Stochastic Daily Rainfall Generation in Southeast Arizona: An Example from Walnut Gulch Experimental Watershed

Huey-hong Hsieh, Jeffry Stone, D. Phillip Guertin, Donald D. Slack

Abstract

The thunderstorm rainfall in southeast Arizona has high spatial and temporal variation. Using areal-averaged rainfall for watershed modeling had caused over or under estimations in erosion and runoff peak. This is a common problem usually faced by watershed researchers. This study had presented a series of analyses to identify spatial characteristics of thunderstorm rainfall based on statistical analyses on Walnut Gulch Experimental Watershed (WGEW) rainfall records. A stochastic daily summer rainfall generator was constructed and calibrated based on derived statistical characteristics and selected events. This rainfall generator can be used for advanced hydrological modeling.

Keywords: thunderstorm rainfall, rainfall generator, hydrological modeling, spatial characteristics

Introduction

Thunderstorm rainfall in semi-arid area has very high spatial and temporal variability (Osborn et al. 1993). Knowledge of the spatial characteristics of thunderstorm rainfall is important for the increasing demands of distributed hydrological modeling. Rainfall data from the semiarid USDA-ARS WGEW are used to investigate the spatial characteristics of thunderstorm rainfall in southeast Arizona and to develop a daily thunderstorm rainfall generator.

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Methods

This study was involved in two major tasks: the first part was the characterization of the spatial patterns of thunderstorm rainfall, which were derived form the statistical analyses on the 40-year WGEW summer rainfall records. The second part was the developments and calibrations of a daily thunderstorm rainfall generator based on the statistical characteristics derived from previous step. The following sections describe the methods in more details.

Characterization of the thunderstorm rainfall patterns

The first objective of this research is to identify the spatial characteristics of the daily summer thunderstorm rainfall on WGEW and TW in order to provide information for the construction of the thunderstorm generator. The spatial characteristics of the thunderstorm rainfall were clarified through the analyses of the historical rainfall events recorded by a dense rain gage network on WGEW and TW. Several distinct characteristics of thunderstorm rainfall were examined, which include: distribution of storm center location, distribution of maximum rainfall depth within a storm cell, shapes and orientations of storms, relation between maximum rainfall depth and storm coverage, and transition probability.

Developments and calibrations of the daily thunderstorm rainfall generator

A stochastic daily thunderstorm rainfall generator was constructed based on the statistical characteristics derived from the previous analysis. The rainfall generator involved the following steps: 1) Generation of a dry/wet sequences using the transition probability derived above, 2) Generation of locations of storm center using derived distribution,

3) Generation of maximum rainfall depth within a storm cell using derived distribution 4) Generation of storm coverage using relation of maximum depth and storm coverage derived above, and 5) Computations of rainfall depth on rain gage location using exponential spread function spreading outward from the storm center.

The simulated results were compared with observations and several adjustments of parameters were made to reduce the differences between simulations and observations.

Events and Data Characteristics

WGEW is located in southeast Arizona and has an area of 148 km^2. It ranges in elevation from 1650 m in the east to 1200 m in the west. The rain gage network consists of 93 weighing bucket recording gages, which record cumulative depth of precipitation on a continuous time base. Location of WGEW and the rain gage network on WGEW are shown in Figures 1 and 2.

Figure 1. Location of WGEW.

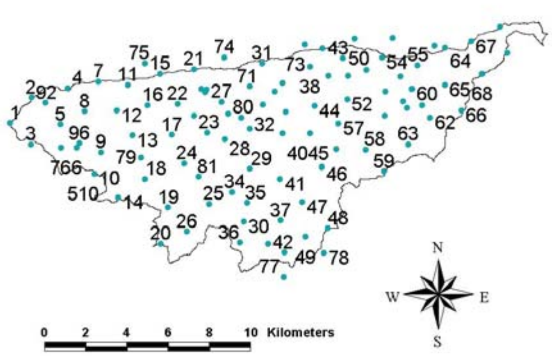

Figure 2. Rain gage network on WGEW.

The comparison study is limited to single events that only one storm event was recorded during a day by event related rain gages. The rain gage network consisted of 93 weighting recording type gages that recorded cumulative depth of precipitation on a continuous time base. The summer (July - September) rainfall records from 1960-95 were converted into an Access database and the selected events were retrieved from the database

Results

Results from this research consist of two parts: the first part is the statistical characteristics of the thunderstorm rainfall in southeast Arizona from the analyses of the rainfall data and the second part is the results of the simulations from the constructed daily rainfall generator. The following articles discuss the results separately.

Statistical characteristics of thunderstorm rainfall

In order to construct a stochastic daily thunderstorm generator, the statistical characteristics of thunderstorm rainfall are examined to provide information for the generator. This study examined several properties of the thunderstorm rainfall, which include: storm occurrence, spatial patterns of storm centers, distribution of maximum rainfall depth within a storm cell, storm shapes and orientations, and relationships between storm coverage and maximum rainfall depth. The following sections describe the approach applied to identify these characteristics.

Storm occurrence

Monthly and bi-weekly transition probabilities (P(D/D) and P(W/W)) and probability of wet (P(W)) at each gage location were calculated. Table 1 presents the average probabilities of 93 gages. The average monthly P(D/D) and P(W) of July and August have no significant differences. On the other hand, the average bi-weekly P(W/W) and P(W) have significant higher values comparing to other periods. The bi-weekly probabilities suggested that the second half of July (July 16-31) is the wettest period in summer. Hence, transition probability was computed on a bi-weekly basis.

Since a bi-weekly simulation period is appropriate, the bi-weekly transition probabilities and probability of wet of the entire watershed were calculated. Table 2 presents the bi-weekly transition probabilities and probability of wet for the entire watershed. The results indicate the storm occurrence has a higher frequency during the last two weeks of July and first two weeks of August than other wet periods, which is consistent with previous studies (Rodriguez et al. 1987).

Table 1. Average monthly and bi-weekly transition probabilities and probability of wet (in percentage) from 93 gages.

Period	Jul	Aug	Sep	Jul 1-15	Jul 16-31
P(D/D)	76.10	76.78	86.63	79.86	68.67
P(W/W)	47.29	40.14	36.48	45.27	51.86
P(W)	23.90	23.22	13.37	25.05	39.06

Period	Aug 1-15	Aug 16-31	Sep 1-15	Sep 15-30
P(D/D)	75.59	80	83	93
P(W/W)	42.98	36	35	35
P(W)	31.28	24	22	11

Table 2. Bi-weekly transition probabilities and probability of wet (in percentage) for the entire watershed.

Period	P(W)	P(W/W)	P(D/D)
Jul 1-15	56	69	61
Jul 16-31	79	82	35
Aug 1-15	67	76	51
Aug 16-31	55	65	61
Sept 1-15	51	67	66
Sept 16-29	25	56	86

Spatial patterns of storm centers on WGEW

In order to identify the spatial patterns of storm centers on WGEW, the locations of storm centers from each selected events were derived and aggregated to a map, which is shown in Figure 3. The nearest-neighbor analysis (NNA) (Davis 1986) was performed to identify the spatial patterns of points on a map.

The NNA compares characteristics of the observed set of distances between pairs of closest points with those that would be expected if the points were randomly placed. The characteristics of a theoretical random pattern can be derived from the Poisson distribution. If the edge effect of the map is ignored, the expected mean distance between nearest neighbors is

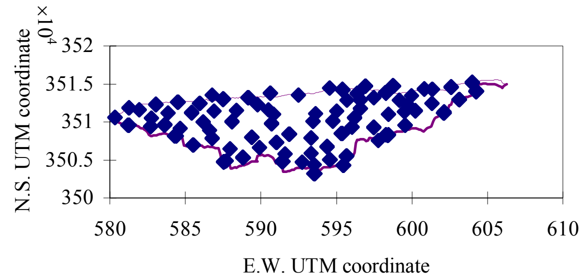

Figure 3. Storm centers derived from single events (374 events from 1970-90).

$$\bar{\delta} = \frac{1}{2}\sqrt{A/n} \qquad (1)$$

where A is the area of the map and n is the number of points.

The expected and observed mean nearest-neighbor distances can be used to construct an index R to identify the spatial patterns of points on a map (Davis, 1986).

$$R = \bar{d} / \bar{\delta} \qquad (2)$$

where $\bar{d}$ is the observed mean distance between nearest neighbors and R is the near-neighbor statistic, range from 0 for a distribution where all points coincide and are separated by distances of zero to 1.0 for a random distribution of points to a maximum value of 2.15 for a uniform distribution. Table 3 presents the results of the nearest-neighbor test. The results show all the R indices fall in the second category that is between 1 to 2.15 and represents a spatial pattern of random distribution, which is a spatial Poisson distribution.

Table 3. Nearest-neighbor analysis for WGEW storm centers.

	Area(sq. km)	n	delta	dbar	R
gages	153	93	640.38	1101.93	1.72
1970-75	162	107	614.47	826.70	1.35
1977-78	154	43	947.48	1285.22	1.36
1980-84	153	120	563.75	706.24	1.25
1985-90	152	104	604.55	881.60	1.46

Distribution of maximum rainfall depth within a storm cell

The rainfall depths are partitioned into several categories according to the amount of the depth. The count of each sub-category is calculated accordingly. The histogram of the depth in each sub-category verses the relative count is made to visualize the possible distributions and the Kolmogorov-Smirnov (K-S) test (Wilks 1995) is performed to test for the possible distributions. The maximum rainfall depth of each event was estimated as the maximum rainfall depth of each event from the observation. Table 4 presents the K-S test for maximum rainfall depth within a storm cell. The tests accepted both lognormal and Gamma distributions. Since the lognormal distribution requires less parameter estimations, hence was used for the generations of the maximum rainfall depth within a storm cell.

Table 4. K-S test for distribution of maximum rainfall depth within a storm cell.

Distribution	Jul 1-15	Jul 16-31	Aug 1-15
Gamma	>0.15	>0.15	>0.15
Lognormal	>0.15	>0.15	>0.15
Exponential	>=0.025&<=0.01	>=0.025&<=0.01	<0.01

Distribution	Aug 16-31	Sept 1-15	Sept 16-30
Gamma	>0.15	>0.15	>0.15
Lognormal	>0.15	>0.15	>0.15
Exponential	<0.01	<0.01	>=0.025&<=0.01

Storm orientations and shapes

The lengths of the major (a) and minor (b) axes and orientations were measured directly from the derived rainfall surfaces. Forty-eight events from the derived rainfall surfaces that had storm centers inside the watershed boundary were selected for the analysis. The ratio r = a/b. Table 5 presents the summary statistics of storm orientation and ratio of the lengths of the major to the minor axes. The mean of the ratio of the major to the minor axes is 1.54, which indicates the shape of storm on WGEW is elliptical rather than circular (consistent with Fogel and Duckstein 1969). As for orientations, the K-S test was performed to test for the possible distributions (Table 6).

Table 5. Summary statistics of storm orientation and ratio of the lengths of the major to the minor axes.

	ratio	orientation
mean	1.54	91.40
Std	0.37	38.27
skewness	0.96	0.06
min.	1.08	0
max	2.50	170
count	39	48

Table 6. K-S test results for the distribution of the storm orientations on WGEW.

Distribution of null hypothesis	P-value
Gamma distribution	>0.15
Lognormal distribution	>0.15
Normal distribution	>0.15

Relation of the maximum rainfall depth within a storm cell and storm coverage

A scatter plot of the storm coverage verses corresponding maximum storm depth using logarithmic scale (Figure 4). The plot shows a linear trend between the depth and coverage after a logarithmic transformation. A regression analysis was performed to obtain the linear relationship between the depth and storm coverage. The R^2 is 0.46 and the slope is 0.93. From the regression, the storm coverage was expressed as $A = 3.18 d_{max}^{0.93} + \varepsilon$, where A is the storm coverage in km^2, d_{max} is the maximum rainfall depth within a storm cell in mm and ε is the error term. The error term is obtained from the error between the prediction from the regression and observation.

Stochastic daily thunderstorm generation

The following section describes the models and algorithms for the various components of the rainfall generator based on the derived statistical characteristics.

Precipitation occurrence

The method uses a two-state Markov chain to generate the number and distribution of precipitation events. The six bi-weekly transition probabilities were calculated and used to provide a transition from one period to another. Random sampling of the bi-weekly distribution is then used to determine the occurrence of a wet or dry day probabilities.

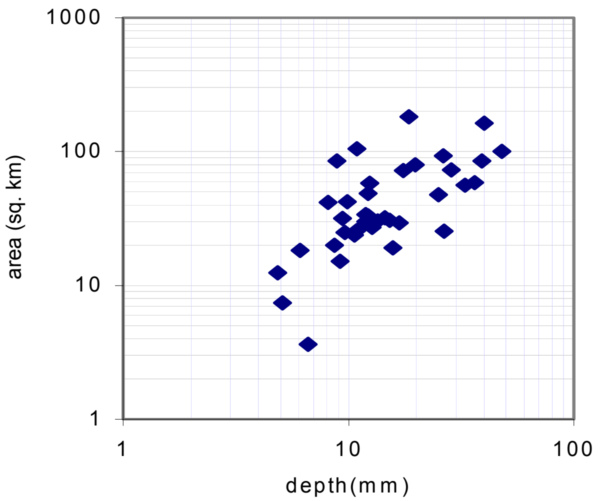

Figure 2. Maximum rainfall depth within a storm cell vs. storm coverage in logarithmic scale.

Precipitation depth

A lognormal distribution is used to represent the maximum precipitation depth within a storm cell. The form of this equation is

$$x = \frac{\log X - \mu}{s} \tag{3}$$

where x is the standard normal deviate, X is the raw deviate, and μ and s are the mean, standard deviation of the raw deviate after a logarithmic transformation, respectively. The mean and standard deviation of the logarithmic daily depths were calculated for each two weeks. Then, to generate a daily depth for each wet day occurrence, a random normal deviate is drawn and the raw variate, X (daily depth), is calculated using Eq. 3.

Storm coverage

The storm coverage is calculated using the corresponding precipitation depth and a random error term. The form of this equation is

$$A = 3.18X^{0.93} \pm \varepsilon \tag{4}$$

where A is the storm coverage in km^2, X is the generated precipitation depth in the storm center (mm) and ε is the random error introduced from the predicted errors of regression of storm coverage and precipitation depth. A uniform variate between 0 and 1 is generated to introduce the random error. The random error is added to the deterministic equation if the random variate is greater than 0.5, otherwise the random error is subtracted from the equation. Then, the storm coverage is calculated using Eq. 4.

Storm orientation

A normal distribution is used to represent the orientation of the major axis of the elliptical shape of the cell. The form of this equation is

$$x_o = \frac{X_o - \mu_o}{s_o} \tag{5}$$

where x_o is the standard normal variate, X_o is the raw variate, and μ_o and s_o are the mean, standard deviation of the raw deviate, respectively. The mean and standard deviation of orientations were calculated in previous chapter. To generate an orientation for each wet day occurrence, a random deviate x_o is drawn and the raw variate, X_o (orientation), is calculated using Eq. 5.

Storm cell axes

The lengths of a generated elliptical storm cell axes (a and b) can be calculated from the storm coverage A. Assuming the ratio of the major to minor axes is r, the form of this equation is

$$A = \pi r b^2 \tag{6}$$

where A is the storm coverage of a wet day event in m^2 and b is the length of the minor axes in m, respectively. A uniform distribution is used to represent the ratio. The form of the equation is

$$r = \frac{R - \mu_r}{s_r} \tag{7}$$

where r is the standard uniform deviate, R is the raw variate, μ_r and s_r are the mean, and standard deviation of the raw variate, respectively. The mean and standard deviation of the ratio were calculated in the previous chapter. Then, to generate ratio for each storm, a random uniform deviate is drawn and the raw variate, R (ratio), is calculated using Eq. 7. Since a = Rb, a and b can be calculated using Eq. 6.

Storm center

A uniform distribution is used to represent the wet day event storm center locations. The form of the equation is

$$z = \frac{Z}{33750} \quad (8)$$

where z is the standard uniform deviate between 0 and 1, Z is the location of storm center. A rectangle simulation space covering the study area (WGEW) which consisted of 33,750 square cells with a resolution of 100m is used for simulations. An index ranges from 1~33,750 is assigned to each cell. To generate the location for each wet day occurrence, a random uniform deviate is drawn and the raw variate, Z (storm center location) is calculated using Eq. 8.

Precipitation depths within storm coverage

The precipitation depth at any location within a storm cell is calculated using a spread function. The spread function is a relation of relative precipitation depth at an arbitrary location with respect to the depth at the storm center and is usually related to the absolute distance between those two points. Two types of spread functions are evaluated in the model: linear and exponential (Fogel and Duckstein 1969) spread functions. The forms of the equations are

$$P_z = \frac{d-x}{d} P_{max} \quad \text{(linear)} \quad (9)$$

$$P_z = P_{max} \exp(-0.27\pi \left(\frac{d}{1600}\right)^2 \exp(-0.0264 P_{max})) \quad \text{(exponential)} \quad (10)$$

where P_z is the precipitation depth (mm) at z, x is the distance between the storm center and z, d is the distance (m) between the boundary of the storm coverage and the storm center along the direction of z and storm center and P_{max} is the precipitation depth (mm) at storm center. Parameters used in equation 4.1.8 are adapted from Fogel and Duckstein (1969). To calculate the precipitation depth at any location within a storm cell, the distance between x and z is obtained, and the precipitation depth P_z is calculated using Eqs. 9 and 10, respectively.

A Fortran code was developed to facilitate the thunderstorm generation. Due to page limitation, the results are summarized in the following sections. For more details, please contact the authors.

Summary and Conclusion

Summary

This research study examined the spatial characteristics of the daily summer thunderstorm rainfall in the southeast Arizona. The following statistical characteristics of daily thunderstorm rainfalls have been identified from an analysis of the WGEW data: the storm center locations on WGEW have a Poisson distribution, the maximum depth within a storm cell has a lognormal distribution, the shape of a storm cell is elliptical with an average major axis length to the minor axis length ratio of 1.55 and the orientation of a storm cell is primarily NW or NE. The storm coverage and the maximum rainfall depth within a storm cell have a linear relationship after a logarithmic transformation. Storm occurrences have higher frequencies during the last two weeks of July and the first two weeks of August than other wet periods and there was no significant trend of transition probability with elevation. The stochastic daily thunderstorm generator is able to produce the statistical daily thunderstorm rainfall characteristics on WGEW.

Conclusion

The daily thunderstorm rainfall generator provides a distributed thunderstorm generator for southeast Arizona. The research of the temporal variation during storms can be further studied.

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