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Modeling and Solving Water Resources Engineering Design Problems as Stochastic Programs to Account for an Uncertain Future

D.S. Yakowitz¹, W. Elshorbagy², and K. Lansey³

Abstract

The future operations of many engineering systems are affected by the uncertainty of many inputs and demands. To account for these elements a two-stage stochastic programming formulation is proposed and illustrated for several engineering problems concerned with water distribution and supply. In all of these applications, the first-stage decisions are the design elements that are to be determined at present. The second-stage decisions involve the future operation, or system response, of the first-stage design after the realization of the uncertain inputs and demands on the system. Previously, the complexity of the problems and lack of good solution procedures led to design strategies that often consider only the expected values of the uncertain inputs or demands. New algorithmic developments in stochastic programming, such as Regularized Stochastic Decomposition or RSD, have made it computationally feasible to consider the more realistic stochastic nature of future operations in the design process. Three water resources design problems are presented to illustrate the solution techniques and the advantages of using a stochastic design approach over more traditional approaches. The problems considered include: 1) the capacity of a reservoir on a river to store water for agricultural or downstream use is sought considering multiple other sources of water such as rainfall, groundwater, or water purchased from an outside source such as the Central Arizona Project (CAP); 2) a regional water supply that seeks the design capacities of recharge facilities and water, wastewater, and tertiary treatment plants while meeting future

¹Research Hydrologist and Systems Engineer, USDA-Agricultural Research Service, 2000 E. Allen Rd., Tucson, AZ 85719

²Research Associate, Department of Civil Engineering, University of Arizona, Tucson, AZ 85721

³Assistant Professor, Department of Civil Engineering, University of Arizona, Tucson, AZ 85721

demands; and 3) the design of a system of irrigation canals that seeks the canal capacities considering several water-deficit scenarios that affect the allocation of water.

Introduction

Design and analysis of engineering systems usually involve many uncertainties. Often some uncertainties can be neglected because they are not known, their values are low, or the uncertain parameters play an insignificant role in the process. However, when uncertainties significantly affect the design they should be taken into account to reduce risk.

In applications that can be modeled as mathematical programming problems, one way to account for uncertainties is to use a probabilistic representation instead of a deterministic one. Some versions of this type of model were introduced in the late 1950's, but most algorithmic development for practically solving stochastic programs has taken place only in the last two decades (see Wets 1982, and Ermoliev 1988). Many engineering applications can be formulated as two-stage stochastic programs with recourse. In this formulation the initial design decisions are made in the first stage and then given those decisions, future operations are considered in the second stage. In this formulation the uncertainties can be accounted for by assigning a probability distribution to the uncertain or random elements of the formulation and generating a realization of these in each iteration. The second-stage decisions may then prompt one to adjust the first-stage decision to improve the overall objective. Although this formulation is well known in the area of operation research, it has not been broadly applied to water resources problems. This is mostly due to the fact that until relatively recent times most solution algorithms were able to handle only small problems, or problems with few possible realizations of the random variables. (Note, a formulation with n constraints whose right-hand sides are independent random variables with m possible realizations for each constraint yields an equivalent deterministic problem with m^n constraints.)

Recent algorithms developed to solve two-stage stochastic linear programs, with recourse involving discrete or continuous random variables, have made it possible to consider this formulation for practical application to real engineering problems. The Regularized Stochastic Decomposition (RSD) algorithm (Yakovitz 1994a) has been applied to several engineering applications and extended to problems in regional water supply planning and irrigation design by Elshorbagy et al. (1995a,b,c). In this paper we illustrate, through the use of several examples, how problems in water resources can be posed as two-stage stochastic programming problems and we highlight some advantages of solving these problems as such. We begin with an outline of the algorithmic details for solving two-stage stochastic programming problems by decomposition style methods such as the RSD algorithm.

Algorithmic Outline for Solving Two-Stage Stochastic Programs by Decomposition

The algorithm involves the solution of a second-stage Subprogram, and a Master Program that consists of the first-stage objective and constraints, with cutting planes generated from the second-stage problem in each iteration. A candidate solution is compared to the current solution and accepted or rejected until the algorithm terminates by satisfying appropriate stopping rules.

Step 0. Initialize with a feasible current first-stage decision (can be obtained by considering only the expected values of the random variables, for example) and set the candidate first-stage decision equal to the current first-stage decision.

Step 1. Randomly generate a single observation of all random variables according to their distributions.

Step 2. Solve the second-stage problem that results from Step 1 at the candidate first-stage decision, and save the solution.

Step 3. Estimate the cutting plane at the current first-stage decision to be added to the Master Program using the current and past solutions to the second-stage problem.

Step 4. Determine if the objective estimate at the candidate solution is significantly lower than the estimate of the objective function at the current first-stage decision. If so, the candidate becomes the current solution.

Step 5. Update, re-evaluate, or eliminate past cutting planes and solve the current Master Program.

Step 6. Determine if stopping criteria are met. If so, stop. If not, return to Step 1.

Examples of Water Resources Problems Formulated as Two-Stage Stochastic Programs

Three examples of engineering problems will be presented, each illustrating different aspects or advantages of formulating the problems as stochastic programs. Since each example is available in other publications, many details will be suppressed in order to simplify the presentation and to highlight key points or features of each problem. We begin with an example from Yakowitz (1994b) to illustrate the progress in each iteration of the RSD algorithm outlined above. In this example, the capacity of a reservoir on a river to store water for agricultural or downstream use is sought considering multiple other sources of water such as rainfall, groundwater, or water purchased from an outside source such as the Central Arizona Project (CAP). This example is followed by a regional water supply problem described in Elshorbagy, Yakowitz, and Lansley (1995a). This seeks the design capacities of recharge facilities

and water, wastewater, and tertiary treatment plants while meeting future demands. Results obtained from an implementation of this problem as a stochastic program indicate a savings over a more conventional deterministic formulation. The third example, which is detailed in Elshorbagy et al. (1995b), examines the design of a system of irrigation canals that seeks the canal capacities considering several water deficit scenarios that affect the allocation of water. This example clearly illustrates a benefit of considering multiple stochastic scenarios.

Example 1. Capacity of a Reservoir

Consider the following situation: a small dam is to be constructed across a river in Arizona providing the facility for the storage of water delivered by means of a canal for agricultural use, or for downstream use by direct releases of the water. Water for agriculture can also be purchased from an outside source, such as the Central Arizona Project (CAP), or pumped from existing groundwater wells (the amount pumped restricted by recharge estimates). The first-stage variables are the capacity of the reservoir and the capacity of the canal. The second-stage variables include the amount of water released from the reservoir for agriculture, the amount released downstream, the amount pumped from groundwater, and the amount of water obtained from the external source (i.e., CAP) for agricultural use. These are determined after the stochastic rainfall, inflow, and downstream demand are realized. The first-stage objective is to minimize the maintenance cost of the dam and canal system. The initial cost of building the system is assumed to be amortized over an extended period and is reflected in the maintenance costs. The second-stage objective is to minimize the cost of purchasing and delivering the water, minus the crop yield revenues, which are assumed to be directly proportional to the water used for irrigation.

In this example, first-stage constraints impose upper and lower bounds on the reservoir and canal capacities, and the initial storage level of the water in the reservoir at the end of stage 1 is assumed to be a fraction (random) of the reservoir capacity.

Second-stage constraints ensure that the capacity of the canal is adequate to handle the flow and guarantee that the storage at the end of the second stage does not exceed the reservoir capacity or drop below a fraction of the reservoir capacity. Another constraint requires that the reservoir capacity be at least a fraction of the total water released for agriculture and downstream use. This constraint is a surrogate for a more complicated constraint system that would insure that peak demand could be met. The water balance equation is also included as a constraint in the second stage, that is, the change in storage must equal the inflow minus the outflow from the reservoir. A constraint that requires that the water released downstream must satisfy a minimum stochastic demand is likewise imposed. A set of constraints also bounds the amount of water for agriculture, including stochastic precipitation, between a minimum and maximum crop requirement. The last inequality restricts the water pumped from groundwater for agriculture to amounts less than the recharge of the

aquifer, which is estimated in this example to be a fraction of the precipitation, and water from other sources applied to the fields. With these constraints the two-stage stochastic program is:

$$\begin{aligned} \text{Min } c_R R + c_C C + E[Q(R, C; \tilde{\Omega})] \\ \text{s.t.} \\ R_{\min} \leq R \leq R_{\max} \quad (\text{bounds on capacities}) \\ C_{\min} \leq C \leq C_{\max} \end{aligned}$$

Where the second stage is:

$$\begin{aligned} Q(R, C; \tilde{\Omega}) = \text{Min } c_e x_e + c_g x_g - r(x_a + x_e + x_g) \\ \text{s.t.} \\ C - x_a - x_e - x_g \geq 0 \quad (\text{adequate capacity}) \\ a_1 R \leq s_2 \leq R \quad (\text{storage restrictions}) \\ R - a_2(x_a + x_d) \geq 0 \quad (\text{reservoir restriction}) \\ s_2 + x_a + x_d = Y + \tilde{s}_1 \quad (\text{water balance}) \\ x_d \geq M \quad (\text{minimum downstream water}) \\ w_{\min} \tilde{P} \leq x_a + x_e + x_g \leq w_{\max} \tilde{P} \quad (\text{min and max crop water}) \\ x_g \leq a_3(x_e + x_a + \tilde{P}) \quad (\text{groundwater constraint}) \\ x_a, x_d, x_e, x_g, s_2 \geq 0. \end{aligned}$$

With:

R: capacity of the reservoir

C: capacity of the canal

c_R : cost per unit of capacity R

c_C : cost per unit of capacity C

x_a : amount of water released from the reservoir for agriculture

x_d : the amount of water released downstream

x_g : the amount of water pumped from groundwater for agriculture

x_e : the amount of water obtained from the external source for agricultural

c_e : the cost of purchase and delivery per unit of external water

c_g : the cost of pumping per unit of ground water

r: crop yield proportionality factor

a_1 : storage capacity fraction

a_2 : reservoir capacity fraction

a_3 : recharge fraction

R_{\max} : the maximum reservoir capacity

C_{\max} : the maximum canal capacity

- R_{\min} : the minimum reservoir capacity
- C_{\min} : the minimum canal capacity
- \tilde{s}_1 : the storage level of the water in the reservoir at the end of stage 1
- s_2 : the storage level of the water at the end of stage 2
- \tilde{Y} : the stochastic inflow to the reservoir
- \tilde{M} : the minimum downstream stochastic demand
- \tilde{P} : stochastic precipitation
- w_{\max} : maximum crop requirement
- w_{\min} : minimum crop requirement.
- Ω : is the set ($\tilde{Y}, \tilde{M}, \tilde{P}, \tilde{s}_1$)

While the convergence results for the RSD algorithm given in Yakowitz (1994a) require that the probability space be compact, most real-world examples are not so obliging. For the above example, gamma distributions were assumed for the annual precipitation and inflow to the reservoir. Cost coefficients, distributions of the random variables, and other parameter values for this example problem and the algorithm implementation appear in Yakowitz (1994b). To illustrate the progress of the algorithm, Figure 1 is a plot from one of five replications of the objective value estimates obtained in iterations 200 through 1519 when the algorithm terminated, under quite tight termination criteria.

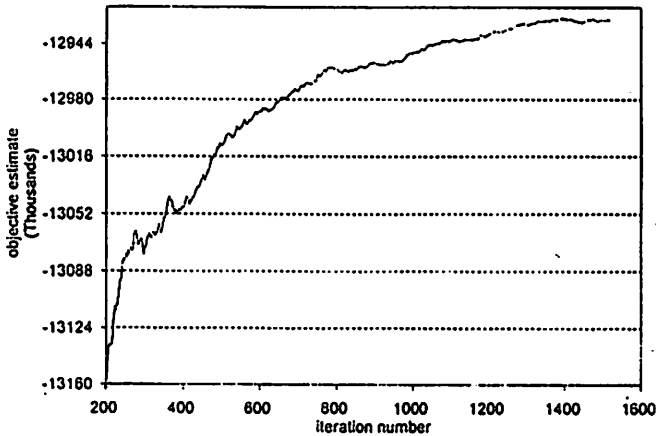


Figure 1. Plot of Objective Estimates for Ex. 1: Iterations 200-1519

Notice that the estimates are not monotonic from iteration to iteration but exhibit an increasing trend. From iteration 1000 until termination the change in objective function was less than 0.02 % of the termination value.

Example 2. Regional Water Supply

Following the apparent success of modeling and solving the problem given in Example 1, Elshorbagy, Yakowitz, and Lansey (1995a) considered a water planning problem as a two-stage stochastic program. The first stage is to determine the design capacities of a recharge basin, water treatment plant, secondary wastewater treatment plant, and tertiary treatment facility for a region which has two communities. Each community (represented in Figure 2 by u_1 and u_2) has demands for both potable water for municipal use, and reused water for irrigation and other purposes. The goal is to design water supply facilities required to satisfy the community demands over two 10-year time periods.

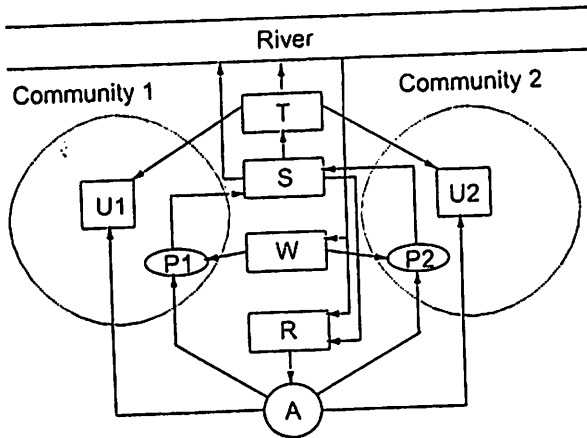


Figure 2. Schematic Representation of Regional Water Supply Problem of Ex. 2

Referring to Figure 2, the demands of potable water at nodes P_1 and P_2 can be met from direct supply from the aquifer (A) and/or treated water from the water treatment plant (W) which is supplied from a surface source (river). The demands of reused water at nodes U_1 and U_2 can also be met from direct supply from the aquifer or from a tertiary treatment plant (T) which is supplied from a secondary wastewater treatment plant (S). The aquifer is recharged through a basin system (R) with water from the river or the wastewater treatment plant after secondary treatment. The second-stage decisions are the water allocations (in million gallons per day, mgd) from the supply facilities to different users during the time periods. The flows indicated by arrows on the system diagram in Figure 2 are the second-stage operation variables for one period. For the two periods of this problem, the total number of the second-stage variables is 34. Available water, potable demands, and reuse demands for the two communities and two time periods are stochastic variables yielding a total of 10 stochastic constraints.

The first-stage objective is to minimize the present construction cost of the four supply facilities, while the second-stage objective is to minimize the expected value of the uncertain operation cost during future time periods. The operation costs include treatment costs and pumping costs. These costs were assumed to be linear functions of the treated and delivered amounts of water, respectively.

The time value of each period is represented in the objective function through the use of equivalent present worth of the operation costs during the period. For the application presented in Elshorbagy, Yakowitz and Lansey (1995a) it was assumed that the operation cost is uniformly distributed along each individual period with constant average annual value. The present worth of each period, given at the beginning of the period, is then calculated using a discounting factor, and a second discount factor is required for the second-period operation variables.

The first-stage constraints are simple bounds to maintain non-negative values of the capacities. The second-stage constraints are divided into eight groups as follows:

- 1) Capacity constraints insure that the total delivered amount of water to any unit during any time period will be less than the capacity of the unit.
- 2) River Availability constraints insure that the available water in the river exceeds the amount diverted to the system during any time period.
- 3) Demand constraints guarantee that the potable demands and the reuse demands are satisfied for the two communities during any period. External water at a penalty cost may be required to maintain feasibility for some demand realizations since demand may exceed the supply.
- 4) Aquifer Storage constraints assure that the amount of water stored in the aquifer at the end of each period is greater than a pre-specified reserve amount. The amount of stored water equals the initial storage, plus entering water, minus withdrawn water, plus external penalty water.
- 5) Reuse Quality constraints maintain a pre-specified ratio of the total reuse demands to be directly supplied from the aquifer.
- 6) Potable Quality constraints maintain a pre-specified ratio of the total potable demands to be delivered from the water treatment plant.
- 7) Temporal Continuity constraints insure that all demands and losses are met using true sources of water. If these constraints are not present, a situation might result in which the model constraints are all satisfied although the true supplies from the river or the initial storage of the aquifer during advanced periods are not sufficient to satisfy the demands.

- 8) Mass Balances constraints preserve the mass balances at different nodes and the accounting of their losses. The nodes of concern are the supplying units and the two nodes of potable demands.

The total number of second-stage constraints in this problem is 42. The stochastic parameters in the right-hand side of the second-stage constraints are the available river water, the potable demands, and reuse demands by the two communities. The number of independent random parameters considered in the two periods in this case is 10. Stochasticity in the treatment costs of the four supplying units, along with the pumping costs which are also considered, represent eight independent random parameters. Continuous normal distributions with a coefficient of variation of 0.25 were assumed for all random parameters.

Design capacities for a specific implementation of the problem with a linear first stage objective function were obtained by Elshorbagy, Yakowitz, and Lansey (1995a) using the stochastic approach described earlier. These results were compared with the value of the stochastic objective function at an optimal deterministic design obtained by evaluating the corresponding uncertain parameters at their mean values only. The result was that the four capacities obtained using the stochastic model were larger than those of the deterministic design. However, there was a 5% improvement in the total objective function due to a reduction in costs from the second-stage decisions given the larger capacities. Even greater improvements (11% and 24% savings) were obtained when two non-linear first-stage objective functions were considered.

Example 3. An Irrigation Canal System

In this example an application from Elshorbagy et al. (1995b) is considered for an irrigation canal system with 9 canals, 6 fields, and one source of water connected to the first canal. The advantages of using stochastic modeling when a number of possible scenarios, in this case levels of drought, is illustrated. The time horizon for this example consists of 10 multiple years with two 6-month growing seasons (periods) in each year. Figure 3 is a schematic of a symmetrical branch system of the canals and the crops grown in each field during the two periods. One crop pattern through the year includes cotton, grape, and maize during the first growing period of May through October. This is followed by wheat and barley during the second growing period of November to April. Data for this application can be found in Elshorbagy et al. (1995b and c) and references therein.

The canal capacities for this example are expected to be small, so that linear construction cost functions covering the range of concern, and minimizing these, are acceptable first-stage objectives. The second-stage objective is to maximize the expected net revenues from the crops. In this study, the first-stage constraints were simple bounds to maintain positive canal capacities, while the second-stage constraints consisted of the following three sets:

- 1) Crop irrigation water demands: The crop demand is the amount of water required by the crop during a period so that the maximum crop yield can be achieved. This set of constraints ensures that the supply is less than or equal to the demand, which will prevent unnecessary over-irrigation.
- 2) Canal capacities limitations: The total flow in any canal during any period should be less than the canal capacity. This flow is comprised of direct deliveries to fields from the canal and flows passed to downstream canals. Conveyance losses are also included in this total flow as a fraction of the total deliveries. Losses typically range between 10 and 20% of the total conveyed water.

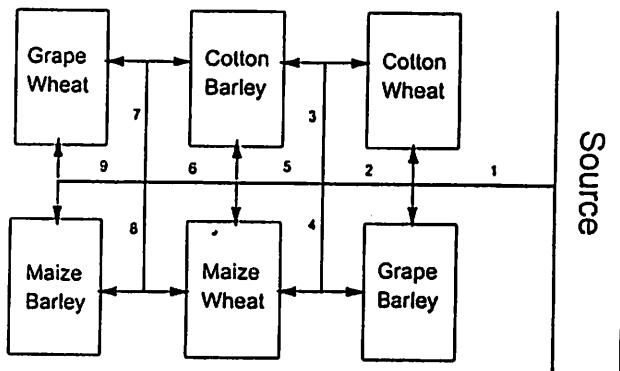


Figure 3. System of Irrigation Canals for Example 3

- 3) Water availability: For application systems where one source is connected to one canal, the total flow in this canal during any period should be less than or equal to the amount of water available during that period. This type of constraint can also be included for multi-source or supply systems. One such constraint is required for each time period.

The stochastic optimization model has two random parameters that appear in the right-hand side of the model constraints: crop demands and available water. The effect of the variability of each type on the canal capacities and future revenues was evaluated using the RSD approach by Elshorbagy et al. (1995b) for different levels of parameter uncertainty. The results then were compared with the optimal deterministic solution which was developed using the mean parameter values.

The effects of the variability of available water, which appears in the right

hand-side of the model constraints, is looked at here. Sufficient amounts of available water during the two periods (condition of zero deficit level) were assumed to follow normal distribution. Design capacities obtained for different coefficients of variation (CV) for the random available water at the zero deficit level were identical to the ones obtained from the deterministic design.

To evaluate the effect of the water shortage on the design, a similar analysis was then repeated for different levels of deficit where the mean availability was set below the sufficient amounts of available water. Table 1 lists the percent increase in net revenue between the designs obtained for the deterministic problem and the stochastic problem for different deficit levels and irrigation systems. Insights into the design of the system under varying levels of water shortage were gained through the use of stochastic programming. The percentage increase in the net revenues of the two seasons increased with the deficit level as well as with the coefficient of variation, CV. For 50% deficit level, the design capacities using the RSD approach dramatically changed, as did the percentage increase in revenues. The last observation points out the importance of using the stochastic approach in design when a significant shortage of available water can be expected.

CV	0.25	0.50	0.75	1.0
Deficit 10%	2.36	2.90	3.30	3.61
Deficit 25%	4.85	8.00	9.89	11.22
Deficit 50%	4.66	10.89	17.34	22.44

Table 1. Percent Increase in Net Revenues,
Deterministic to Stochastic Solution in Example 3

Summary

We have illustrated the formulation of stochastic programs for three water resources engineering examples. The methodologies applied indicate that it is now possible to explicitly consider uncertainties in parameters of the mathematical models. Regularized Stochastic Decomposition has been successfully applied to these three problem formulations. In Example 1, the progress of the RSD algorithm as it zeros in on an optimal solution was illustrated for a reservoir and canal capacity problem. Example 2 indicated the advantages of considering the stochastic variability in a regional water supply problem over designing based only on the expected values of stochastic parameters. Finally, multiple drought scenarios were examined for an irrigation system. Insights into the design of the system under varying levels of water shortage were gained through the use of stochastic programming.

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