AREAL EFFECTIVE INFILTRATION DYNAMICS FOR RUNOFF
OF SMALL CATCHMENTS

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INTRODUCTION

Soils of natural catchments are characterized by spatially variable infiltration properties, often with significant differences over relatively short distances. Because of the highly nonlinear nature of the infiltration process, the areal mean value of infiltration parameters cannot describe the net performance of a catchment. (Woolhiser and Goodrich, 1988; Binley et al., 1989).

We studied the net effective time pattern of infiltration of a small catchment with randomly distributed infiltration rate, subject to a spatially uniform but time varying rainfall. Important spatial interactions may be observed if areas with high infiltration rates lie downstream of areas with low infiltration rates. We examined these effects, and compared several methods to treat runoff from variably infiltrating catchments. Our objective is to compare several different methods used to represent a catchment with spatially variable runoff, in light of the interaction of rainfall intensity, mean infiltration, and variability shown by previous investigators (Smith and Hebbert, 1979; Woolhiser and Goodrich, 1988).

METHODS

An experimental version of the runoff model KINEROS was employed for these studies (Woolhiser et al., 1990). Using this runoff model, the spatial variation of infiltration was treated in different ways, described below. KINEROS accurately reflects the surface hydraulics for cascaded networks of rectangular shaped surfaces, provided the criteria for kinematic hydraulics are met.

We assume that for any point on the area of interest the infiltration pattern is described by the analytically derived (Smith and Parlange, 1978) function

\[ f = K_s \exp(I/G)/[\exp(I/G)-1] \]  

where \( f \) is infiltration flux, \( I \) is water depth infiltrated since the beginning of the event, \( K_s \) is effective saturated hydraulic conductivity, and \( G \) is a parameter reflecting capillary properties and initial water content. This study focuses on the effect of spatial variations in \( K_s \), because it is the most sensitive infiltration parameter. The joint distribution of \( G \) and \( K_s \) is not considered, since Springer and Cundy (1987) showed for runoff from hillslopes with spatially-variable infiltration parameters that \( G \) could be set to mean values with essentially no effect if the variation in \( K_s \) is properly represented. Field measurements indicate that a lognormal distribution is acceptable for representing the spatial variability of \( K_s \) (Nielsen et al., 1973; Rogowski, 1972; Sharma et al., 1980).
Composite (non-interactive) Analysis. We can obtain analytic results if we assume that runoff from one area will not affect the effective infiltration at another. When a uniform rainfall rate, \( r \), falls on a surface with \( K_s \) distributed with density function \( p(K_s) \), part of the area will have \( K_s > r \) and will have no runoff, and part will have runoff defined by Eq. (1). The expected value of \( K_s \) over the area is denoted by \( K_e \), and is found as

\[
K_e = [1 - P_k(r)]r + P_k(r)E_r(K_s)
\]

(2)

where \( p_k \) is the cumulative distribution of \( K_s \), and \( E_r(K_s) \) is the expected value of \( K_s \) over the region \( r > K_s \). With \( p(K_s) \) representing the probability density function of \( K_s \), \( E_r(K_s) \) is found to be

\[
E_r(K_s) = \frac{\int_r^\infty p(K)KdK}{P_k(r)}
\]

(3)

If Eq. (1) applies at each point, we can represent the net dynamic infiltration curve for the area with a Latin hypercube method. In this method, the ensemble net value is found as the sum of \( n \) parts, each represented by a value of the distributed variable \( K_s \) which corresponds to the mean of a fraction \( 1/n \) of the cumulative distribution function. We define a scaled infiltration rate \( f_s = (f-K_e)/(r-K_e) \). Then from the numerical results we find an ensemble effective value for \( f_s \) to be

\[
f_s = \frac{f-K_e}{r-K_e} = \left[ 1 - \frac{(r-K_e)[\exp(I/G)-1]}{K_e} \right]^{-1/\alpha}
\]

(4)

in which \( \alpha \) is \( 1.5/c_v \), and \( c_v \) is the coefficient of variation of \( K_s \). \( c_v \) cannot be zero, but Eq. (1) is the limit of Eq. (4) as \( c_v \) approaches zero. Figure 1 shows this curve for several values of \( c_v \). Sivapalan and Wood (1986) obtain comparable results for uniform rain on variable soils using a modified version of Philip's infiltration relation. Some previous studies of variable runoff surfaces have taken local runoff directly to the stream with some time delay but without runon interaction (Binley, et al., 1988, and Freeze, 1980). Those simple routing methods are related to this composite analysis, with the addition that the composite function Eq. 4 is effectively convoluted by a uniform distribution representing the probability of a given point being a certain distance from the bottom of the runoff area. Eq. (4) is not theoretically suitable for unsteady rainfall rates, and does not include runon interaction, so we do not expect it to compare favorably with methods applicable to variable rain patterns on a surface.

A runoff simulation method for spatially variable \( K_s \) which treats variable rain rates but does not simulate runon interaction was used by Woolhiser and Goodrich (1988). The Latin hypercube division of the cumulative distribution is used on the runoff surface to divide the flow surface into \( n \) equal strips, each exhibiting a homogeneous subvalue of \( K_s \). Runoff from the surface is obtained by summing the runoff from each strip. This method, hereinafter referred to as W-G strips, effectively composites runoff after nonlinear surface routing, rather than at the point of production, as in Eq. (4). Using this method in a research version of KINEROS, we determined
that 10 strips is an adequate number to represent the distribution of $K_e$ and is within a few percent of the results using 50 strips. The conceptual weakness of the W-G strip model of a catchment surface is that it presumes homogeneity along flow paths, such that the lowest $K_e$ sites will be located along one path, and conversely all the high $K_e$ sites will be similarly situated.

Interactive Surface Runoff Simulation. In this model $K_e$ varies with the same distribution along the flow path as it does across flow paths. KINEROS was modified to allow specification of a lognormally distributed random $K_e$ for each of as many as 50 points along the length of surface flow, and using as many as 20 strips to represent separate flow portions of a catchment surface. The parallel strips are not required for representing a distribution, as for the W-G method, but reduces sampling statistical error. Provision was made to allow spatial correlation along the flow path if desired. The same variability of $K_e$ can be applied to this model as to the W-G model for comparison of a range of storms and relative values of storm intensity. We wished to see if the result of Woolhiser and Goodrich (1988), in which bias was inverted for relatively large storms as compared with small ones, was true for interactive model representations of variable $f$.

RESULTS

We did not examine enough variables to make general conclusions regarding treatment of variability in runoff modelling. Some of the biases of simpler methods can nevertheless be illustrated here. The following illustrations use two storms measured on the Walnut Gulch, Arizona experimental watershed.

Figure 2 shows comparison of hydrographs calculated using the three methods of treating variability. The catchments used here are hypothetical simple planes, divided into 10 strips for the W-G strip method, and into 10 strips of 30 points each for the runon model.

Compared to the interactive model simulation, the uniform model underestimates the runoff peak and the composite model overestimates it. Because runon can effectively accelerate ponding in high $K$ zones from upslope zones of lower $K$, the composite model cannot properly treat runoff from surfaces with distributed infiltration. Use of the W-G strip model may overemphasise the nonlinearity of the runoff production process, and overestimate the peak and runoff volume from the variable surface. The band of shading around the results from the interactive model in Figure 2 illustrates the spread of different results from 10 different realizations of the 600 points used on the surface. This shows that even with 20 strips, the sampling error for any random realization can effect the results when runoff is a small fraction of the rainfall.

Woolhiser and Goodrich (1988) demonstrated a "crossover" effect when hydrographs obtained by the W-G method were compared with uniform ($C_e=0$) surfaces. For some storms the uniform surface produced higher peaks than the W-G method for low mean infiltration rates, and lower peaks when the infiltration rate was increased. Figures 3 and 4 illustrate cases taken from Figure 12 of Woolhiser and Goodrich (1988). The catchment length is 91.4 m., and $K_e$ was 3.4 or 0.35 mm/hr. In these fig-
ures, c, is either 0 or 1 (in the earlier work, c, was 0, 0.4, or 0.8). Note that the interactive method does not exhibit the "crossover" effect. In all cases the interactive method leads to higher peak rates than the uniform surface. In Figures 3 and 4 the W-G method uses 10 strips instead of 5 in the earlier work, and some modifications have been made in the infiltration algorithm. Although the "crossover" effect is still apparent, it is considerably reduced compared to the effect demonstrated in the earlier paper.

Another factor in the relation between spatial variability and runoff is the ratio of the surface length to the soil correlation length. The spatial interaction of runoff on a variably infiltrating plane serves to attenuate the differences in infiltration between points. Attenuation of peaks in slope hydrographs is also effected by plane length, so an interaction of plane length and variability would be expected. Further, the longer surface will exhibit a greater number of sample infiltrating points, given a constant soil correlation length, and thus have a greater probability of interactive attenuation. Figure 5 shows hydrographs obtained from uniform surfaces of 30.5 and 91.4 m. length, as well as variable infiltrating surfaces of the same length, using storm 4. These tests maintained an effective correlation length for the surface soil of 4.5 m, consistent with published values (Wagenet, 1981). Hydrograph peaks are considerably greater for the short planes, in either case. Addition of spatial variability, however, causes greater changes in hydrograph shape and peak location for the short planes than for the long ones.

CONCLUSIONS

We have extended somewhat our understanding of the role that spatially variable infiltration plays in watershed response, and its relation to slope length, and ratio of storm intensity to infiltration capacity. Earlier results, showing that effects of variability are greater for relatively lower intensity storms are confirmed by this work. Using a non-interactive runoff model Xs gives results considerably different from non-interacting models. Length of runoff surface interacts with correlation scale of Ks in determining the effect of variability on the hydrograph for a given storm. These results suggest the study of natural distributions of lengths of surface flow prior to flow concentration, as related to correlation scales of infiltration properties. Flow concentration, such as in small surface rills, should limit the runon type of interaction, and determine the extent to which catchment runoff is influenced by the interactive effects demonstrated here. We plan to pursue studies of such microtopographic influences, hoping to be able to characterise the significant variables involved in the role that small scale variability plays in shaping surface hydrology.
REFERENCES


Figure 1. Example curves of composite infiltration for lognormally distributed \( K_s \). \( C_V \) of \( K_s \) varies from 0. to 1.
Fig. 2. Comparison of methods to simulate spatially variable infiltration in surface runoff.

Fig. 3. Effect of spatial variation for the Woolhiser-Goodrich (1988) case, with lower mean $K_s$.

Fig. 4. Same storm as Fig. 3, except that $K_s$ is larger, and runoff is more sensitive.

Fig. 5. Comparative effect of slope length and spatial variation on simple surface runoff.