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### A BASINWIDE STOCHASTIC MODEL FOR EPHEMERAL STREAM RUNOFF IN SOUTH-EASTERN ARIZONA

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#### ABSTRACT

A stochastic model for generation of synthetic data on basins of 100 mi<sup>2</sup> or less is developed using data from the Walnut Gulch Experimental Watershed in south-eastern Arizona. Variables describing the intermittent and independent runoff events are: start of runoff season, number of runoff events per season, time interval between events, beginning time of runoff event, volume of runoff and peak discharge. Each of these variables is generated from its own probability distribution. The means and standard deviations of the various distributions form the set of parameters that define the stochastic model. Some parameters are expressed as functions of drainage area and some are assumed constant for the range of basin areas used in the study. The means of the runoff variables vary with basin area while the standard deviations appear to be independent of basin area. By describing the variation of the model parameters with basin area, a model for a specific basin is developed into a model of a general basin for runoff events.

#### RÉSUMÉ

Un modèle stochastique pour la production de données synthétiques sur des bassins de 100 mi<sup>2</sup> ou moins a été développé en utilisant les données du Bassin Expérimental de Walnut Gulch au Sud-Est de l'Arizona.

Les variables qui décrivent les événements de ruissellement intermittent et indépendant sont : le début de la saison de ruissellement, le nombre d'événements d'écoulement par saison, l'intervalle de temps entre les événements, le temps initial de l'événement d'écoulement, le volume d'écoulement et le débit maximum. Chacune de ces variables est produite par sa propre distribution de probabilités.

Les moyennes et les déviations normales des diverses distributions forment l'ensemble des paramètres qui définissent le modèle stochastique. Certains paramètres sont exprimés en fonction de la superficie de drainage et certains sont présumés être constants dans le domaine des bassins versants utilisés dans cette étude. Les moyennes des variables de ruissellement varient avec la superficie du bassin, alors que les déviations normales paraissent être indépendantes de la superficie. En décrivant la variation des paramètres du modèle avec la superficie du bassin, un modèle pour un bassin spécifique est généralisé en un modèle d'un bassin général pour les événements de ruissellement.

#### LIST OF SYMBOLS

- A Area of basin in square miles;
- B Coefficient in equation for number of events per season;
- C Constant in equation for number of events per season;
- D Time interval in days between runoff events;
- K Coefficient in equation for peak discharge of runoff event;
- L Logarithm of runoff volume per event measured in 10<sup>-6</sup> in. units;
- M Coefficient in equation for parameters of stochastic model;

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- $N$  Number of runoff events per season;
- $n$  Exponent in equation for peak discharge of runoff event;
- $P$  Peak discharge of runoff events in cfs;
- $R$  Parameter of stochastic model;
- $r$  Simple correlation coefficient;
- $S$  Starting date of runoff season (number of days from beginning of year);
- $T$  Beginning time of runoff event during the day in hours;
- $U$  Uniformly distributed random number in the range 0 to 1;
- $V$  Volume of runoff per event in inches;
- $X$  Random variable of an arbitrary distribution function.

**INTRODUCTION**

Stochastic hydrological models are used to generate synthetic hydrological data that preserve the statistical properties of the original data observed in the real system. The parameters of the various distributions associated with the original data are input to the stochastic models. The corresponding model's output is in the form of sequences of synthetic hydrological data that may be used for design purposes. The model's goodness of fit is judged in terms of the agreement between values of statistical parameters derived from the synthetic data and the corresponding parameters of the observed data, including some parameters not used as input.

Stochastic models are a valuable tool in runoff data studies because they make it possible to extend existing data and generate large amounts of data needed for the study and comparison

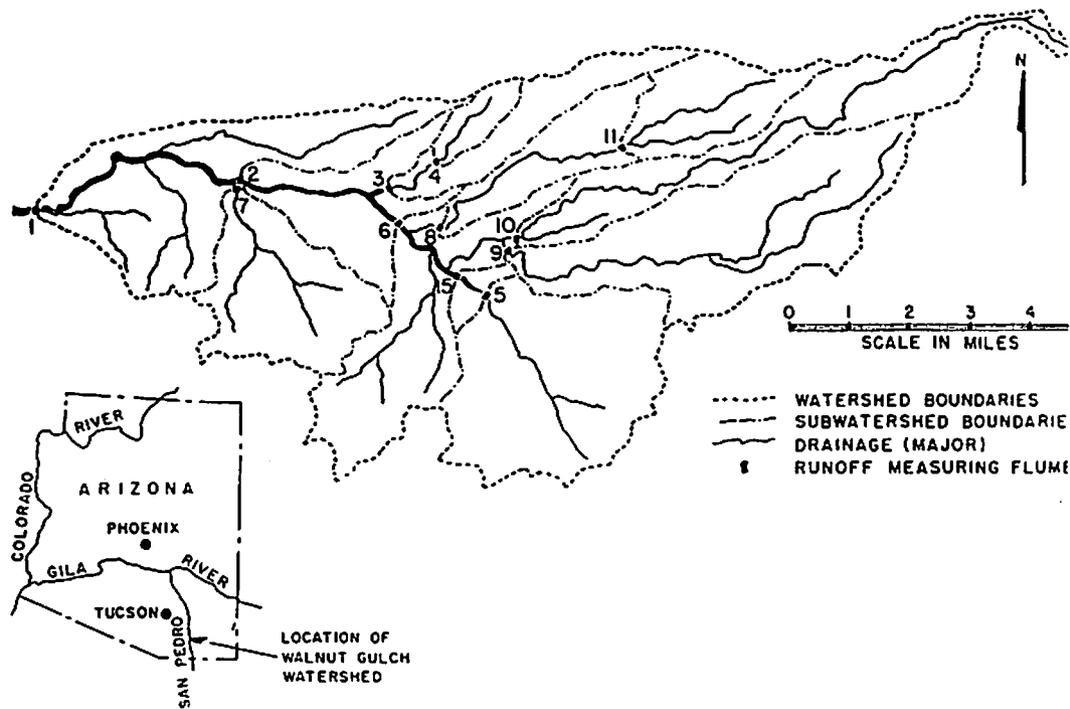


Fig. 1 — Drainage map of Walnut Gulch watershed showing flow measuring stations.

of water use schemes for the basins concerned. Stochastic models are more valuable if they are developed so that synthetic data can also be generated for basins for which no runoff observations are available (Soil Conservation Service, 1970). This may be possible by developing regional or basinwide stochastic models for basins located within a given region where climatic and runoff-producing factors are fairly uniform.

A regional stochastic model is based on the hypothesis that the parameters describing the various distributions vary systematically with the size of the basins, and possibly with some other measurable physical features of the basins within the region. The functional relationships between the various model parameters and the physical characteristics of the basins will constitute, in this case, part of the regional stochastic model. If the above hypothesis is valid, the parameters of the distributions describing runoff events for any ungauged basin in the region could be predicted from the relationships included in the regional model. The predicted parameters can then be used to generate sequences of synthetic data for the ungauged basin.

The feasibility of a regional stochastic model for runoff events for semi-arid basins was investigated using data available for a number of basins in south-eastern Arizona. The basins studied form part of the Walnut Gulch Experimental Watershed (Fig. 1), operated by the Southwest Watershed Research Center, Agricultural Research Service, USDA. The results of the investigation and a description of the model developed are reported herein.

The regional model is developed from a stochastic model for the generation of synthetic runoff data on a single semi-arid basin (Diskin and Lane, 1970). Since the proceedings of the workshop at which the stochastic model was presented are not generally available, it is described briefly as part of the description of the regional model.

The general procedure followed in developing the stochastic model is outlined in Fig. 2, which is self-explanatory. Briefly, the procedure includes deriving statistical distributions to describe the variables selected to represent the runoff events, testing the goodness of fit of the distributions selected, and then comparing statistical parameters of synthetic data generated by

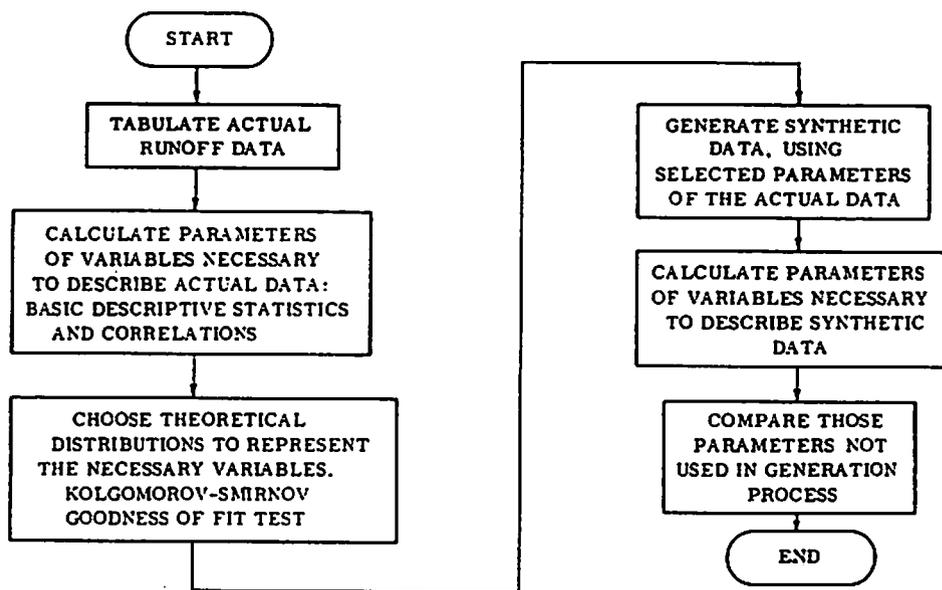


Fig. 2 — Flow chart of stochastic modelling process.

the model with the original values of the same parameters. The Kolmogorov-Smirnov goodness of-fit test was used to test the hypotheses that the sample data came from the statistical distributions assumed for the various variables.

#### DESCRIPTION OF WATERSHEDS STUDIED

Walnut Gulch is an ephemeral channel tributary to the San Pedro River. The watershed contributing to Walnut Gulch ranges in elevation from just under 4000 ft at the outlet to about 6300 ft at the upper boundaries of the watershed. It is predominantly a brush- and grass-covered rangeland basin with brush dominating the lower portions and grass dominating the upper portions. The basin is bounded on the east by the lower Dragoon Mountains, on the south by the Tombstone Hills, and on the north by low alluvial hills.

The experimental watershed is defined by a flow-measuring flume on the main channel of Walnut Gulch approximately 2.5 miles from its junction with the San Pedro River. The area of the basin is 57.7 mi<sup>2</sup>. It is further divided into a number of sub-basins by measuring flumes located at various points on the channel network (Fig. 1). The experimental basin is operated by the Southwest Watershed Research Center, located at Tucson, Arizona, some 70 miles from the basin, with a field station located at the city of Tombstone near the center of the basin itself.

Runoff from the entire basin and from the main sub-basins is measured with pre-rated, supercritical depth flumes described by Gwinn (1964). The 12 flumes on the basin are equipped with continuous water-level recorders. Rainfall on the basin is measured by 95 well distributed recording rain gauges. Precipitation data, such as averages, refer to simple averages from the dense network which yields results comparable to Thiessen weighting.

Runoff records from five flumes, those defining the main basin and four sub-basins, were used in the study to derive the relationship needed for the regional model. The records of two more flumes were used to check the results obtained. The main physical characteristics of the basins used in the study are listed in Table 1. The last two digits in the watershed number corre-

TABLE 1  
*Physical characteristics of basins used in study*

Watershed number	Area (mi <sup>2</sup> )	Length of main channel (mi)	Range of elevation (ft)	Period of record (years)	Mean annual runoff (in.)
63.001	57.7	20.3	4000-6300	13	0.21
63.006	36.7	13.5	4360-6300	8	0.25
63.015	9.24	3.2	4450-5220	5	0.17
63.008	5.98	8.0	4420-5040	7	0.28
63.007	5.22	3.8	4220-4970	4	0.07
63.011	3.18	4.0	4680-5040	7	0.75
63.004	0.88	2.2	4460-4620	12	0.59

spond to the flume numbers shown in Fig. 1. Records from watersheds 63.007 and 63.015 were used for verification purposes. Data from five watersheds were not included in this study. This was partly because some of the periods of record were too short and partly because some were thought to contain systematic errors due to uncertainties in the calibration curves.

The major part of the basins studied is covered by the Tombstone pediment, which is a deep Quarternary and Tertiary alluvial fill of disconnected lenses and layers of sands, gravel, and conglomerate. The soils of Walnut Gulch are mostly primary, moderately to poorly developed, and thus exhibit characteristics of the parent rock of geological alluvium. Almost 80 per cent of the soil surface is developed on geological alluvium derived from limestone, Cambrian to Carboniferous in age. The other 20 per cent consists of lithosols on limestone itself, granite rock, and granodiorite. With a few exceptions in this latter group, all the soils are base saturated, if not calcareous, throughout the profile and have sandy to gravelly sandy loam surface textures.

By the use of the Canfield line intercept method (National Research Council Publication, 1962), it was shown that the vegetation cover on the main basin is predominantly desert shrubs such as whitethorn, creosote bush, tarbush, and mertonias. The part classified as brush covered occupies about 65 per cent of the area. It has 23 per cent brush cover (crown spread measurement) and 2 per cent grass cover (basal area measurement). Thirty-five per cent of the area is classified as grassland, with about 20 per cent grass cover and 5 per cent shrub cover. Dominant grasses are black grama, curly mesquite grass, sideoats grama, blue grama, and tobosa grass. In both cases the vegetation covers only about 25 per cent of the area, leaving 75 per cent as bare or stone-covered soil.

More information about the characteristics of the Walnut Gulch Watershed is given in a data compilation published by the USDA (Hobbs, 1964) and in a recent paper by Renard (1970). The latter outlines also the hydrology of the area and discusses special problems associated with the study of semi-arid watersheds.

#### AVAILABLE DATA

Hydrological data have been collected on the Walnut Gulch Experimental Watershed since 1954. The network of rain gauges and measuring flumes has been developed over a number of years so that records for various locations are of unequal length. The periods of record for the basins included in the study are given in Table 1.

Precipitation in south-western Arizona is characterized by convective thunderstorms in the summer and low-intensity frontal storms in the winter (Sellers, 1960). The thunderstorms are characterized by their short duration, limited areal extent, and high intensity. An analysis of 11 years of record from 30 recording rain gauges on Walnut Gulch (Osborn and Hickok, 1968) produced a mean annual precipitation of 11.2 in. and a mean summer (June-September) precipitation of about 8.0 in. About 90 per cent of the runoff-producing rain falls in July-August, with the remainder occurring in June and September. Significant winter runoff has not been recorded on any sub-basins larger than 20 ac during the 11 years of record.

The stream channels are usually dry, with runoff occurring for short periods of time following intense rainfalls and totalling only a few hours per year. For example, in the 13 years of record from the 57.7-mi<sup>2</sup> Walnut Gulch Experimental Watershed, the mean duration of runoff per event was 181 min (3.0 hr), with a standard deviation of 155 min. The range in duration of runoff values was from 23 min for a trace of runoff to 760 min for an unusual convective-frontal storm producing a multi-peaked outflow hydrograph.

The total number of hours of runoff in the 13 years was about 495 or some 38 hr per year. The annual volume of runoff from the basins studied is small in comparison to the annual rainfall. Values of mean annual runoff given in Table 1 for the various basins range from 0.07 in. to 0.75 in.

Runoff data available for the study included stage hydrographs and tabulation of discharge values at close time intervals throughout the runoff event, as well as the runoff volumes produced in each event. The data have been tabulated by a digital computer programme from water-level readings prepared manually from the recorder charts. Rating tables for the various flumes are based on tests of scale models of the measuring flumes carried out at the Stillwater, Oklahoma Water Conservation Structures Laboratory of the US Department of Agriculture.

STRUCTURE OF STOCHASTIC MODEL FOR ONE BASIN

The stochastic model for runoff events from a single basin, which forms the basis of the regional model, produces as output, sequences of runoff events. One sequence is produced for each runoff season. The season is defined as the time period starting with the first runoff event and ending with the last runoff event during the summer months of June–October.

Examination of the runoff records from the five experimental basins described above indicated that the runoff season for each basin can be described in terms of the starting date of the season, the number of runoff events that occur at the outlet per season, and the time intervals between successive runoff events. Runoff events can be characterized by their time of occurrence during the day, the volume of runoff, and peak discharge. The shape of the runoff hydrograph was not included in the simulation process because of the short duration of each runoff event and the relative similarity of the runoff hydrographs. The observed hydrographs on Walnut Gulch are similar in that they are characterized by rapid rises and steep recessions. Furthermore, they exhibit a high degree of correlation between peak rate and volume of runoff primarily due to characteristics of the runoff-producing thunderstorm rainfall (Osborn and Hickok, 1968; Renard, 1970). Runoff events for all the basins studied usually last less than 6 hr and, in a great many cases, less than 4 hr. The time of rise is very short, 0.3–0.6 hr, and the first portion of the recession curve is also fairly steep (Renard and Keppel, 1966).

Each of the variables needed to describe the runoff season and runoff events has its own statistical distribution. These distributions must be specified to generate synthetic data. The correlations between variables and the serial correlations for all variables must also be known if they are significantly different from zero. The statistical distributions that gave the best fit for each variable and for all basins included in the study are listed in Table 2. The distributions were chosen using the Kolgomorov–Smirnov goodness of fit test as the objective method of accepting appropriate statistical models.

TABLE 2  
*Probability distributions adopted for runoff variables*

Runoff variable	Symbol used	Theoretical distribution	Parameters
Start of runoff season	<i>S</i>	Normal	Mean, standard deviation
No. of events at outlet per runoff season	<i>N</i>	Normal	Mean, standard deviation
Begin time of each event	<i>T</i>	Normal	Mean, standard deviation
Logarithm of volume of runoff for each event	<i>L</i>	Normal	Mean, standard deviation (of logarithms)
Interval between events	<i>D</i>	Exponential	Mean

Correlation between variables was significant (statistically) for only one pair of variables, the volume of runoff and the peak discharge of the runoff event. Therefore, the stochastic model generated only the runoff volume. The peak discharge was obtained from a regression equation expressing the peak discharge as a function of volume plus a normally distributed random component equal to the standard error of estimate of the regression equation. This technique allowed the preservation of both the means and standard deviations of the volume and peak rate as well as their correlation.

The meteorological factors that govern the occurrence of thunderstorm rainfall during the summer season in south-eastern Arizona are such that it is reasonable to assume that the beginning date and end date of the season are independent events. The spacing of storms throughout the season is also independent of the starting date so that it may be expected that the number of runoff events per season would decrease if the starting date is late. The relationship between the number of events per season and the starting date of the season was assumed to be a simple linear equation despite the nonsignificance of the statistical correlation between the two variables. These linear relationships, when incorporated into the generation scheme, improve the agreement between the synthetic data and the observed data.

Tests for serial correlations carried out during the study indicated the absence of such correlations for any of the variables considered. This result is consistent because the runoff events are produced by convective storms independent of each other.

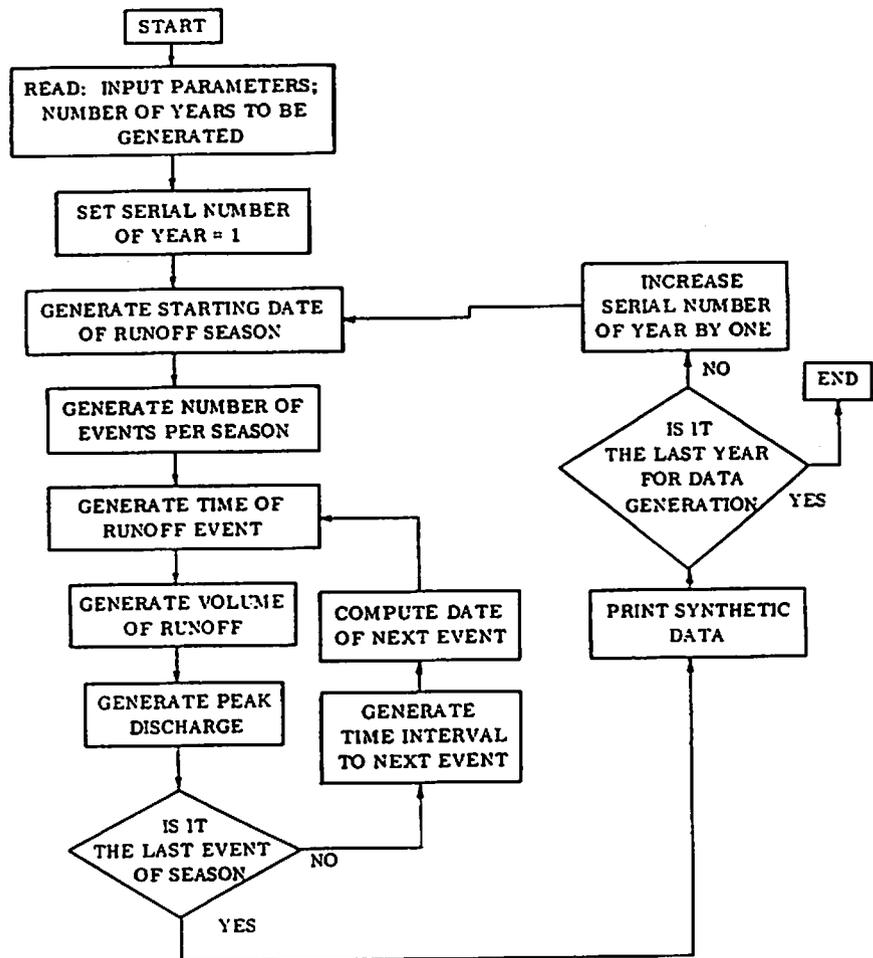


Fig. 3 — Flow chart of stochastic model for runoff events.

With the distributions and correlations discussed above for the variables needed to describe the runoff events, the model structure expressed by the procedure for generation of synthetic data, is outlined in Fig. 3. The flow chart gives the order in which the various variables are generated from their distributions, using the parameters of these distributions, or from the regression equations to other variables, previously generated. If a regression equation was used, a random component, assumed to be normally distributed, was added. The random component had zero mean and a variance equal to the variance about the regression line expressing the relationship between the two variables.

The synthetic data were produced by generating uniformly distributed random numbers and then transforming the values so that the desired distributions were obtained.

The procedure developed for converting the uniformly distributed numbers to numbers having a specified distribution was based on successive approximations to the number  $X$  for which the cumulative distribution function is

$$F(X) = U \quad (1)$$

where  $U$  is the uniformly distributed number. The cumulative normal distribution curve was initially approximated by 5 straight line segments. The correction applied with each successive approximation for the number  $X$  was computed by Newton's method.

#### INFLUENCE OF WATERSHED AREA ON PARAMETERS OF RUNOFF VARIABLES

Extension of the stochastic model for a single watershed to a regional model requires knowledge of the relationships between the parameter values of the model and the watershed physical characteristics. For the model discussed herein, the parameters involved are the mean and standard deviation of the independent variables and the regression equation constants and the standard deviations of residuals for the variables considered as dependent variables. The mean and standard deviation for all variables, both dependent and independent, are listed in Table 3 for the five watersheds used to derive the basinwide model. The only basin characteristic considered in the study was the areas of the basins as given in Table 1.

Values given in Table 3 are mean values for each basin. Individual values of the starting date of the runoff season ( $S$ ) ranged from June 21 ( $S = 172$ ) on watershed 63.001 to August 24 ( $S = 236$ ) on watershed 63.004 although the average beginning dates for these basins were July 18 and 28, respectively. Individual values of the number of events per season ( $N$ ) ranged from 2 to 21. Again, the smallest basin 63.004 had the fewest events in one year, and the largest basin 63.001 had the most events in one year. The interval between events, ( $D$ ) defined as the number of days between successive events within the runoff season, ranged from zero (two events on the same day) to 69 days.

Of the variables listed in Table 3, the time of the beginning of runoff ( $T$ ) appears to be independent of the size of the watershed. Runoff events in south-eastern Arizona tend to occur, because of local meteorological conditions, in late afternoon. Actual beginning time for individual runoff events throughout the day ranged from 0 to 2400 hours. The average starting time of about 1800 hours was obtained by adopting a day which started at 6 a.m. which was the hour with the least number of runoff events.

The variation of the mean values of the variables with the area of the watershed is shown in Figs. 4, 5, and 6. Figure 4(a) shows the relationship between the starting date of the runoff season and watershed area. The equation of the line of best fit is:

$$S = -5.45 \log_{10} A + 205.4 \quad (2)$$

where  $S$  is the average starting date of the runoff season counted in days from the beginning of the year and  $A$  is the area of the watershed in square miles. Figure 4(b) shows the relationship

TABLE 3  
Comparison of parameters of observed runoff data from five basins

Variable	Parameters	Watershed no.					Weighted mean
		63.001	63.006	63.008	63.011	63.004	
$S^*$	Mean	198.7	196.2	196.9	201.7	208.8	—
	Standard deviation	13.1	8.1	10.5	10.2	13.8	12.0
$N$	Mean	13.4	11.6	9.4	7.9	6.1	—
	Standard deviation	6.0	3.5	3.4	2.5	3.9	4.3
$T$ (hr)	Mean	18.4	17.8	18.0	18.2	17.8	18.0
	Standard deviation	4.0	4.2	4.0	4.1	4.0	4.0
$V$ (in.)	Mean	0.016	0.022	0.030	0.095	0.098	—
	Standard deviation	0.035	0.050	0.059	0.204	0.227	—
$L$	Mean	3.33	3.18	3.62	4.02	3.96	—
	Standard deviation	1.0	1.3	1.0	1.1	1.1	1.1
$P$ (cfs)	Mean	524	495	244	346	92	—
	Standard deviation	1260	1110	610	720	210	—
$D$ (days)	Mean	5.3	5.6	6.4	7.4	8.1	—
	Standard deviation	8.2	6.4	5.9	8.0	9.6	7.8

\* Variables defined in List of Symbols.

TABLE 4  
Regression parameters for correlated variables

Watershed no.	log $P$ vs. log $V$			$N$ vs. $S$		
	Correlation coefficient	Regression parameters		Correlation coefficient	Regression parameters	
	$r$	$K$	$n$	$r$	$C$	$B$
63.001	0.94	7620	0.66	-0.45	54	-0.20
63.006	0.95	5060	0.66	-0.53	56	-0.23
63.008	0.98	3990	0.83	-0.29	28	-0.09
63.011	0.98	3170	0.91	-0.56	36	-0.14
63.004	0.97	618	0.78	-0.48	34	-0.14

between the number of events per runoff season and watershed area. The equation of the line of best fit is

$$N = 3.82 \log_{10} A + 6.2 \quad (3)$$

where  $N$  is the average number of runoff events in the runoff season.

As stated above, better synthetic data were obtained when a relationship between number of events per season and the starting date were incorporated into the model instead of assuming the two variables to be independent. The adoption of such a relationship follows the assumption mentioned above that the spacing of events within the season is independent of the starting

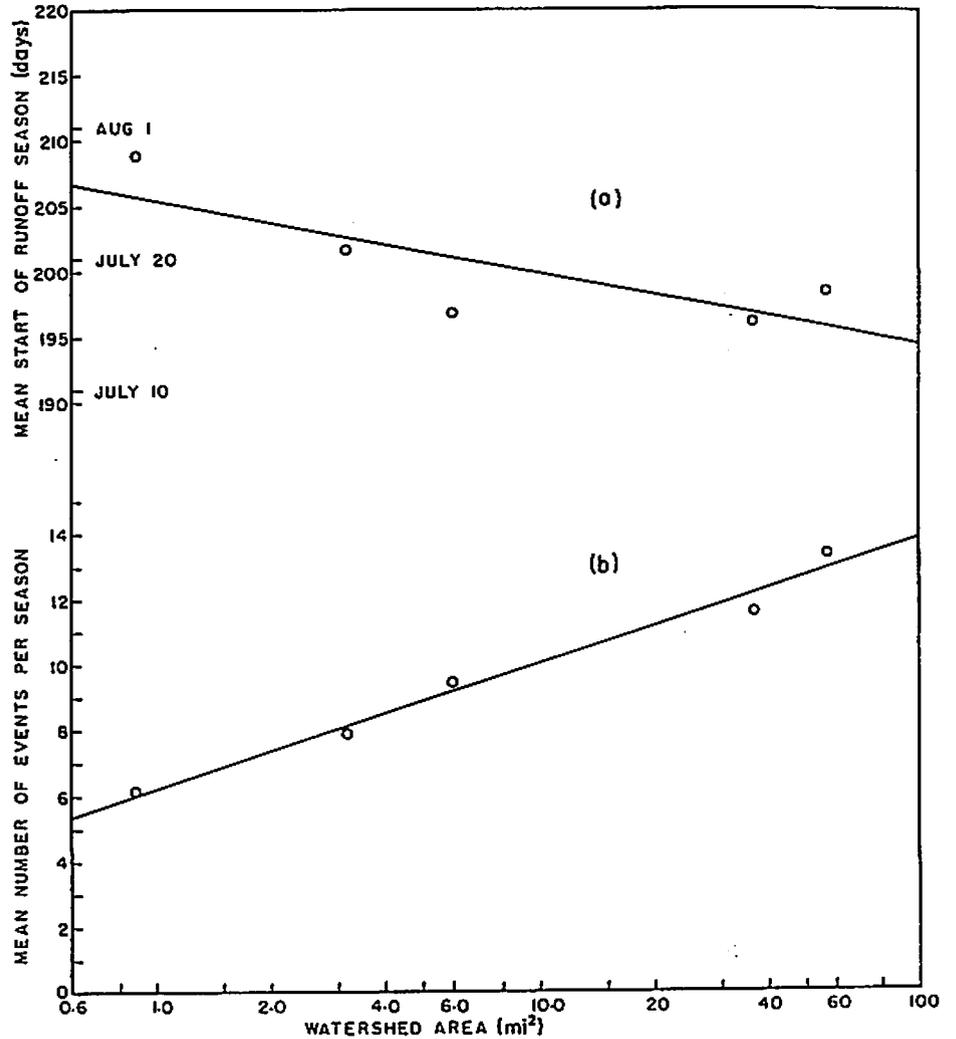


Fig. 4 — Relationship between (a) start of season and (b) number of events per season and the watershed area.

date. The relationship between the two variables was assumed to be in the form

$$N = BS + C \quad (4)$$

Values of the constants  $B$  and  $C$  for the various basins are given in Table 4. The constants were computed by a least squares method. The relationships represented by regression equations using these constants are, however, not significant statistically. Using the  $F$  test of significance gave  $F$  values smaller than those significant at the 5 per cent level which was adopted throughout

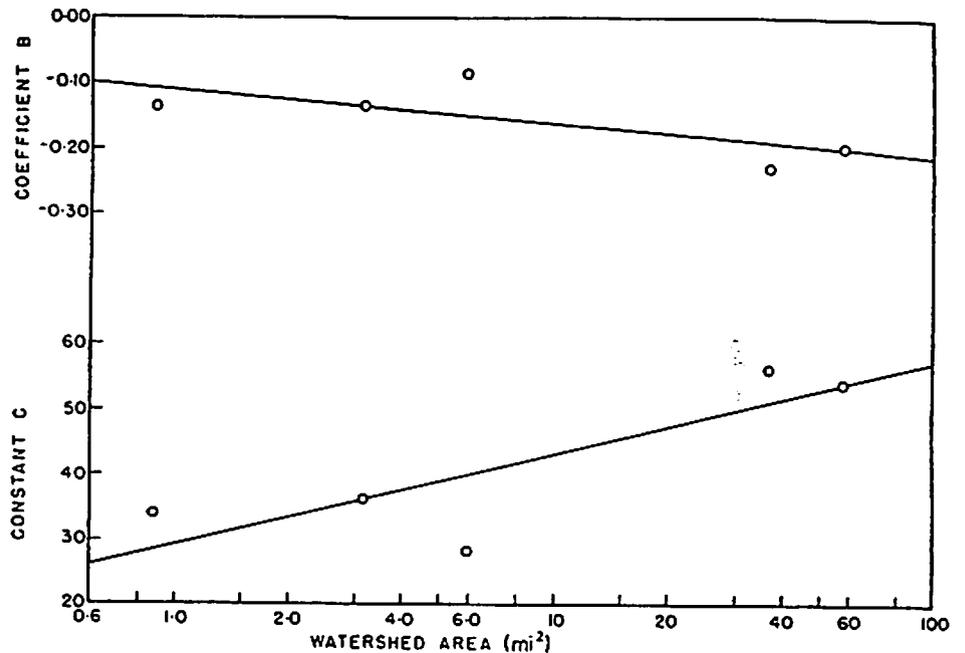


Fig. 5 — Constants in equation for number of events.

the study as the critical level for tests of significance. It should be noted that the values of the correlation coefficient ( $r$ ) given in Table 4 are *not* significant at the 5 per cent level. The lack of significance may, in part, be due to the small number of points used to derive the values of the constants. Values of the constants for ungauged basins in the region may be taken from Fig. 5. The standard error of values estimated by the regression equations for the different basins varied between 2.3 and 5.6 days. For ungauged basins, a mean value of 4.0 days may be taken.

The relationship between the mean interval between events and basin area is shown in Fig. 6(a). The equation of the line of best fit is

$$D = -1.55 \log_{10} A + 7.98 \quad (5)$$

where  $D$  is the average interval between events in days.

Figure 6(b) shows the relationship between the mean of the logarithmic transformation of the volume and the watershed area. The equation of the line of best fit is

$$L = -0.46 \log_{10} A + 4.03 \quad (6)$$

where  $L$  is the average value of the logarithm of the runoff volume in  $10^{-6}$ -in. units.

It was noted previously that the peak discharge  $P$  was highly correlated to the volume of runoff  $V$  in the events. The relationship between the two variables was expressed by the equation

$$P = KV^n \quad (7)$$

which is linear in the logarithmic form

$$\log P = \log K + n \log V \quad (8)$$

Values of  $K$  and  $n$  were derived by the least squares method applied to  $\log P$  and  $\log V$  as variables. The value of the constants listed in Table 4 are for values of  $P$  in cfs and values of  $V$  in inches. The relationship between these two constants and the size of the watershed is shown in Fig. 7. The root mean square deviation of values estimated by regression equation varied for different basins between 0.18 and 0.32 (in  $\log P$  units). A mean value of 0.23 (log units) may be taken as the standard deviation of the random component to be added to values given by the prediction equation for ungauged basins.

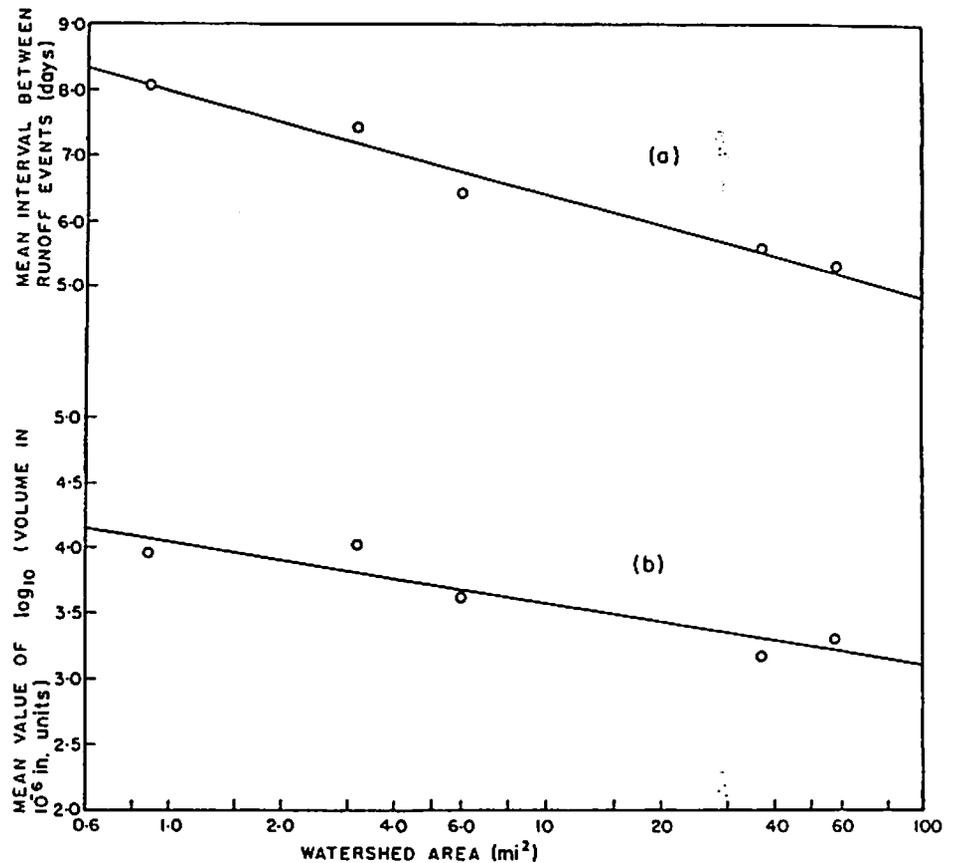


Fig. 6 — Relationship between (a) mean interval between runoff events and (b) mean value of log volume of runoff and watershed area.

The standard deviations of the variables entering the stochastic model did not appear to be a function of basin size. Weighted average values of the standard deviations are therefore recommended for ungauged basins. The weighted means of the standard deviations for the various

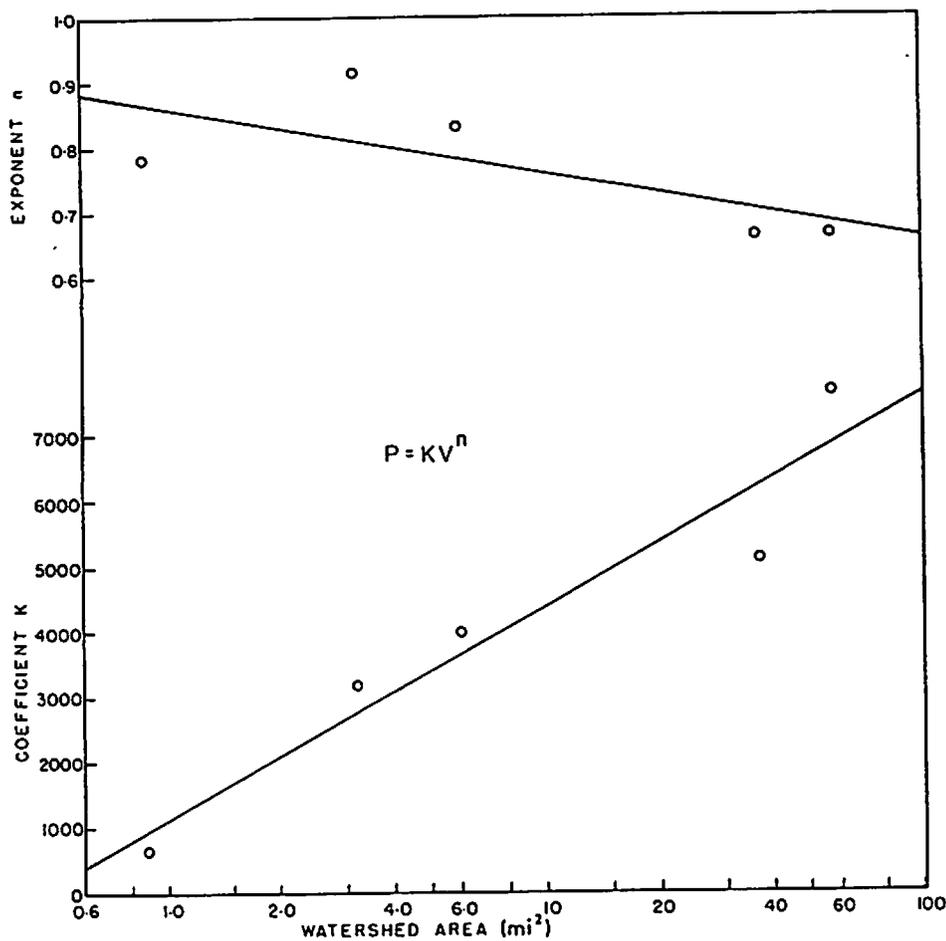


Fig. 7 — Regression constants for peak discharge-runoff volume equation.

variables are included in Table 3. Weights assigned in computing the weighted means and standard deviation were according to the total number of events (observations) on each watershed.

#### COMPARISON WITH OTHER BASINS

The relationships between the parameters of the stochastic model and the size of the watershed, obtained with data from five basins, was checked with data available for two other basins at Walnut Gulch. However, the test is not as objective as would be desired because the

additional basins form part of the set of basins used in deriving the relationships in the first place.

The two basins used in the comparisons are watersheds 63.007 and 63.015. The first is a sub-basin of watershed 63.001. The second is a sub-basin of both 63.001 and 63.006 (Fig. 1). These two basins do not contain in them any of the other basins used in the study. The areas

TABLE 5  
Predicted and observed runoff parameters for watersheds 63.007 and 63.015

Parameter	Values of parameters				Source of Predicted value
	Watershed 63.007		Watershed 63.015		
	Predicted	Observed	Predicted	Observed	
Mean start of runoff season ( <i>S</i> )	201.5	196.9	200.1	206.4	Equation 2
Standard deviation of start of season	12.0	10.7	12.0	14.5	Table 3
Coefficient <i>B</i> in equation for number of events	-0.15	*	-0.16	-0.17	Figure 5
Constant <i>C</i> in equation for number of events	39.0	*	43.0	41.0	Figure 5
Standard deviation of random component in number of events equation	4.0	*	4.0	1.3	—
Mean time of runoff events ( <i>T</i> )	18.0	16.6	18.0	16.8	Table 3
Standard deviation of time of runoff	4.0	3.5	4.0	4.6	Table 3
Mean interval between runoff events ( <i>D</i> )	6.9	9.3	6.5	10.6	Equation 5
Standard deviation of interval between events	7.8	9.9	7.8	11.9	Table 3
Mean logarithm of runoff event ( <i>L</i> )	3.70	3.53	3.59	3.57	Equation 6
Standard deviation of log volume	1.1	0.77	1.1	1.38	Table 3
Coefficient <i>K</i> in equation for peak discharge	3420	9370	4250	3680	Figure 7
Exponent <i>n</i> in equation for peak discharge	0.79	1.05	0.76	0.87	Figure 7
Standard deviation of random component in peak volume equation	0.23	0.16	0.23	0.18	—
Mean number of events per season ( <i>N</i> )	9.0	7.5	9.9	5.0	Figure 4
Standard deviation of number of events each season	4.3	2.4	4.3	2.7	Table 3

\* No regression equation could be derived for the short period of record available (four years).

of these two basins are 5.2 and 9.2 mi<sup>2</sup> respectively. Both basins have relatively short periods of records amounting to four years for watershed 63.007 and five years for watershed 63.015.

Values of the various parameters needed for the stochastic model were estimated for the two basins from their size, using the relationships derived previously. The predicted values are listed in Table 5 together with the values of the same parameters derived from observed runoff data for the two basins.

The agreements between the predicted and observed values are good for some parameters and fair to poor for others. Some of the observed parameter values are less than the predicted values for one of the basins and more than the predicted values for the second basin. Considering the short period of records, it may be concluded that parameter values predicted by the proposed regional model give a fairly good estimate of the parameters needed for generation of runoff data for ungauged basins in the region considered.

#### CONCLUSIONS

Synthetic runoff data for basins of less than 100 mi<sup>2</sup> in south-eastern Arizona and probably in other semi-arid regions, where runoff events are intermittent and independent of each other, may be generated with the aid of a stochastic model described herein. A further constraint on the model involves limiting it to areas where the runoff is primarily due to thunderstorm rainfall. The variables chosen to describe the runoff events are: start of runoff season, number of runoff events per season, time interval between events, beginning time of runoff event, volume of runoff, and peak discharge. Each of these variables is generated from its own distribution, but two were considered to be dependent variables and are generated from regression equations as functions of other independent variables.

The means and standard deviations of the various distributions and the constants in the regression equations form the set of parameters that define the stochastic model. Some of these parameters were expressed as a function of the area of the basin, and some were taken to be constants for the range of basin areas used in the study. The relationship adopted for those parameters varying with the area was of the type

$$R = M \log A + C$$

where  $R$  stands for the parameter,  $A$  is the basin area, and  $M$  and  $C$  are constants. The following observations, based on data for the periods of record shown in Table 1, summarize the relationships found for the means of the distributions.

1. The mean starting date of the summer runoff season is earlier with increasing watershed area up to 60 mi<sup>2</sup>.
2. The mean number of events per runoff season increases with increasing watershed size.
3. The mean time interval between runoff events during the season decreases with increasing watershed area.
4. The mean beginning time of runoff events during the day is essentially independent of watershed size but rather is controlled by meteorological conditions associated with convective thunderstorm development.
5. The mean logarithmic transformed volume of runoff from each event, measured in inches over the area of the watershed, decreases with increasing watershed area.

The standard deviations for the above variables were found to be fairly constant and independent of the area of the watershed. It is hoped that additional data will confirm the tentative relationships established for the means and enable the standard deviations to be related to watershed characteristics.

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