

CATCHMENT GEOMETRIC REPRESENTATION AND IDENTIFICATION OF SEDIMENT YIELD PARAMETERS IN A DISTRIBUTED CATCHMENT MODEL

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Abstract

In recent years, distributed (spatially explicit) models have become more widely used, in part because of the availability of geographic information systems. However, advances in the understanding of hydrologic and geomorphic processes in concert with more readily available spatial data sets make these models more useful. In particular, distributed catchment models have the potential to predict where and when runoff is generated and sediment entrainment or deposition is occurring. To ensure that the models have the potential to produce physically realistic sediment yield predictions, it is necessary to identify unique sediment contributions from raindrop impact and entrainment by flowing water; which are highly scale-dependent processes. In this study, we used four different representations of a 4.4 ha experimental catchment with the most complex representation preserving the channel network detail as observed in the field. Our objective was to determine whether it is possible to identify the relative contributions of sediment entrainment by raindrop impact and by flowing water. We used a distributed catchment model (KINEROS2) and a multiplier parameter identification approach to preserve the spatial variability of the runoff-erosion process. We found that as geometric complexity decreased, the ability to model sediment yield decreased, while runoff could still be modeled effectively. Furthermore, as geometric complexity decreased more entrainment by raindrop impact was required, while less entrainment by flowing water was required to simulate sediment yield.

INTRODUCTION

In recent years, distributed catchment models based on hydrodynamic principals have successfully been used to model runoff and sediment yield on small plots (e.g. Lopes and Lane, 1990), and runoff on small catchments (Goodrich, 1990; Lopes and Lane 1990). However, successful applications of runoff-erosion models on small catchments are few (Wicks and Bathurst, 1996; Jetten et al., 1999; Smith et al. 1999). The hydrodynamic approach to runoff-erosion modeling offers a number of benefits over more empirical methods. One of the major benefits is that, these models can be used to describe response to a single event, because they describe the physics of water and sediment movement on a catchment. In contrast, the most widely used empirical method for estimating soil loss [the Universal Soil Loss Equation - USLE (Wischmeier and Smith, 1978)] can be used to estimate annual soil loss, but not to describe erosional processes on an event basis. In addition, the USLE does not describe detachment by flowing water, or consider the subtractive effect of infiltration on overland flow. Another benefit of physically-based distributed runoff-erosion models is the potential to describe where

and when erosion and deposition occur (Nearing et al., 1994). Furthermore, physically-based distributed runoff-erosion models have the potential to describe the movement of sediment-borne contaminants, and the effect of management practices (Jensen and Mantoglou, 1992) and climate change (Hawkins et al., 1991) on erosion and sediment yield.

Typically, the hydrodynamic approach to runoff-erosion modeling recognizes two distinct sediment entrainment processes on a hillslope: sediment entrainment by raindrop impact (sometimes called rainsplash erosion), and entrainment by flowing water. Entrainment by flowing water also occurs in rills and channels. One problem that arises with this approach is that it may be impossible to identify the relative contribution of these two processes to the sediment measured at the outlet of a small catchment.

Parameter identification is an automated process by which model parameters are identified (or calibrated). It comprises the following three major components: (1) an objective function that determines how well model estimates fit the observed data, (2) a search algorithm that selects possible parameter values to compare, and minimize or maximize an objective function, and (3) a means to determine if the selected parameter values are physically realistic. Researchers have found that for sediment entrainment some search algorithms and objective functions work better than others (Freedman et al. 1998), and that some types of sediment entrainment equations have more identifiable parameters than others (Freedman et al. 2001).

A difference between this study and previous studies is that this study uses observed data from an experimental catchment rather than data from rainfall simulators on small plots. This study further differs from previous studies in that it begins with a spatial representation of the catchment that includes all the channel network complexity as observed in the field up to and including the rills on hillslopes. We chose to use this representation in an attempt to minimize the potential effect of process-scale interaction by using a field-identifiable measure of process scale (i.e. the location on the catchment where flow entrainment processes are dominant enough to incise the hillslope producing a rill).

Entrainment by flowing water and entrainment by raindrop impact are scale-dependent processes, and as such we hypothesized that simplifying catchment geometry may affect the relative fluxes of these two processes needed to predict sediment yield on a small catchment. We started with a catchment representation that included all the rills observed in the field, and then simplified the representation.

The objective of this study was to (1) determine the effect of simplification of channel network complexity on parameter identifiability, and (2) determine the ability of a distributed event based catchment model to predict runoff and sediment yield under simplifying catchment geometric representations.

Study Area

The study was conducted on a 4.4 ha experimental catchment (Lucky Hills 104) located within the Walnut Gulch Experimental Watershed in southeastern Arizona and operated by the United States Department of Agriculture – Agricultural Research Service (USDA-ARS) Southwest Watershed Research Center in Tucson, Arizona. The 149 km² Walnut Gulch Experimental Watershed is representative of approximately 60 million hectares of brush and grassland found throughout the southwestern United States.

Vegetation on the catchment is creosote bush and acacia, which are typical increasing species on rangelands in the southwestern United States. Hydrology and scale issues related to runoff have been studied extensively on this catchment (Goodrich et al., 1995; Faures et al., 1995; Goodrich 1990). Yitayew et al. (1999) studied erosional processes on Lucky Hills 104 using RUSLE (Renard et al. 1997). Some results of sediment yield modeling on Lucky Hills 104 are described elsewhere (Canfield, 1998). Sediment yield modeling has also been studied on the paired catchment Lucky Hills 103 (Lopes, 1987).

METHODS

The KINEROS2 Model

KINEROS2 (Smith et al. 1995; Smith and Quinton 2000), is a distributed runoff-erosion model based on Hortonian overland flow theory, and therefore, well-suited for describing the hydrodynamics of runoff and erosional processes on semiarid catchments, where infiltration rates are low, and rainfall is infrequent but intense. Studies have shown that KINEROS2 has performed well in estimating sediment yield in comparison to other models (Smith et al. 1999). The model allows for spatially variable rainfall input, channel transmission losses, and spatial variability of catchment characteristics (soils, slopes, vegetation, etc.). Catchment geometry is represented in KINEROS2 as a combination of overland flow plane and channel elements with plane elements contributing lateral flow to the channels or to the head of first order channels (Figure 1a). Each plane may be described by its unique parameters, initial conditions, and precipitation inputs. Runoff is treated in KINEROS2 with a one-dimensional kinematic-wave approximation to the full dynamic continuity equation applicable to both overland flow and channel flow:

$$\delta A/\delta t + \delta(Q)/\delta x = q_L(x,t) \quad (1)$$

where q_L = local rate of lateral inflow [L^2T^{-1}], A = local cross-sectional area of flow [L^2], Q = local discharge [L^3T^{-1}], t = time from start of runoff [T], and x = distance in the direction of flow [L]. Equation (1) is combined with a normal flow relation for local velocity $v(h)$. Solution of equation (1) is then obtained by a four-point finite difference method. Channel segments may receive uniformly distributed but time-varying lateral inflow from adjacent contributing overland-flow planes on either or both sides of the channel, or from one or two channels at the upstream boundary, or from a plane at the upstream boundary. Infiltration is calculated interactively with runoff calculations, to simulate infiltration losses during recession flow, when rainfall has ceased, or to simulate runoff advancing down an ephemeral channel.

Sediment entrainment and transport on hillslopes and channels is treated as an unsteady, one-dimensional convective transport phenomenon, using a continuity equation similar to that for runoff (Eq. 2):

$$\delta(cA)/\delta t + \delta(cQ)/\delta x = \phi_s(x,t) \quad (2)$$

where $\phi_s = \Sigma\phi_{si}$ = sediment flux [$ML^{-1}T^{-1}$], and c = sediment concentration [ML^{-3}]. A flow reach is conceptualized as a string of computational elements of length Δx , linked sequentially to one another via the mechanism of flow and sediment transport. Sediment concentration in KINEROS2 is calculated using the Engelund and Hansen (1967) transport capacity relationship.

Figure 1b illustrates the sediment entrainment and deposition processes as simulated in KINEROS2.

Sediment flux on a hillslope is composed of two independent sources, raindrop-induced entrainment q_r [$ML^{-1}T^{-1}$], and flow-induced entrainment q_f [$ML^{-1}T^{-1}$] (Eq. 3).

$$\phi_s = q_r + q_f \quad (3)$$

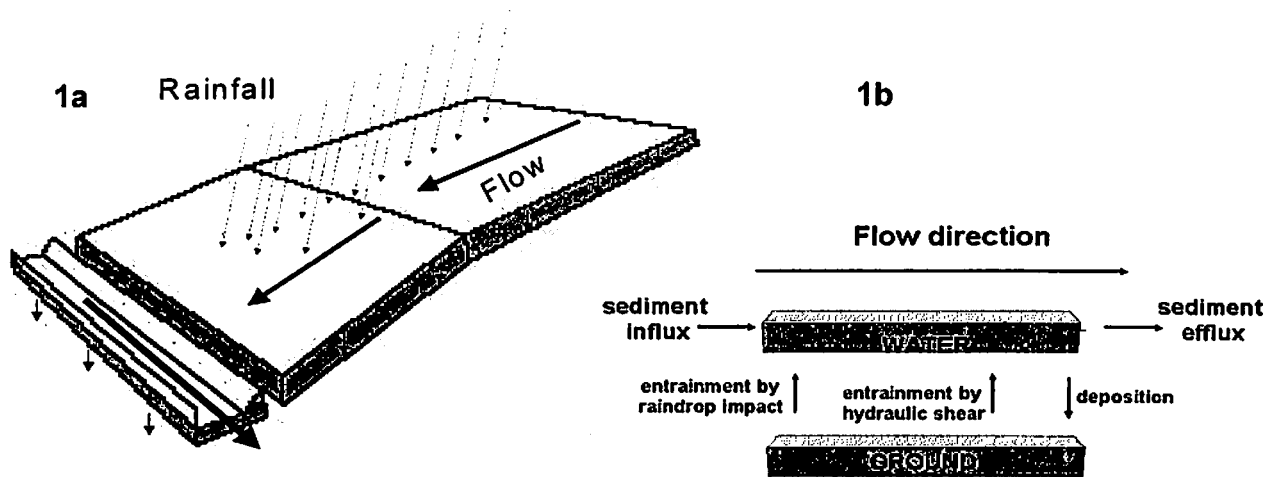


Figure 1– 1a.) Plane and Channel Representation: Hillslopes in KINEROS2 are represented as a series of cascading planes. Channels are represented as trapezoidal troughs. 1b.) Representation of the sediment entrainment by raindrop impact and flowing water simulated in KINEROS2

Sediment entrainment by raindrop impact is described by the following relationship (Eq. 4):

$$q_r = K_i i^2 e^{-mh} \quad (4)$$

where, K_i = parameter describing the susceptibility of the soil particles to be detached and entrained by raindrop impact, i = rainfall rate [LT^{-1}] m = parameter describing the attenuation effect of flow depth on raindrop impact. Flow-induced sediment entrainment for a particle size (i) is treated as the net difference between simultaneous entrainment q_{ei} [$ML^{-1}T^{-1}$], and deposition q_{di} [$ML^{-1}T^{-1}$] (Eq. 5):

$$q_{fi} = q_{ei} - q_{di} \quad (5)$$

Deposition rates for a particle size class (i) is assumed to be directly estimable from particle settling velocity v_{si} [LT^{-1}] and sediment concentration (Eq. 6):

$$q_{di} = w v_{si} C_i \quad (6)$$

in which w = flow width [L]. Sediment contributions to a channel element from surrounding hillslopes are treated as either an upper boundary condition or distributed lateral inflow.

Catchment Representation and Initial Parameter Estimates

The general approach used to obtain initial spatial estimates of parameter values for KINEROS2 was to gather data on the landscape form and materials and relate these to hydrologic and erosional processes. Specifically, landscape form was characterized using topographic surveys to produce a 2.5m x 2.5m DEM, and the materials on the landscape were characterized using soil particle size analysis. Initial estimates of sediment entrainment parameters were based on the spatial variability of particle size data. The raindrop-impact entrainment parameter was estimated from K_s values using the methods described by Ben-Hur and Agassi (1997) who provide several different equations based on the kinetic energy of raindrops for the original WEPP model (Lane et al. 1987). The details of this parameterization are described elsewhere (Goodrich, 1990; Canfield, 1998).

The result of these initial parameter estimation techniques is to produce parameter values on a 2.5m x 2.5m grid cell scale. These grid cell estimates can then be averaged for parameter estimates for hillslope and channel model elements. We used the TOPAZ DEM processing program (Garbrecht and Campbell, 1997) to produce four spatial representations of Lucky Hills 104. The most complex representation was based on field-identified channel heads and included all channels identifiable in the field. The upslope contributing source area varied from about 90 square meters to 350 square meters and had an average upslope contributing source area of about 200 square meters (Figure 2a). The least complex representation had a contributing source area (CSA) of approximately 5000 square meters (Figure 2d). Two intermediate levels of complexity with contributing source areas of 1200 square meters (Figure 2b) and 500 square meters (Figure 2c) were also used. These scales were selected because KINEROS and KINEROS2 have been shown to effectively reproduce hydrographs using this range of complexity on the Lucky Hills 104 catchment, though different runoff parameters are required for each complexity (Goodrich, 1990; Canfield, 1998). Six events with good sedigraphs and hydrographs are available for parameter identification from the 1980s. These events are summarized in Table 1.

Date	Rainfall (mm)	Runoff Volume (mm)	Peak Discharge (mm/hr)	Hydrograph Model Efficiency 200 m ² CSA	Hydrograph Model Efficiency 5000 m ² CSA	Peak Sediment Discharge (Kg/s)
30-Jul-85	24.4	3.5	18.7	0.98	0.98	1.72
6-Aug-88	25.3	5.5	29.4	0.99	0.99	6.14
25-Aug-84	12.4	1.8	12.0	0.95	0.97	2.13
10-Sep-83	26.9	3.9	19.7	0.97	0.98	1.55
11-Sep-82	24.0	7.9	35.9	0.98	0.97	3.32
20-Sep-83	18.1	2.2	16.3	0.93	0.93	0.45

Table 1 - Event Characteristics and Hydrograph Model Efficiencies for Six Events Used for Parameter Estimation. Note that there is virtually no degradation in the hydrograph fit in using the less complex catchment representation (5000 m² CSA).

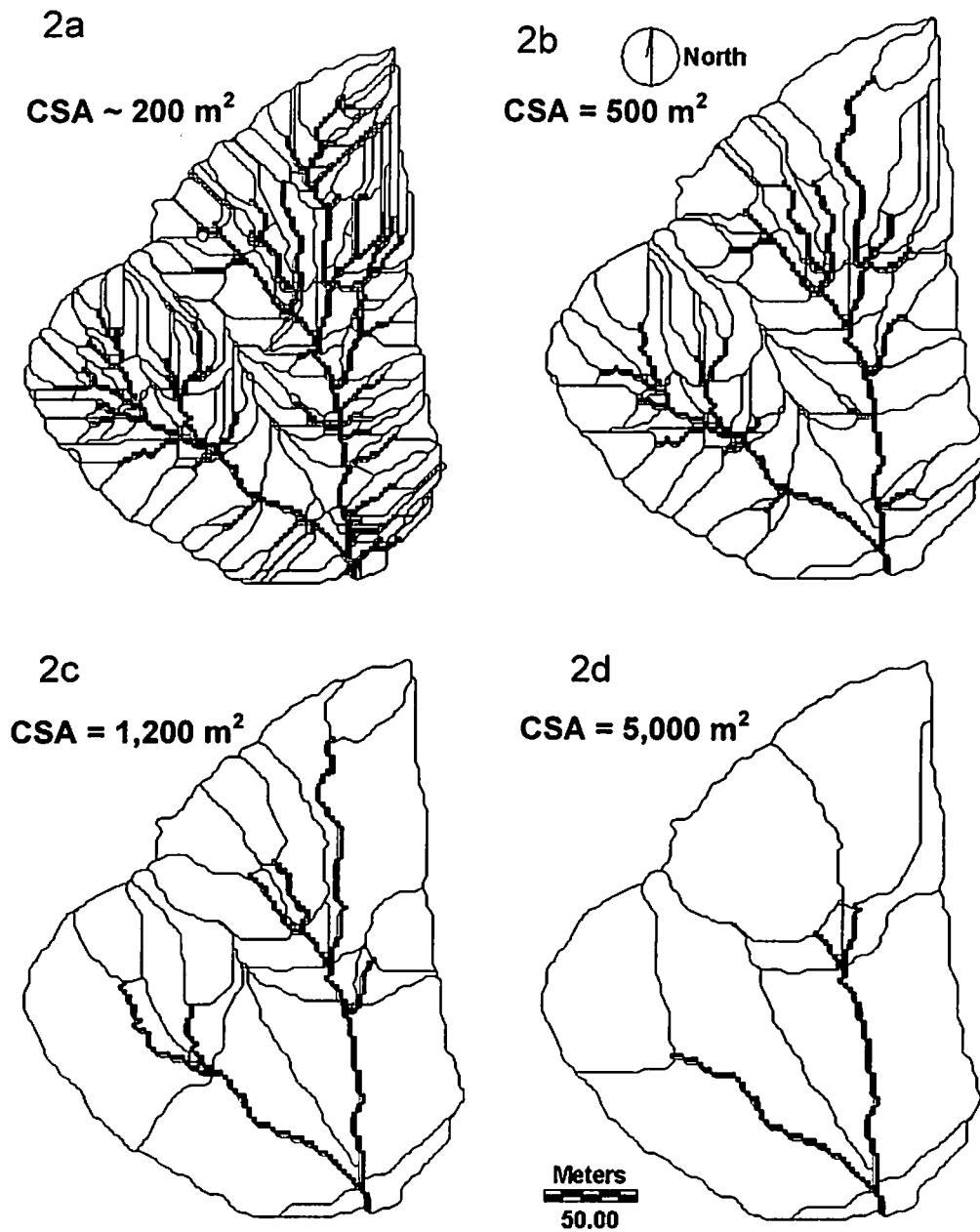


Figure 2 – Channel network Representations: The most complex case (2a), has a mean contributing source area (CSA) of 200 square meters, reflecting the channel network complexity observed in the field. The least complex channel network complexity (2d) is the least complex representation at which simulated hydrographs fit observed hydrographs well, and has a contributing source area of 5000 square meters. Figures 2b and 2c show intermediate complexities with contributing source areas of 500 and 1200 square meters, respectively.

The Parameter Identification Process

Rather than estimating distributed parameter values, the parameters were adjusted up or down by multiplying the spatially distributed initial parameter values by a multiplier (Figure 3). In this approach parameter identification was done in a two-step process. Parameters for hydrology were identified first, and once these were selected, parameters for sediment were identified. For hydrology, multipliers for Manning's n , K_s and the coefficient of variability of K_s (CV_{ks}) were used as fitting parameters.

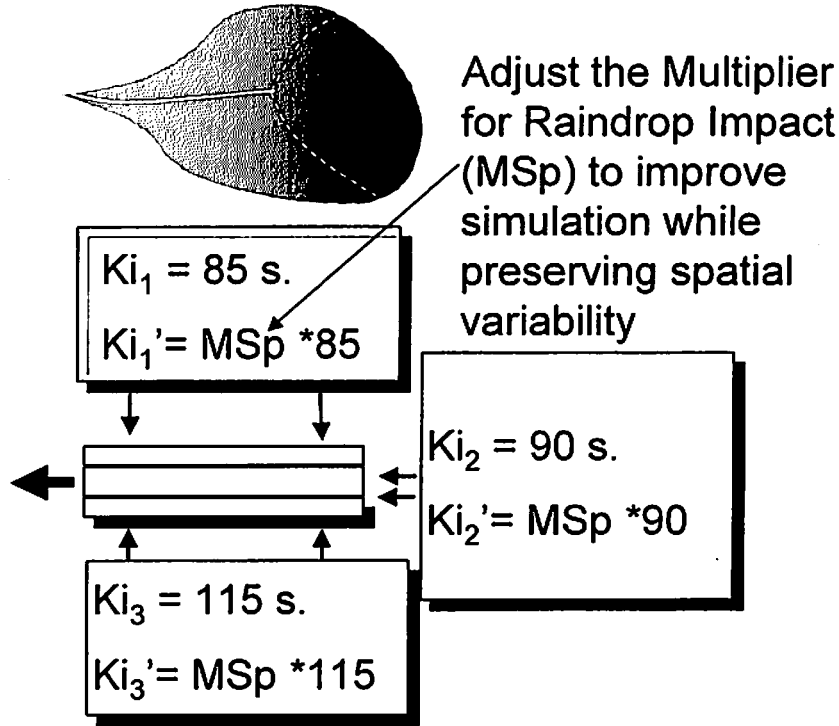


Figure 3 – The Multiplier Approach Used in KINEROS2. Parameters are calibrated by multiplying all elements by a single multiplier, therefore maintaining the spatial complexity observed in the field while constraining the free parameter dimensional space.

The multiplier for raindrop-induced sediment entrainment is (Eq.7):

$$q_r = MSp * K_{i1}^2 e^{-mh} \quad (7)$$

where, MSp = multiplier for raindrop-induced sediment entrainment. This description implies that a new value of K_i (e.g. K_i') is simply a linear multiple of the original K_i . The multiplier for sediment entrainment by flowing water is (Eq.8):

$$q_{fi} = \beta_i w (MTC * c_{mx(i)} - c_{s(i)}) \quad (8)$$

where, MTC = multiplier on sediment concentration determined by transport capacity, β_i = erosion rate coefficient for particle size class (i), w = width of flow, $c_{mx(i)}$ = sediment concentration at transport capacity for particle size class (i), and $c_{s(i)}$ = sediment concentration for

particle size class (i) entering the node.

While the SCEUA automatic parameter identification technique (Duan et al., 1992) was used, the primary means of identifying the parameter values was the two-dimensional error response surface generated by the search algorithm. Essentially, the SCEUA is a search algorithm, that is an extension of the simplex method (Nelder and Mead, 1965). The primary benefit to the SCEUA over the standard simplex method is that it is better able to find a global minimum, when there are multiple minima in the sample space. The SCEUA has been found to be a useful technique for complex parameter identification problems in distributed hydrologic modeling (Eckhardt and Arnold, 2001).

For both runoff and sediment, the observed value for each measured time was compared with the simulated value for that time. In this way, the full hydrograph or sedigraph was fit, rather than simply optimized for peak or runoff volume. Both the total sum of squared residuals (TSSR) and the Nash and Sutcliffe (1970) statistic were used as objective functions. The total sum of squared residuals objective function (TSSR) required fewer shuffling loops to find the optimum parameter set than the Nash and Sutcliffe (1970) objective function, and was, therefore, selected for this study.

RESULTS

Canfield and Lopes (2001) have shown that only larger events sufficiently activate both the flow-induced sediment entrainment and raindrop impact induced entrainment in order to identify the fluxes from these two sources. For the sedigraphs, one event displayed identifiable characteristics (August 6, 1988), which was the event that produced the most sediment of all the six events used in this study. With the total sum of squared residuals, the larger events will tend to dominate the determination of error more than the smaller events. The error response surface for all six events for the most complex catchment representation is dominated by the August 6, 1988 event. The multiplier for raindrop impact (M_{Sp}) is 2.1 and the multiplier for concentration determined by transport capacity (MTC) is 2.75. A twofold increase in raindrop impact is reasonable considering that raindrop impact entrainment coefficients can vary by an order of magnitude using the infiltration-based estimates used to parameterize the raindrop impact component of the model (e.g. Ben-Hur and Agassi, 1997).

In this case, the error response surface tends to be “flat,” but highly pitted in the region of the minima. The high degree of pitting was not observed when synthetic data were used, suggesting that the pitting can be a function of input, output and structural errors in the model. As such the location of the minimum may be different for another data set from the same catchment, and interpreting the best fit as “optimal” under-represents the degree of uncertainty in the estimate. As the catchment complexity is decreased, the error response surface shows a decrease in the flow-induced entrainment multiplier, and an increase in the raindrop-impact induced entrainment multiplier (Figure 4). Furthermore, as complexity is decreased, the TSSR at the minimum increases, indicating that simplification of the catchment geometry significantly reduces the ability of the simulation to describe the observed sediment-yield process (Table 2). The corresponding Nash-Sutcliffe statistic for each catchment complexity is shown in Table 3.

Table 3 - The Sedigraph Model Efficiency for Each Event for Each Model Complexity. The simulations from the most complex geometry have much improved model efficiencies than the less complex geometries.

CSA	200 m ²	500m ²	1200 m ²	5000 m ²
Date	Efficiency at Minima	Efficiency at Minima	Efficiency at Minima	Efficiency at Minima
30-Jul-85	0.71	0.88	0.77	0.79
6-Aug-88	0.96	0.91	0.89	0.68
25-Aug-84	0.56	0.38	0.59	0.60
10-Sep-83	0.86	0.66	-2.40	-1.31
11-Sep-82	0.51	0.45	0.33	0.41
20-Sep-83	0.81	0.60	-1.37	-7.08

Figure 4 -- Error Response Surface for Four Catchment Geometric Representations: The lines represent a contour of Total Sum of Squared Residual (TSSR) 20 (kg/s)² above the optimal fit given by the dot. The most complex representation is blue (200m² CSA), while the least complex is green (5000 m² CSA) and the intermediate representations are black (500 m² CSA) and brown (1200 m² CSA). (The contours have been smoothed to limit the effect of pitting.)

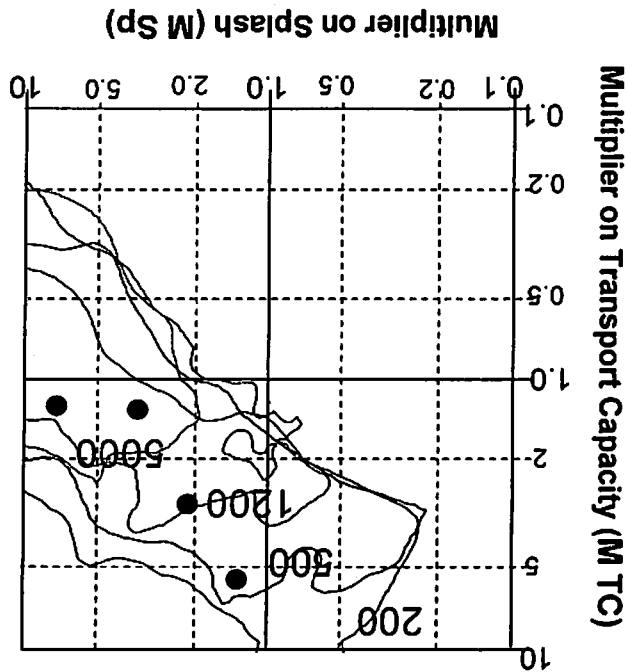


Table 2 - The Multipliers and Total Sum of Squared Residuals at the Minimum: Note that error increases as catchment geometric complexity decreases.

CSA	200 m ²	500m ²	1200 m ²	5000 m ²
Splash Multiplier (M Sp)	2.1	1.41	3.10	5.62
Transport Capacity Multiplier (M TC)	2.75	5.12	1.35	1.29
Total Sum of Squared Residuals at Minima	5.01	6.61	13.89	18.51

DISCUSSION AND CONCLUSIONS

While catchment geometric representations of Lucky Hills 104 with CSAs varying between 200 m² and 5000 m² can be used to model runoff, sediment yield estimates become poorer as more simplified representations of the catchment are used. Between CSAs of 500 m² and 1200 m² the ability of the model to produce simulations like the observed degrades as indicated by an increase in the total sum of squared residuals (TSSR). As channel network complexity decreases, the multiplier on entrainment by raindrop impact needs to be increased at the expense of the multiplier on transport capacity in order to produce the same amount of sediment. In part, the systematic decrease in the transport capacity multiplier is due to the fact that simplification removes concentrated flow areas from the catchment representation. The total area of the channels in the most distributed case is about 780 square meters, while in the least distributed case the total area of channels is about 450 square meters; a decrease of 42 %. The entrainment by raindrop impact component must be increased on hillslopes because channels are being removed from the catchment representation.

While simplification to a contributing source area of 5000 m² does not adversely affect complex hydrograph simulations, simplification greatly reduces the ability of the model to simulate complex sedigraphs as illustrated for the September 20, 1983 event (Figure 5).

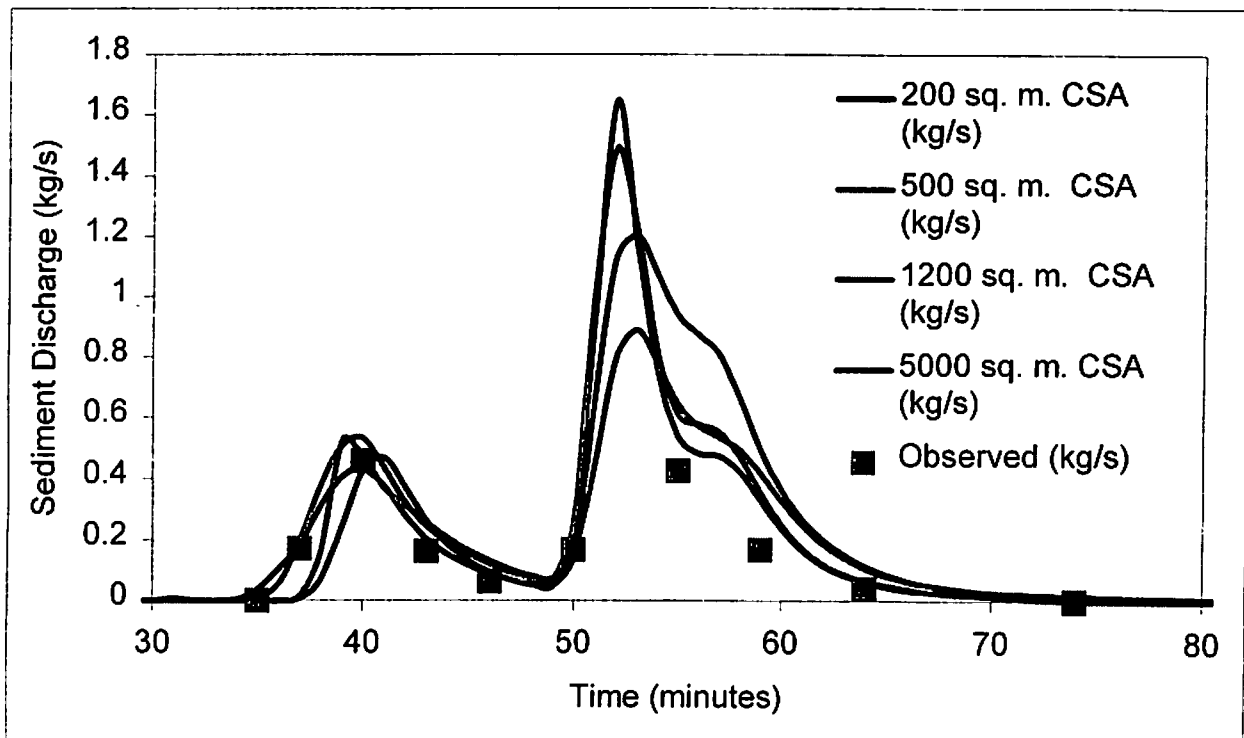


Figure 5 - Effect of catchment geometric representation on simulations of complex sedigraphs as illustrated for the September 20, 1983 event.

The study indicates that reducing catchment geometric complexity reduces the capability of the model to simulate observed sedigraphs. Furthermore, in order to make up for reduced sediment from channels that are modeled as hillslopes in less complex geometries, the contribution from

raindrop impact needs to be increased at the expense of the contribution from entrainment by flowing water. Additional experimental data are needed to further evaluate these conclusions.

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