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DESIGN OF ENGINEERING SYSTEMS USING STOCHASTIC DECOMPOSITION: WATER SUPPLY PLANNING APPLICATION

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To effectively design engineering systems, the future operation of the system which usually involves many uncertainties must be considered. A two-stage stochastic programming formulation can aid in satisfying this requirement. The first stage of this formulation represents the design criteria at the present time when a decision must be made. The second stage represents the future operation or the system response to the design where other actions (recourse decisions) are to be made after observing the random input. To solve this type of problem, the Regularized Stochastic Decomposition (RSD) algorithm, which allows the consideration of continuous random variables, was employed and extensions to better handle real engineering problems were investigated. The algorithm is applied to a regional water supply problem that seeks the optimal design capacities of water treatment plants, secondary and tertiary wastewater treatment plants, and recharge facilities while meeting future demands. Results are generated based on different forms of uncertainties for both linear and nonlinear first-stage objective functions. The advantages of using stochastic programming in engineering decision making are evaluated.

Keywords: Stochastic programming; decomposition; water supply; planning; operation; uncertainty

INTRODUCTION

Design and analysis of engineering systems usually involve many uncertainties. Often these uncertainties are neglected because they are

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unknown or their contributions are insignificant. However, if the uncertainties can significantly affect the system behavior, they should be taken into account. One way to account for uncertainties is to use a probabilistic representation instead of deterministic estimates of the uncertain coefficients. Some versions of this type of modelling were introduced in the late 1950s by Dantzig [2] and Charnes and Cooper [1]. One of them is chance constrained programming which has been widely used in engineering practice. Another is multi-stage programming. In fact, many engineering problems can be formulated as multi-stage programs such that initial design decisions can be made while considering the future system operations. In this case, the uncertainties can be accounted for by assigning a probability distribution to some of the uncertain coefficients. Although this modelling formulation is well known in areas of operations research, such as capacity expansion or facility location (Louveaux and Peeters [19]) and energy planning (Louveaux [18], Pereira and Pinto [22]) it has not been broadly applied to more generally described engineering problems and water resources applications in particular (Yeh [31] and Reznicek and Cheng [24] provide a few references). This is mostly attributed to the fact that most algorithms developed to date (including those in the references above) are limited to problems with few random variables that are discretized in order to effectively apply the solution procedures. An exception is Yakowitz [30] who considers a hypothetical reservoir planning problem affected by continuously distributed random variables.

This study considers the algorithm known as Regularized Stochastic Decomposition (RSD) (Yakowitz [29]). RSD has previously been applied to solve some simple test engineering applications (Yakowitz [29,30]), for the case of random right hand side coefficients only. Like most of the solution algorithms developed to date (Van Slyke and Wets [26], Louveaux [18]), RSD is decomposition based. But, unlike most, it is not limited to discrete, or discretized, random variables. The present work considers RSD and some enhancements so that it can be applied to a wider and more practical range of applications. A regional water supply planning problem in several forms is considered.

This paper is organized as follows. First, a review of the main approaches used to solve the two-stage stochastic programs with recourse is presented. Next, the RSD approach is described in detail in order to understand how the function approximations are made and explain the modifications needed. Finally, a regional water supply problem example is introduced and the results for different types of uncertainties are shown.

BACKGROUND

Two-stage stochastic LP with recourse problems have a first-stage set of variables representing decisions that must be made at present. A set of second-stage variables must also be determined in the future based on the uncertain future conditions and satisfying restrictions resulting from the first-stage decisions. A general formulation of the problem is:

$$\text{Min } f(x) = cx + E_{\tilde{\omega}}[Q(x, \tilde{\omega})] \tag{1}$$

Subject to:

$$x \in X \subseteq R^{n_1} \tag{2}$$

where:

$$Q(x, \tilde{\omega}) = \text{Min } qy \tag{3}$$

Subject to:

$$Wy = \tilde{\omega} - Tx. \tag{4}$$

The problem consists of the following components: 1) a first-stage objective function, cx, with its associated n_1 first-stage decision variables, x, associated cost vector c, and first-stage constraint set X, assumed to be convex and bounded; 2) a second stage objective function $Q(x, \tilde{\omega})$, with second-stage variables, $y \in R^{n_2}$, with associated n_2 cost vector, q, and second-stage constraints for an observation of $\tilde{\omega}$. Here W is an $n_2 \times m_2$ matrix; T is an $n_1 \times m_2$ matrix. The random m_2 vector, $\tilde{\omega}$, is defined on a probability space (Ω, A, P) where Ω is a compact set. The probability distribution function, $F_{\tilde{\omega}}$, is associated with $\tilde{\omega}$, and $E_{\tilde{\omega}}[.]$ is the mathematical expectation with respect to $\tilde{\omega}$. With these conditions, the total objective function will be a piecewise linear convex function of x.

The stochastic program is said to have the property of complete recourse if the second-stage is feasible for all values of $x \in X$. A problem has a simple recourse property if $W=(I,-I)$ (Wets [27]). This type of problem is very common in many practical applications and because of its simple formulation, many algorithms have been extensively developed for its solution. The literature reviewed in this section focuses on the case of fixed recourse, where the matrix W is general but not stochastic.

Dantzig and Madansky [2] introduced the decomposition approach to solve stochastic programs. This approach takes advantage of the structure of the dual problem. Later, Van Slyke and Wets [26] extended the approach by developing the L-shaped method to solve problems with discrete random variables.

Fraunendorfer and Kall [26] introduced an approach that solves the problem by successive partitioning schemes using the function's upper and lower bounds. These bounds, used to approximate the objective function, were first introduced by Madansky [20]. He used a discrete random variable $\tilde{\omega}$ which attains values at the vertices of the rectangle bounded by $[a_l, b_l]$, where a_l, b_l define the interval bounds of each component $\tilde{\omega}_l$ for $I=1,\ldots,m$. The stochastic function defined at the new random variable was an upper bound of the original function, i.e. $E[Q(x,\tilde{\omega})] \leq E[q(x,\tilde{\omega})]$. Other bounding schemes can be found in Dupacova [4], Gassman and Ziemba [13], Dula [3] and Frauendorfer [11]. Recently, Edirisinghe and Ziemba [5] developed upper and lower bounds of the stochastic function using first and cross moments. They considered the case of bounded random right-hand side and objective coefficients of the second-stage problem. Later Edirisinghe and Ziemba [6] developed these bounds in cases of unbounded domains of the random parameters and proposed and order-cone decomposition scheme to solve the stochastic problem.

The Stochastic Quasi-Gradient Method' SQM' is another approach for solving two-stage stochastic problems. The method is statistically based, with roots originating in the work of Robbins and Monroe [23] and Kiefer and Wolfowitz [15], who proposed a method for unconstrained unidimensional optimization. Fabian [10] and Ermoliev [7] later developed the SQM to solve constrained optimization problems. A survey of many numerical efforts in this area can be found in Ermoliev [9]. The method uses statistical estimates for the values of the functions and derivatives rather than exact values and is not hampered by continuous distributions of the random parameter. Ruszczynski [25] updated the SQM by obtaining the step directions by minimizing a linear approximation to the objective function plus a quadratic proximal term. A few difficulties arise with SQM. The function projection P on X, required by the algorithm, is easy only for problems with simple structure. The choice of an efficient step-size and algorithmically implementable stopping criterion are still open questions. Details of the method and proofs are found in Ermoliev [8, 9] and Wets [27].

Higle and Sen [14] introduced a Stochastic Decomposition (SD) approach that combines many of the strengths of the decomposition based algorithms and the stochastic gradient method. SD produces a piecewise linear approximation of the objective function then solves one subproblem and one master program at each iteration. A major problem with the algorithm is the progressively increasing size of the master program as a result of the new cut (constraint) added at each iteration after solving the subproblem. This problem can result in severe computational effort especially for large problems with many random parameters.

Yakowitz [28] considered an SD approach with an exact penalty term in the objective to handle cases in which the recourse problem appears in the constraint set. Recently, Yakowitz [29] introduced a quadratic regularizing term in the SD master program that limits the movement of the solutions to a region where the function estimates are assumed to be adequate. This term also allows the size of the master program to be limited without compromising the convergence theorems by introducing a cut-dropping scheme similar to that given in Mifflin [21] and Kiweil [16]. Theoretical developments and/or proofs of the regularized stochastic decomposition (RSD) algorithm are not included in the discussion to follow. For details on SD and RSD, the reader can consult Higle and Sen [14] and Yakowitz [29], respectively.

REGULARIZED STOCHASTIC DECOMPOSITION (RSD)

To briefly summarize the RSD algorithm (Yakowitz [29]), a random realization $\tilde{\omega}$ is generated at each iteration k and a master program is solved at the current solution x_{k-1}, called the current incumbent, producing a direction d_k. Adding d_k to the current incumbent x_{k-1} results in a new point z_k called the candidate solution ($z_k = x_{k-1} + d_k$). The total objective function is computed at z_k, x_{k-1}, and other points to check if the current incumbent solution should be replaced by the candidate or stay as it was. Termination criteria are then checked to decide whether to stop or to proceed with new random generations. The master program $\mathbf{M}^k$ is formulated as:

$$(\mathbf{M}^k)\ \ \mathrm{Min}\,[0.5\,\|d_k^2\| + v_k(d_k)] \tag{5}$$

$$x_{k-1} + d_k \in X \tag{6}$$

subject to:

$$v_k(d_k - \max\{f_k^j(x_{k-1} + d_k) \quad \forall\ j \in J^k \tag{7}$$

The function f_k^j is a linear approximation of the objective function given in Eq. (1) at $(x_{k-1} + d_k)$ and known as a cut or support. The superscript j defines the iteration at which the cut was first developed while the subscript k defines the iteration at which the cut was last updated, usually the current iteration. The set J^k is redefined in each iteration according to a cut-dropping scheme that acts to limit the size of the master program. It is defined as:

$$J^k = J^{k-1} \bigcup \{\gamma_k, k\} \tag{8}$$

where J^{k-1} represents the indices of active constraints (cuts) obtained from the master program solution at the previous iteration and characterized by having positive Lagrange multipliers. Index γ_k is the index of the cut associated with the current incumbent, and k is the index of the cut associated with the current candidate.

The following discussion of how the cuts are generated is a summary of the development in Yakowitz [29].

A cut at any point x is defined in terms of the linear coefficients α and β as:

$$f_k^j(x_k) = \alpha_k^j + (c + \beta_k^j)x_k \tag{9}$$

To obtain expressions for α and β, the dual of the second-stage stochastic subproblem is required. The subproblem (S^k), at the current candidate is:

$$(S^k) \qquad Q(z_k, \omega_k) = \text{Min } qy \tag{10}$$

subject to

$$Wy = \omega_k - Tz_k \tag{11}$$

The dual to this problem (DS^k) is:

$$(DS^k) \qquad Q(z_k, \omega_k) = \text{Max } \pi(\omega_k - Tz_k) \tag{12}$$

subject to

$$\pi \in \Pi = \{\pi : \pi w \leqslant q\} \tag{13}$$

At any iteration k, an estimate of the objective function at any point x is given by:

$$f_k^k(x) = cx + \frac{1}{k} \sum_{t=1}^{k} \pi_t^k(\omega_t - Tx) \tag{14}$$

Equating Eq. (14) with Eq. (9), the expressions for α and β are obtained:

$$\alpha_k^k = \frac{1}{k} \sum_{t=1}^{k} \pi_t^k \omega_t \tag{15}$$

$$\beta_k^k = -\frac{1}{k} \sum_{t=1}^{k} \pi_t^k T \tag{16}$$

Eq. (14) indicates that at a certain x, all π's for all previously generated random observations are required, which implies solving the subproblem (DS), k times at iteration k. This is avoided by the stochastic decomposition approach by exploiting some properties of the constraint set, Π, to avoid much of that burden. Assuming Π is a non-empty closed convex polyhedral set and noting its independence of the random vector $\tilde{\omega}$, let set V denote the set of all extreme points (vertices) of the set Π. V^k is a subset of V and represent the vertices that have been found up to the current iteration k. At each iteration k, the subproblem (DS^k) needs to be solved once using the last generated realization to get one π_k^k that is used to update V^k. The other π_t^j for $t = 1, 2, \ldots, k-1$ are then obtained from the set V^k using the following simple argmax operation:

$$\pi_t^k \in \operatorname{argmax}\,[\pi(\omega_t - Tz_k) : \pi \in V^k] \tag{17}$$

The cuts that were previously generated, will lack information gained from subsequent sampling of the random variable $\tilde{\omega}$. Therefore, the coefficients of these cuts are updated in each iteration by the current dual solution according to the following formula justified in Yakowitz [29]:

$$\alpha_k^j = \frac{k-1}{k}\,\alpha_{k-1}^j + \frac{1}{k}\,\pi_k^k\,\omega_k \tag{18}$$

$$\beta_k^j = \frac{k-1}{k}\,\beta_{k-1}^j - \frac{1}{k}\,\pi_k^k\,T \tag{19}$$

Since a particular solution may remain as the current incumbent for many iterations, its associated cut, which was first developed in iteration γ_{k-1}, may give a looser estimate of the objective function than is possible in the current iteration k. So, a re-estimation of that cut is necessary to guarantee that the function estimate at an incumbent solution converges (with a probability of 1) to the actual value. The re-estimation is done by computing new cut coefficients (α, β) at the current incumbent using the current set of subproblem dual vectors, V^k. This step is required if either of the following two conditions is satisfied at any iteration:

$$f_k^k(x_{k-1}) - f_k^{\gamma_{k-1}}(x_{k-1}) > 0, \text{ or} \tag{20}$$

$$k - \tau_{k-1} = \tau_0 \tag{21}$$

where $f_k^k(x_{k-1})$ is the objective estimate at the current incumbent using the cut derived at the current candidate. Eq. (21) means that τ_0 iterations (defined as an input parameter) have passed since iteration, τ_{k-1}, in which the cut associated with the incumbent x_{k-1}, was last evaluated.

A test should be made at each iteration to decide if the new candidate z_k should replace the incumbent solution x_k. Satisfying this condition means that a sufficient fraction of reduction in the objective value is attained using the new candidate. If this acceptance condition is satisfied, then z_k replaces x_k. Otherwise, x_{k-1} remaines as x_k. This condition is defined as follows:

$$[f_k^k(z_k) - f_k^{\gamma_{k-1}}(x_{k-1})] < \mu\,[v_{k-1}(d_{k-1}) - f_k^{\gamma_{k-1}}(x_{k-1})] \tag{22}$$

where μ is a fixed parameter such that $0 < \mu < 1$. The right hand side represents the anticipated amount of descent of the objective function in moving from x_{k-1} to z_k, while the left hand side represents that descent after updating (i.e. updating the cuts with last generated realization at iteration k).

A sub-sequence of the incumbent solutions, $\{x_k\}$, produced by the algorithm was shown to accumulate at an optimal solution of the two-stage problem [29].

Termination Criteria

Several termination rules can be used for this type of problem. Three rules, however, are used in the present algorithm.

1. The number of vertices found in V_k is observed. The algorithm should terminate if that number does not change within a prescribed number of iterations, KP.
2. The stability of the objective function is monitored using an exponentially smoothed average, η_k. Given a tolerance value ε, the algorithm should terminate if the following condition is satisfied:

$$\frac{|f_k^{\gamma_k}(x_k) - \eta_k|}{f_k^{\gamma_k}} < \varepsilon \tag{23}$$

Choosing an appropriate value of η_0 with $\lambda \in (0,1)$, η_k is defined as:

$$\eta_k = \begin{vmatrix} \lambda f_k^{I_k} + (1-\lambda)\eta_{k-1}, & \text{if } k \in K^* \\ \eta_{k-1} & \text{otherwise} \end{vmatrix} \tag{24}$$

K^* is a set of indices which defines a sub-sequence of iterations along which the incumbent solutions are accumulating at the optimal solution. This sub-sequence is needed when the incumbent solution changes infinitely often. It is defined as:

$$K^* = \{k : \| d_k \| \leq \delta_{k-1}\} \tag{25}$$

where δ_k defines the monotonic sequence $\{\delta_k\}_{k=1}^{\infty}$ that converges to zero with probability 1. Define $\mu_2 < \mu_1 < 1$ and starting with sufficiently large δ_0, δ_k is given according to:

$$\delta_k = \begin{vmatrix} \mu_1 \delta_{k-1}, & \text{if } \|d_k\| < \mu_2 \delta_{k-1} \\ \min[\delta_{k-1}, \|d_k\|] & \text{otherwise} \end{vmatrix} \tag{26}$$

3. Check the stability of descent (the regularizing term) d_k, $\rho_k < \varepsilon$. A statistic ρ_k is defined similar to η_k by:

$$\rho_k = \begin{vmatrix} \lambda \|d_k\| + (1-\lambda)\rho_{k-1}, & \text{if } k \in K^* \\ \rho_{k-1} & \text{otherwise} \end{vmatrix} \tag{27}$$

The above three criteria are used together in the algorithm with the highest priority given to first criterion which must be satisfied before any termination caused by the other two criteria.

The RSD Algorithm

The main steps of the solution method can be summarized as follows:

0. Define all required parameters, such as $KP, \varepsilon, \mu_1, \mu_2, \eta_0$, and ρ_0. Initialize other variables at $k=0$, such as $V^0 = 0, d_0 = 0$, and $z_1 = x_0$ where x_0 is the initial solution of the problem defined at some ω_0. One way to obtain x_0 is to externally solve the corresponding deterministic problem whose random parameters are set at their mean values.

1. For $k = k + 1$, randomly generate an observation ω_k according to the specified probility distribution of the random parameters.
2. Solve the subproblem (DS^k) and get the dual vector $\pi(z_k, \omega_k)$. Modify V^k by adding the new π if it has not already been included. Form the candidate cut according to Eqs. (14) and (17).
3. Determine the set J^{k-1} of active constraints distinguished from the previous master program solution as those having positive Lagrange multipliers. Update these constraints according to Eqs. (18) and (19).
4. Check the re-estimation conditions (20) and (21). If satisfied, re-evaluate the cut at the current incumbent using the new V^k.
5. Check the new incumbent condition (22). If satisfied, replace x_k with z_k and γ_k with k. Otherwise, keep x_k and γ_k as they are. Determine J^k according to Equation (8).
6. Solve the master program M^k to obtain d_k, $v_k(d_k)$ and Lagrange multipliers λ_k of the relevant constraints. Set $z_{k+1} = x_k + d_k$.
7. Check the termination criteria. If satisfied, stop. Otherwise, go to step 1.

NONLINEAR FIRST-STAGE OBJECTIVE FUNCTION

The RSD approach, has been applied to linear two-stage problems with uncertainly only in RHS of the second-stage constraints. Since many engineering problems behave in a nonlinear manner, the algorithm was adapted to handle the nonlinearity of the first-stage objective function. The assumption of convexity of the overall problem is violated, in this case, especially with problems in which the first-stage objective function is non-convex. Therefore, global optimality of the optimal solution is no longer guaranteed. Prudent selection of the initial point can potentially improve the solution and bring it closer to the global optimal solution for the case of non-convexity of the objective function.

The nonlinearity in the first-stage objective is easily handled by including this nonlinear function, $g(x_k)$, in the objective of the master program rather than as part of the cut generation. Cuts identified with this objective will approximate the second-stage stochastic function only. The new objective, Eq. (28), replaces Eq. (5) and the cut expression is also modified from Eq. (9) to Eq. (29). All other definitions and equations remain the same. The new forms of Eqs. (5) and (9) are:

$$\text{Min}\ [0.5\|d_k\|^2 + g(x) + v_k(d_k)] \tag{28}$$

$$f_k^j(x_k) = \alpha_k^j + \beta_k^j x_k \tag{29}$$

GRG2 (Lasdon [17]), a nonlinear programming model, was used to solve the master problem for the case of the non-linear first stage objective. GRG2 applies the generalized reduced gradient method as a basis for solving the NLP. A small subroutine linked to GRG2 is required to describe the first-stage objective function and constraints of the investigated problem. Restarting the algorithm at several initial starting points is suggested to improve the objective for the case of multiple local optima.

STOCHASTIC COEFFICIENTS OF THE SECOND-STAGE OBJECTIVE FUNCTION

Both the SD (Higle and Sen [14]) and the RSD (Yakowitz [29]) algorithms were developed considering only the case of stochastic right-hand sides of the second-stage constraints. In many applications, the coefficients of the second-stage objective funciton include future revenues and/or prices that might be subject to a great deal of uncertainty. Therefore, it was necessary to consider modifications necessary for the algorithm to handle the stochasticities of the second-stage objective function. Most algorithm developments address only the case of random right-hand sides, noting that the dual of the subproblem with random objective coefficients has random right-hand side coefficients. It is important to note that the major difference in tackling this problem is that when the random components appear in the second-stage objective, the first-stage decision and these random variables are de-coupled. In both the primal and dual problems, one appears in the objective while the other appears in the constraints. Issues of feasibility of the previously produced dual variables require modifications to steps of RSD in order to produce the best feasible cuts.

To consider random components in the second-stage objective function, the subproblem and its dual must be redefined to include deterministic RHS and stochastic objective coefficients. The subproblems, (S^k) is now:

$$(S^k) \qquad Q(z_k, \tilde{q}_k) = \text{Min}\,(\tilde{q}_k)\,y \tag{30}$$

subject to

$$Wy = h - Tz_k \tag{31}$$

and its dual, (DS^k), is:

$$(DS^k) \qquad Q(z_k, \tilde{q}_k) = \text{Max}\,\pi(h - Tz_k) \tag{32}$$

subject to

$$\pi \in \Pi = \{\pi : \pi W \leq \tilde{q}_k\}, \tag{33}$$

where, h is a deterministic vector in R^{n_2} and random n_2 vector $\tilde{q}$, is defined on a probability space (Ω, A, P) where Ω is a compact set. The probability distribution function, $F_{\tilde{q}}$, is associated with $\tilde{q}$, and $E_{\tilde{q}}[.]$ is the mathematical expectation with respect to $\tilde{q}$.

Recall, that the set Π in the case of random RHS was independent of the random realization ω_k. This observation made it possible to use Eq. (17) to select any $\pi_t \in V_k$ to correspond to ω_t for $t = 1, 2, \ldots, k-1$ to produce or update the cuts to the master program. In the case of random second-stage objective coefficients, Π is no longer independent of the random observations and the argmax operation alone does not guarantee the feasibility of the selected π. Cuts produced according to Eq. (17) for the case of random objective coefficients could be invalid. Another approach to determine the correct feasible multipliers to be used in the cut generation and cut updating is needed and is described below.

Since $\tilde{q}$ does not appear in the subproblem dual objective function, $\pi(H - Tz_k)$, maximizing this function over the current set of dual solutions can be accomplished as follows:

1) Arrange the multipliers, π, in V^k in descending order according of the value $\pi(H - Tz_k)$.

2) For each $q_t, t = 1,\ldots,k-1, \pi_t^k$ is selected by testing one by one, in order, the sorted multipliers in V^k until Eq. (33) is satisfied.
3) The cut is then defined according to Eqs. (9), (15) and (16).

This method select a multiplier for each realization $q_t, t = 1,\ldots,k-1$, from the multipliers generated so far, that maximizes the dual objective function while also being feasible to Eq. (33). The result is a feasible cut estimate given the current information at the current incumbent solution.

Although the modification was successfully implemented, computational problems during execution point out the difficulty in practical usage of the algorithm in certain situations. The sorting-testing scheme should perform well if the number of multipliers in V^k is finite and not too large.

APPLICATION

To illustrate the RSD method and modifications for the case for stochastic second-stage objective coefficients, a water planning problem is presented.

Consider a region that has two communities. Each community has demands for both potable water P_i for municipal use, and reclaimed water U_i for irrigation and other uses. The goal is to design water supply facilities for satisfying the community demands over a 20-yr time horizon which is divided into two 10 year periods. The demands of potable water can be met from direct supply from the aquifer Q and/or treated water from the water treatment plant W which is supplied from a surface source V (see Fig. 1). The demands for reused water can be also met from direct supply from the aquifer or from a tertiary treatment plant T which is supplied from a secondary wastewater treatement plant S. The aquifer is recharged through a basin system R with water from the river or the wastewater treatment plant after secondary treatment.

The planning problem is to determine the design capacities of the recharge basin, water treatment plant, secondary wastewater treatment plant, and tertiary treatment facility. These decisions represent the first-stage decision variables in the two-stage formulation. The second-stage variables represent the water allocations (in million liters

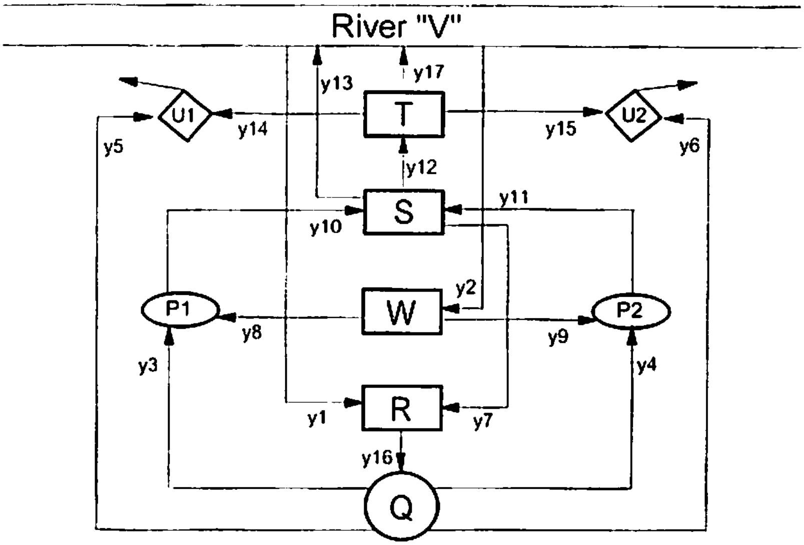

FIGURE 1 Diagram of the water planning application system for communities 1 and 2.

per day, mld) from the supply facilities to different users during different time periods. The variables y_1 to y_{17}, shown on the system outline of Fig. 1, are the second-stage operation variables for the first 10 years period. The total number of the second-stage variables for the two 10 year periods is 34.

The first-stage objective funciton represents the present construction cost of the four supply facilities. These costs were first assumed to be linear functions of the design capacities with linear cost coefficients given in Table I. The second-stage objective represents the expected value of the uncertain operation cost during the future time periods. The operation costs include treatment costs and pumping costs. These costs were assumed to be linear functions of the treated and delivered amounts of water, respectively. The linear coefficients of these costs are given in \$ per million liters per day and listed in Table II.

To obtain a good design, the two components of the objective function need to be represented in appropriate measures. Therefore, the time value of each period was considered through the use of equivalent present worth of the operation cost during the period. It was assumed that the operation cost is uniformly distributed along

TABLE I Construction cost coefficients for the first-stage decision variables ($/mld)

Unit Tertiary Coefficient	*Recharge*	*Water Treat.*	*WastewaterTreat.*
15,900	2,650	80,000	40,000

TABLE II Cost coefficients for the second-stage objective function ($/mld)

Treatment Cost Coefficients				
Unit	*Recharge*	*Water Treat.*	*WastewaterTreat.*	*Tertiary*
Coef. in Period I	0.026	2.643	1.586	0.264
Coef. in Period II	0.032	3.171	1.718	0.317

Piping and pumping Cost Coefficients

Route	V-R	V-W	R-Q	W-P	P-S	T-U	S-T	Q-U	Q-P	S-R	S-V	T-V
Period I	1.32	0.00	5.30	5.30	5.30	1.32	13.21	7.93	2.64	5.30	0.00	0.00
Period II	1.60	0.00	5.80	5.80	5.80	1.50	14.50	9.25	3.20	5.80	0.00	0.00

**The route V-R means from the component V (river) to the component R (recharge).

each individual period with constant average annual value. This annual value was obtained by multiplying the allocation variable (mld) by 365 days/year by the corresponding average operation cost coefficient. The present worth of each period k, P_k, is given at the beginning of the period. The first 10 year period is assigned a discounting factor of 6.145 corresponding to a 10% discount rate. A second discount factor of 0.386 is used for the second period operation variables to present cost.

The water supply planning problem can be expressed as the following two-stage stochastic program:

$$\underset{\{x,y\}}{\text{Min}} \sum_{l=1}^{4} c_l * x_l + 6.145 * 365 * E\left[\sum_{r=1}^{17} q_r^1 * y_r^1 + 0.386 * \sum_{r=18}^{r=34} q_r^2 * y_r^2\right]$$

$$+ q_e * \left[\sum_{\xi=1}^{2}\left[eq^{\xi} + \sum_{v=1}^{2}(ep_v^{\xi} + eu_v^{\xi})\right]\right] \qquad (34)$$

Subject to

First-stage constraints

$$x_l \geqslant 0, \quad l \in \{1,4\} \tag{35}$$

Second-stage constraints (For $\xi = 1,2$)

1) Canal Capacity

$$\sum y^{\xi}_{\to x_l} \leqslant x_l, \quad l \in \{1,4\} \tag{36}$$

2) Water Availability

$$\sum y^{\xi}_{\leftarrow V} \leqslant AV^{\xi} \tag{37}$$

3) Potable and Reuse Demands

$$\sum y^{k}_{\to P} + ep^{\xi}_{v} \leqslant DP^{\xi}_{v} \quad v = 1,2 \tag{38}$$

$$\sum y^{k}_{\to U} + eu^{\xi}_{v} \leqslant DU^{\xi}_{v} \quad v = 1,2 \tag{39}$$

4) Aquifer Storage

$$QI^{\xi} + \sum y^{\xi}_{\to Q} - \sum y^{\xi}_{\leftarrow Q} + eq^{\xi} \geqslant QS^{\xi} \tag{40}$$

5) Quality of Reuse demands

$$y_{Q \to U_v} \geqslant PCU * DU^{\xi}_{v} \quad v = 1,2 \tag{41}$$

6) Quality of Potable Demands

$$y^{\xi}_{W \to P_v} \geqslant PCP * DP^{\xi}_{v} \quad v = 1,2 \tag{42}$$

7) Temporal Continuity

$$QI^{\xi} + \sum y^{\xi}_{\leftarrow V} + \sum_{v} (ep^{\xi}_{v} + eu^{\xi}_{v}) + eq^{\xi} \geqslant (1 + loss_{avg}) * \sum_{v} DP^{\xi}_{v} + DU^{\xi}_{v} \tag{43}$$

8) Mass Continuity

$$(1 - loss_j) * \sum_j y^{\xi}_{\to j} = \sum_j y^{\xi}_{\leftarrow j} \quad j \in \{R, W, S, T, P1, P2\} \tag{44}$$

where x_i is the design capacity of the supply units with x_1, x_2, x_3 and x_4 being capacities of the recharge basin (R), water (W), secondary (S), and tertiary (T) treatment plants, respectively. Objective function coefficient, q^1_r is related to the allocation, y^1_r (the superscript 1 indicates the first period), and reflects treatment and pumping costs. The unit price, q_e, of the penalty water used to maintain feasibility is explained later. The first-stage constraints are only simple bounds to maintain non-negative values of the capacities. The subscript of y on the second-stage constraints identifies the allocated water. For example, $y_{\to U}$, defines all y's entering the unit U, and $y_{Q\to U1}$ defines the allocated water from unit Q, recharge storage, to unit U1, reclaimed water user one. The second-stage constraints, are divided into eight groups and explained below.

1) Capacity constraints ensure that the total delivered amount of water to any unit during any time period, ξ, will be less than the capacity of the unit.
2) River availability constraints insure that the available water in the river, AV, exceeds the amount diverted to the system during any time period, ξ. The average amounts of available water during the two periods was assumed to be 120 mld.
3) Demand constraints guarantee that the potable demands, DP, and the reuse demands, DU, are satisfied for the two communities during any period, ξ. The quantities ep_v and eu_v are external water at a penalty cost required to maintain feasibility during times when demand exceeds the supply. Table III lists the values of the demands used in this application.

TABLE III Demands for different users in million liters per day (mld)

User	*P1*	*P2*	*U1*	*U2*
PERIOD 1	150.0	190.0	114.0	132.0
PERIOD 2	190.0	227.0	150.0	170.0

4) Aquifer storage constraints assure that the amount of water stored in the aquifer at the end of each period is greater than a pre-specified reserve amount, *QS*. The amount of stored water equals the initial storage, *QI*, plus entering water minus withdrawn water plus external penalty water.
5) Reuse quality constraints maintain a pre-specified ratio of the total reuse demands *PCR* to be direct supply from the aquifer.
6) Potable quality constraints maintain a pre-specified ratio of the total potable demands *PCR* to be delivered from the water treatment plant.
7) Temporal continuity constraints ensure that all demands and losses are met using true sources of water. If these constraints are not present, a situation might result in which the model constraints are all satisfied although the true supplies from the river or the initial storage of the aquifer during advanced periods are not sufficient to satisfy the demands.
8) Mass balance constraints preserve the mass balances at different nodes and accounting of their losses. The nodes of concern are the supplying units and the two nodes of potable demands (*P*).

The total number of second-stage constraints in this problem is forty two . When uncertainities in the right side of the second-stage constraints were considered, the uncertain parameters were the river supply, *AV*, and the four demands given by *DP* and *DU* for the two periods. The mean values of these parameter are listed in Table III. They were assumed to follow normal distributions with a coefficient of variation equal to 0.25. The number of independent random parameters considered in this case is 10.

In cases with uncertainty in the objective function the treatment cost coefficients represent the treatment and pumping costs from the recharge wells, and water, wastewater, and tertiary treatment plants. The mean costs for the four costs are listed in Table II for periods (eight independent random parameters). Continuous normal distributions with a coefficient of variation of 0.25 were also assumed in this case.

RESULTS AND DISCUSSION

The problem was solved using the RSD model for the following three cases:

1) Linear first-stage objective function and stochastic RHS.
2) Non-linear first-stage objective function and stochastic RHS.
3) Linear first-stage objective function and stochastic second-stage objective function.

Demands, available water, and treatment costs with the related pumping costs were considered random parameters for the appropriate cases. To assess the improvement in the objective function using the stochastic design (using the RSD model), the stochastic program objective function was evaluated at the optimal deterministic design which considers only the mean values of the random parameters.

In case 1, the design capacities using the stochastic approach were obtained after 15.75 hours. The four capacities obtained using this design were larger than those of the deterministic desing with about 5% improvement in the total objective function.

In case 2, a power form for the first-stage objective function was assumed. Two separate designs were examined using different values of the power function exponent. Table IV lists the stochastic and deterministic designs for these two cases along with the total function improvement obtained when the stochastic design was used instead of the deterministic one. Results show that the stochastic design in both linear and concave non-linear first-stage objectives (power coeff. = 0.80) enlarged the facility capacities, while it reduced the capacities in case of the convex non-linear case (power coeff. = 1.50). In both cases, the stochastic problem decreased the overall system cost compared with the deterministic solution.

The solution in the third case with stochastic function coefficients, was exactly the same as that obtained from the deterministic model (given in Tab. IV, case of linear first-objective function). This solution was reported on the results of the algorithm after a large number of iterations. This means that the variability in the objective coefficients for this particular setting of the problem had no effect on the first-stage decisions.

A major problem related to the execution time required to solve the third problem was encountered. The number of dual vertices was growing similar to the number of iterations. This resulted in marked delay in the progress of the algorithm. While 940 iterations were solved during the first 24 hours, only 200 iterations were done in the following 24 hours.

When a new dual variable is identified at nearly every iteration, the first termination criterion cannot be satisfied and suggests that for this

TABLE IV Results for the case of stochastic RHS

Unit	R (mld)	W (mld)	S (mld)	T (mld)	Obj. Fn. (10^6 dollars)	NK[1]	NV[2]	KP[3]	Time (hrs)
Case of Linear First-Objective Function									
Det. Soln	612.36	146.89	333.03	181.22	83.511				
Stoch. Soln	824.90	160.44	363.53	190.41	79.497	2719	198	100	15.75
Improvement in Obj. Fn. = 4.014 (million $) = 5.05 %									
Case of Nonlinear First-Objective Function with Power Coeff. = 0.80									
Det. Soln	587.50	146.89	332.99	236.99	68.514				
Stoch. Soln	937.94	177.13	401.52	255.53	60.888	1284	139	50	2.50
Improvement in Obj. Fn. = 7.627 (million $) = 11.13 %									
Case of Nonlinear First-Objective Function with Power Coeff. = 1.50									
Det. Soln	609.91	146.89	332.99	42.57	285.324				
Stoch. Soln	586.78	75.68	189.20	0.00	217.122	954	39	50	2.33
Improvement in Obj. Fn. = 68.20 (million $) = 23.90%									

[1]Number of iterations.
[2]Number of Vertices.
[3]Number of iteration required by termination criteria.

case, one might want to neglect the first termination criterion and terminate the algorithm if the other two criteria are satisfied while the incumbent does not change for a certain number of iterations. When incorporating this modification to the present program, the application problem still did not satisfy any of the termination criterion. The model was stopped after four days of execution with about 1500 iterations. We expect that violations of the model conditions on the random variables and the dual solution set, II, were responsible for these difficulties.

SUMMARY AND EXTENSIONS

The original RSD algorithm considers only the case of stochastic right-hand sides of the second-stage constraints in linear problems. Two extensions of the algorithm were implemented to account for other conditions. First, the ability to consider a non-linear first-stage objective function was incorporated. This modification was successfully demonstrated in the application. Global optimality might be sacrificed based on the given application. To consider stochasticity of the second-stage objective function coefficients a new cut generation procedure was introduced. Although this enhancement is theoretically valid and the modification was successfully programmed, the computations showed that the algorithm cannot be efficiently applied to general problems that may violate the assumptions of the random variables and dual solutions. In this case, having stochastic objective coefficients may yield an infinite number of dual multipliers for the subproblem. This results in significantly longer execution times per iteration as the iterations progress. These observations indicate that other approaches to handle the stochasticity of the second stage objective coefficients such as a modification of stopping rules and/or the selection of dual variables should be investigated.

Finally, the results of the application system indicate that significant improvements over deterministic solutions can be obtained with maximum improvement, for this example, for the case of a convex nonlinear objective function.

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