meaningful distribution for summer frontal occurrence other than the constant probabilities assumed in the model.

There is even less information on the effects of Pacific tropical storms on rainfall in the southwest. However, more recently, satellites have provided better definition of these storms, and some estimate of occurrences, other than the constant probability in the model, might be used to estimate the variability of summer rainfall occurrence in time.

The high correlation between rainfall persistence and elevation is probably primarily a question of whether or not rainfall can reach the ground at stations at lower elevations. The system persists independently of elevation, even though the rainfall is correlated with elevation.

The question of persistence of SE rainfall will be discussed in a later section.

Actually, the three systems normally do not develop independently (See any text on Meteorology.). Moist air moves into the Southwest from the Gulf of Mexico and/or the Pacific after the prevailing path of frontal systems has moved northward. However, NWS weather maps do suggest the possibility of joint occurrence of such systems, although the probabilities and results of such occurrences are uncertain. A mixture of SE and SW moisture may be more common in the Southwest, although identifying the differences in results may be even more difficult.

The 22 stations used in developing the model were considered representative of their geographic and topographic location. However, there may be anomolies in Arizona and New Mexico that are not explained by the model. For example, annual and seasonal rainfall differ considerably at the same elevations just southwest and northeast of the Mogollon Rim in central Arizona. Presently, there are insufficient data available to determine whether thunderstorm frequency also varies significantly from that predicted by the model.

## ALL EVENTS

The average number of events in a season (N) was determined from 15 years of record at 22 rain gage locations in Arizona and New Mexico (Figure 1, Table 1). Through regression analysis using elevation, latitude, and longitude as independent input variables, we developed two equations. The first equation is:

$$E(N) = 196 + 0.00398h + 0.811\,\ell_\alpha - 1.99\,\ell_o, \tag{10}$$

where $R^2 = .87$ and SEE = 2.65,

$\ell_\alpha$ = latitude in degrees

$\ell_o$ = longitude in degrees

applies to Regions I and II, New Mexico and the upper Gila and Little Colorado River basins in Arizona.

The second equation is:

$$E(N) = 333 + 0.00467h - 3.11\,\ell_\alpha - 1.97\,\ell_o, \tag{11}$$

where $R^2 = .98$ and SEE = 1.90,

applies to Region III, the remainder of Arizona. In general, observed and predicted values vary appreciably only at a few stations (Yuma, Winslow, Carlsbad, Las Cruces, and Zuni) out of the 22 used in the analysis (Table 1).

Estimates based on equation 11 were compared with a study of the effects of elevation on rainfall in the Catalina Mountains of southern Arizona (Duckstein et al., 1973; Battan and Green, 1971). Based on seven seasons of recording rain gage records in the Catalina Mountains, Duckstein et al. (1973) found that the number of events per season was strongly correlated with elevation as by:

$$E(N) = 12.44 + 3.12\,h, \tag{12}$$

where $R^2 = .88$ and SEE = 2.15 and

where h = elevation in 1,000 feet.

For the seven seasons, there was an average of 23 events at the Tucson International Airport. For 15 years of record, there were 28 events, or roughly 20% more than were estimated from the shorter record. Other records in the vicinity also indicated a larger average number of events. When the average number of seasonal events were increased by 20%, equation 12 becomes

$$E(N) = 17 + 3.87\,h, \tag{13}$$

and assuming one latitude and longitude for the Catalina Mountains, and h in thousands of feet, equation 11 becomes:

$$E(N) = 15 + 4.5\,h. \qquad (14)$$

TABLE 1.

Average number of observed and predicted rainy days, June through September, for 22 selected stations in Arizona and New Mexico.

| Station | Elev. (ft) | Long. (°) | Lat. (°) | Observed Rainy Days (N) | Predicted Rainy Days E(N) |
|---|---|---|---|---|---|
| Albuquerque | 5310 | 106.6 | 35.0 | 32 | 33 |
| Alamogordo | 4350 | 106.0 | 32.9 | 30 | 29 |
| Carlsbad | 3230 | 104.3 | 32.3 | 22 | 27 |
| Clayton | 4970 | 103.1 | 36.4 | 41 | 40 |
| Deming | 4300 | 107.7 | 32.2 | 26 | 25 |
| Las Cruces | 3880 | 106.7 | 32.4 | 28 | 25 |
| Las Vegas | 6860 | 105.2 | 35.7 | 43 | 43 |
| Roswell | 3640 | 104.5 | 33.4 | 30 | 30 |
| Truth or Conseq. | 4820 | 107.3 | 33.2 | 27 | 29 |
| Tucumcari | 4050 | 103.6 | 35.2 | 34 | 35 |
| Zuni | 6440 | 108.8 | 35.1 | 31 | 34 |
| San Simon | 3880 | 109.1 | 32.2 | 24 | 20 |
| Black River Pump | 6040 | 109.8 | 33.5 | 30 | 29 |
| Ajo | 1760 | 112.9 | 32.4 | 17 | 18 |
| Douglass | 4100 | 109.6 | 31.5 | 36 | 38 |
| Flagstaff | 7000 | 111.7 | 35.1 | 36 | 36 |
| Ft. Huachuca | 4660 | 110.3 | 31.6 | 39 | 39 |
| Kingman | 3360 | 114.0 | 35.2 | 15 | 15 |
| Phoenix | 1120 | 112.0 | 33.4 | 14 | 14 |
| Tucson | 2580 | 110.9 | 32.1 | 28 | 27 |
| Winslow | 4900 | 110.7 | 35.0 | 26 | 29 |
| Yuma | 194 | 114.6 | 32.6 | 4 | 7 |

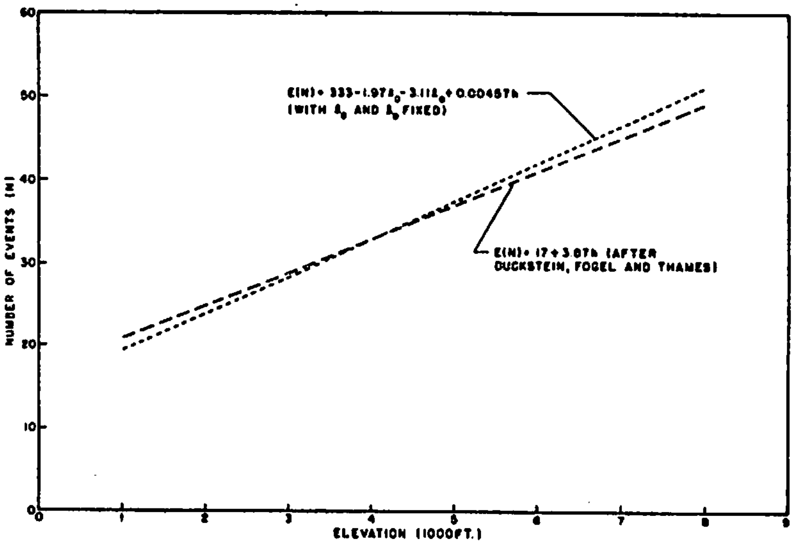


Fig. 7. Comparison of 2 equations for estimating the number of summer rains in the Catalina Mountains of southern Arizona.

The curves are similar (Figure 7), suggesting the equations developed from NWS stations may provide good estimates of summer rainfall occurrence in other mountainous regions of the Southwest, and not just for the populated "valleys."

Estimates of the number of seasonal occurrences can be used as a check of the equations within the rainfall occurrence model. For example, $P_{SE}(N)$ for any station can be estimated by subtracting $P_f(N)$ and $P_{SW}(N)$ from P(N) (Table 2). The resulting values for $P_{SE}(N)$ should equal average seasonal values predicted with equations 6, 7, 8, and 9.

TABLE 2.

Average frequency of storms/season for eight selected Arizona and New Mexico rain gage locations, June through September (events).

| Station | E(N) | P(N) | $P_fN$ | $P_{SW}N$ | $P_{SE}N$ |
|---|---|---|---|---|---|
| Douglas | 36 | .30 | <.01 | .02 | .27 |
| Flagstaff | 36 | .30 | .04 | .05 | .21 |
| Tucson | 28 | .23 | <.01 | .03 | .19 |
| Phoenix | 14 | .13 | <<.01 | .02 | .12 |
| Albuquerque | 32 | .26 | .10 | 0 | .16 |
| Las Cruces | 28 | .23 | .05 | 0 | .18 |
| Roswell | 30 | .25 | .07 | 0 | .18 |
| Tucumcari | 34 | .28 | .14 | 0 | .14 |

## PERSISTENCE

Several investigators have used Markov Chain models to predict point rainfall occurrence. Smith and Schreiber (1973) assumed all events were of the same population and successfully fitted daily rainfall occurrence at three stations in southeastern Arizona with a segmented first-order Markov Chain model. Woolhiser (1975) has proposed a three-parameter mixed-exponential Markov Chain model of daily rainfall, based primarily on data from the Great Plains area. Possibly this model, or a variation, could be adapted to the Southwest as a substitute for the more cumbersome empirical equations that are presented here.

Other investigators, like Allen and Haan (1975), have used Markov Chain models to fit rainfall distributions in the eastern United States, and some of these may have application in the Southwest.

Separate equations were developed in the model to account for persistence in frontal activity and moisture from the southwest. No persistence equation was included for moisture from the southeast.

Moisture from the southeast particularly dominates summer rainfall in southeastern Arizona (Figures 2 and 5). Osborn, Mills, and Lane (1973) modeled storm occurrence as a seasonal Bernoulli random variable, based on occurrence of storms of more than 0.2 inch on Walnut Gulch (located between Tucson and Douglas, Arizona). A comparison of 12 years of simulated and actual storms of 1 inch or greater on Walnut Gulch indicated that the occurrence of these larger (major runoff producing) events on Walnut Gulch on successive days were similar for simulated and actual data. Although there was no statistical difference in the persistence pattern for the major events between simulated and actual data, the model seemed to simulate greater persistence than the actual data. Therefore, since the principle purpose of the regional rainfall model is to simulate occurrences that can be used to predict runoff, the model does not include a persistence equation for southeast moisture.

## SUMMARY

A regional model based on NWS and ARS precipitation and climatalogical data in the Southwest is being developed. The model includes independent outcome of 3 types of rainfall, as well as any combination of the 3 types. The model can be used to predict the occurrence of rainfall for engineering and watershed design purposes.

## REFERENCES CITED

Allen, David M., and C. T. Hann. 1975. Stochastic simulation of daily rainfall. Research Report #82, Water Resources Res. Inst., Univ. of Kentucky.

Battan, L. J., and C. R. Green. 1971. Summer rainfall over the Santa Catalina Mountains. Univ of Arizona, Atmos. Phys. Tech. Rep. #22. 11 p.

Duckstein, Lucien, M. M. Fogel, and J. L. Thames. 1973. Elevation effects on rainfall: A stochastic model. Jour. of Hydrology, 18:12-35.

Hales, John E., Jr. 1973. Southwestern United States summer monsoon source--Gulf of Mexico or Pacific Ocean. NOAA Tech. Memo NWS WR 84. 26 p.

Osborn, H. B. 1971. Some regional differences in runoff-producing thunderstorm rainfall in the Southwest. American Water Resources Association, Arizona Section, Arizona Academy of Science, Hydrology Section, Hydrology and Water Resources in Arizona and the Southwest. IV:144-157.

Osborn, H. B., L. J. Lane, and R. S. Kagan. 1971. Stochastic models of spatial and temporal distribution of thunderstorm rainfall. Proc. Symposium on Statistical Hydrology, USDA-ARS Miscellaneous Publication #1275:211-231, issued 1974.

Osborn, H. B., and E. M. Laursen. 1972. Thunderstorm runoff in southeastern Arizona. J. Hydraulics Div., ASCE 98(HY7):1129-1145.

Osborn, H. B., W. C. Mills, and L. J. Lane. 1972. Uncertainties in estimating runoff-producing rainfall for thunderstorm rainfall-runoff models. Proc. Int'l. Sym. on Uncertainties in Hydrologic and Water Resource Systems, I(2.6)1:189-202.

Osborn, H. B., W. C. Mills, and L. J. Lane. 1973. Reply to discussion of above paper. III:1410-1413.

Petterson, Sverre. 1969. Introduction to Meteorology. Third edition. McGraw-Hill. 333 p.

Smith, R. E., and H. A. Schreiber. 1973. Point processes of seasonal thunderstorm rainfall events. Water Resources Research, AGU 9(4):871-884.

Woolhiser, D. A. 1975. Personal communication.