INTRODUCTION

The magnitude, frequency, duration, and time of occurrence of rainfall on small watersheds and the consequent runoff are of interest to many. In particular, the magnitudes of flood peaks that may be expected with varying probability are often the characteristic of concern to the designer. For small watersheds, especially in the southwest, the runoff-producing rain giving rise to "floods" is the thunderstorm. In this paper, peak discharges from thunderstorm rainfall for small watersheds (100 sq miles (260 km²) and less) in southeastern Arizona are investigated, and the applicability of the results for design purposes is considered.

CURRENT PRACTICES IN SMALL WATERSHED RUNOFF DESIGN

Very Small Watersheds.—Designers today may use any of several methods to determine flood peaks and frequencies. For predicting peak discharge from very small watersheds, the "rational formula" is still the most widely used design equation. Linsley, Kohler, and Paulus (10) pointed out that the rational formula has the advantage of simplicity and its physical meaning is reasonably clear. The rational method is indeed rational if the watershed is small enough so that the rain is fairly uniform over the entire area and lasts long enough so that the runoff rate is equal to the rainfall excess. Since these restrictive conditions are seldom met, judgments in choosing the C values must involve many factors, few of which are simple and straightforward. However, many...
of the same arguments hold for most prediction methods, and the confidence that can be placed in the predictions is dependent on both the amount and the immediacy of actual measurements available for verification.

Chow (2) listed over 100 equations for determining peak discharges for very small drainage basins and commented on their differences, similarities, and uses. About 25 of the equations were essentially in the form of the rational formula, while the remainder were largely empirical and were developed for specific situations. The usefulness of any of these methods is directly proportional to the amount of local data that has been used to verify a particular method—and usually this amount of data is inadequate.

Small Watersheds [About 3 sq miles to 100 sq miles (7.8 km$^2$ to 260 km$^2$)]—Several Federal agencies have developed methods appropriate to their problems and available data. For example, there are two Soil Conservation Service methods—one for watersheds greater than about 2,000 acres, the other for those less than 2,000 acres (7.8 km$^2$). The SCS methods for estimating volume and peak discharges on small watersheds are largely dependent on a determination of the relationship between rainfall and rainfall excess for various soil types and precipitation distributions. Details of these methods can be obtained from the SCS Hydrology Handbook (6,20).

Regional flood frequency curves have been developed by the U.S. Geological Survey for areas of greater than 50 sq miles (130 km$^2$) (6,16,20). Patterson and Somers presented this method for the Colorado River Basin which includes drainages in much of the southwest (16). The USGS procedure combines data for many stations by the regionalization concept. First, the mean annual flood (MAF) is found graphically as a function of watershed size, hydrologic region, and other factors. Then, the regional magnitude-frequency relationship, normalized on the MAF, is also determined graphically. Since the base data are generally for larger watersheds, the user is usually cautioned against extrapolating the curves to smaller areas.

For small watershed runoff design, a technique that is often used to predict peak discharges on watersheds with a known runoff record is to extrapolate the peak discharge-frequency curve as plotted from one of several statistical distributions or plotting position formulas. The log-Pearson Type III distribution has been recommended for standard use by all Federal agencies, but several other distributions and plotting position formulas are also widely used. Most drainage basins with any length of record are large, and the statistical distributions that have been suggested to fit flood data have been used primarily with these large watersheds.

Comparison of Methods.—Eight methods, including the SCS method for larger watersheds, were compared by predicting flood peaks for 10 yr, 25 yr, 50 yr, and 100 yr frequencies for the 58-sq mile (150 km$^2$) Walnut Gulch watershed in southeastern Arizona (Table 1). The predictions based on 15 yr of runoff data from Walnut Gulch, for a recurrence interval of 100 yr, range from 12,500 cfs (350 m$^3$/s) by the USGS graphical method to 35,000 cfs (990 m$^3$/s) according to the Log-Gumbel distribution. The average 100-yr flood peak for the eight methods is 18,500 cfs (520 m$^3$/s).

Subjectively, a designer might decide that since the Log-Pearson Type III estimate and the average were about the same, an estimate of 20,000 cfs (570 m$^3$/s) looks like a pretty good design figure. However, the possible error in
assuming 20,000 cfs (570 m³/s) could be easily as much as 50%, and the actual frequency of a 100-yr peak discharge based on 15 yr of good record within 65% confidence limits is somewhere between 25 yr and 300 yr. Therefore, although using 15 yr of record to predict the 10-yr storm may be reasonable, using the same record to predict more and more unlikely events is less and less reasonable.

Limits.—Cost-benefit analyses are an important part of any engineering project and usually determine whether the project will be considered. When such analyses are based on predictions that may include large uncertainties and errors, the definition of the risk associated with rare floods becomes far from simple. Furthermore, the question of a “larger-than-100-yr” flood must often be considered since a 100-yr flood has an excellent chance of occurrence in say a 50-yr life. Although even an estimate of the 100-yr flood may be beyond the credible

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Note: 1 cfs = 0.0283 m³/s.

limits of the available data, the statistical distributions in use can all be extrapolated to predict larger and larger flood peaks for rarer and rarer floods.

Therefore, the perplexing issue of the economic analysis of a project based on risk, uncertainty, and error is probably the strongest incentive for devising a means to predict rare floods independent of the extrapolation of measured data for a “short” period at the project site.

The most common “other way” is to examine known rainfall data for a region or area to determine what conditions will produce an extreme event, or to search runoff records for a region to determine what has occurred within the known record. The use of records from elsewhere is not without uncertainty, but the final product is a limit which hopefully has zero probability of occurrence, however uncertain the value of the limit may be.

Among all the uncertainties in the art and science of hydrology, there is one certainty—neither the rainfall intensity nor the rate of runoff can possibly
be infinite. No matter how uncertain the estimate of the limiting finite flood is, there is no uncertainty that there is a finite limit. The question is not whether there are natural limits, but how, or if, one can use this knowledge to estimate limits for specific cases.

Wilson (21), in response to a paper by Yevjevich (22), stated:

"It is believed that no one has succeeded in assigning meaningful probabilities to transposed rare events. This possibility should perhaps be investigated, but, in the meantime, it seems reasonable to examine the record of storms and other information with which to decide that this storm could happen here while there is no evidence that a much larger storm can occur."

Without rainfall data, a search for an estimate of the "rare event" for a watershed or region must be based on available runoff data or a hydrometeorological study based on possible precipitable water. However, if the volume and duration of rainfall from, or for, an exceptional event can be routed through a watershed or directly correlated to peak discharge from a watershed, then perhaps the rare, or limiting, flood event can be estimated more accurately.

**Walnut Gulch**

**Watershed Characteristics.**—Walnut Gulch is an ephemeral Stream located in the San Pedro River Drainage in southeastern Arizona. The Southwest Watershed Research Center of the Agricultural Research Service established a runoff-measuring station in 1954 on Walnut Gulch about 2 miles above the confluence of Walnut Gulch and the San Pedro River (Fig. 1). Hereafter, the drainage area above this station [58 sq miles (150 km²)] is referred to as the Walnut Gulch watershed, or simply Walnut Gulch. Other stations were established on the principal channel system of Walnut Gulch after 1954, with 11 stations now in operation.

Walnut Gulch is similar to much of the brush-grass rangeland in the southwest. The lower two-thirds is largely brush covered; the upper one-third largely native grasses. Almost all of the watershed is grazed year round, and there is no irrigated or dry farming. The watershed ranges in elevation from just under 4,000 ft (1,200 m) to slightly over 6,000 ft (1,800 m), MSL. The channels abstract large volumes of surface runoff from the relatively few events occurring on summer afternoons and evenings during the thunderstorm season. The instrumentation and makeup of Walnut Gulch is described elsewhere (17).

**Data.**—Fair to good runoff and rainfall records are available from 1955 to the present for Walnut Gulch as a whole and for subwatershed 5 [8.5 sq miles (22 km²)]. Good runoff and rainfall records have been collected on subwatershed 6 [37 sq miles (96 km²)] since 1962. At present there are 95 recording rain gages on or immediately adjacent to Walnut Gulch.

**Rainfall Model.**—Quantitative analysis of the thunderstorm rainfall-runoff relationship is difficult for several reasons. For one thing, rainfall is not uniform in time or space, and rainfall input can only be estimated from rainfall measurements within certain limits of accuracy and precision. Furthermore, peak discharge does not increase proportionally to rainfall intensity. Also, channel ab-
stractions may account for much or all of the initial onsite runoff. For example, average annual thunderstorm runoff from the Walnut Gulch watershed is only about 5% of the total thunderstorm rainfall, although for the largest events it may be over 25% of the total.

The more intense, longer lasting thunderstorms have a well defined core of runoff-producing rainfall. The correlation between this core of rainfall and peak discharge improves with increasing magnitude of the core, primarily because channel abstractions do not increase at the same rate, and therefore the dif-

![Figure 1](image1.png)

**FIG. 1.**—Location of Walnut Gulch Experimental Watershed Near Tombstone, Ariz.

ferences in abstractions for comparable events become less important. Runoff-producing rainfall generally lasts less than 30 min at any one point and seldom exceeds 40 min. Furthermore, runoff-producing rainfall over watersheds the size of Walnut Gulch generally lasts less than one hour and seldom exceeds 70 min. The importance of the relationship between the core of runoff-producing rainfall and peak discharge allows for simplification of an otherwise complex relationship.

The direction that the storm moves, or propagates, and the location of the
storm center should affect both the magnitude and time of peak discharge. However, observations of real events indicate that storm direction and location (assuming that the storm is largely contained within the watershed) may not significantly affect peak discharges from the larger runoff-producing events. The greatest abstractions occur with the advancing flood front, and wave fronts of later contributions move more rapidly through the already wetted channel. Contributions from subwatersheds during the same storm tend to accumulate as one peak at the watershed outlet. If the storm moves downstream with the flood, the peaks from the several tributaries coalesce. If the storm moves upstream the flows from the upstream tributaries tend to catch up to the flows from the downstream tributaries and in effect tend to coalesce at the watershed outlet. Thus, within the error of the field measurements the magnitude of the flood peak seems to be relatively unaffected by the order in which runoff from each subwatershed enters the main channel.

The volume of maximum 30-min rainfall was chosen as the value that best represented the core of runoff-producing rainfall for correlating major rainfall-runoff events on Walnut Gulch. Maximum 30-min rainfall was arbitrarily divided into estimates of volume between 0.25 in. and 0.75 in. (6 mm and 19 mm), 0.75 in. and 1.25 in. (19 mm and 32 mm), 1.25 in. and 1.75 in. (32 mm and 45 mm), and so on, which were designated $V_{0.25}$, $V_{10}$, $V_{15}$, and so on, respectively. These estimates were determined from isohyetal maps of maximum 30 min storm rainfall sampled by the Walnut Gulch precipitation network.

Rainfall-Runoff Correlation.—All events with peak discharges of 700 cfs (20 m³/s) or greater for Walnut Gulch and 300 cfs (8.5 m³/s) or greater for subwatershed 5 were used in the analysis. For Walnut Gulch, there were 30 peak discharges of more than 700 cfs (20 m³/s) with a maximum recorded peak discharge of 11,500 cfs (330 m³/s). For subwatershed 5 there were 23 peak discharges of more than 300 cfs (8.5 m³/s) with a maximum recorded peak discharge of 5,300 cfs (150 m³/s). The means of the major peak discharges for Walnut Gulch and subwatershed 5 were 1,100 cfs (31 m³/s) and 900 cfs (26 m³/s).

Because of previous success (12,19) a stepwise multiple linear regression program (MLR) was used initially to correlate rainfall and runoff variables. The dependent variable was peak discharge, in cubic feet per second per acre ($Q_a$). Maximum 30-min rainfall was the precipitation input.

Several watershed variables (distance from storm center, distance from storm edge, channel length, and antecedent rainfall) were initially entered into the program, but only an index of antecedent channel condition (ARI) which was added later, appeared to add significantly to the basic rainfall-runoff regression relationship. The ARI was based on antecedent streamflow, estimates of channel abstraction, and potential losses from the channel alluvium through deep percolation and evapotranspiration. A detailed explanation and justification for the ARI on Walnut Gulch was presented by Osborn and Renard (14).

For the Walnut Gulch watershed, the regression equation with significant variables was:

$$Q_a = 0.019 + 0.095 V_{20} + 0.010 V_{10} + 0.007 \text{(ARI)}$$

with $R^2 = 0.78$ and SEE = 0.035 cfs/acre (0.24 m³/s/km²).

For subwatershed 5, the regression equation was
with \( R^2 = 0.92 \) and \( \text{SEE} = 0.085 \) cfs/acre \((0.59 \text{ m}^3/\text{s/km}^2)\). The poorer correlation for Walnut Gulch as compared to subwatershed 5 was probably because none of the runoff-producing storms covered the entire watershed (many covered less than half), and because of greater channel abstractions in the principal channels compared to abstractions in the smaller channels of subwatershed 5.

Eqs. 1 and 2 were modified by trial and error for Walnut Gulch and subwatershed 5 to include the antecedent channel condition as a multiplier rather than a regression variable, which is more reasonable physically, and then a similar equation was developed for subwatershed 6. The equations for the watersheds in order of decreasing size (Walnut Gulch, subwatershed 6, and subwatershed 5) were:

\[
Q_a = (0.02 + 0.010 V_{10} + 0.010 V_{15} + 0.03 V_{20} + 0.15 V_{25}) (R) \quad (3)
\]

\[
Q_a = (0.03 + 0.015 V_{10} + 0.020 V_{15} + 0.05 V_{20} + 0.25 V_{25}) (R) \quad (4)
\]

\[
Q_a = (0.05 + 0.10 V_{10} + 0.15 V_{15} + 0.40 V_{20} + 0.60 V_{25}) (R) \quad (5)
\]

in which \( R \) is proportional to ARI and equals one under dry antecedent conditions \((R > 1)\). The predicted versus actual (measured) peak discharges for the three watersheds including \( \text{SEE} \) and \( R^2 \) are shown in Fig. 2.

Increasing coefficients for decreasing watershed size can be rationalized because of limited channel abstractions and a shorter time base for the flow with decreasing watershed size. The shorter time base could mean that a higher-intensity, shorter-duration core should be used for the smaller watersheds. However, the precision of the data was not sufficient to isolate this possible factor.

The equations and available data suggest that there are greater differences in rainfall-runoff characteristics between the 8.5-sq mile \((22-\text{km}^2)\) and 37-sq mile \((96-\text{km}^2)\) watersheds than between the 37-sq mile \((96-\text{km}^2)\) and the 58-sq mile \((150-\text{km}^2)\) watersheds. Unfortunately, only short records are available on Walnut Gulch for watersheds between 8.5 sq miles and 37 sq miles \((22 \text{ km}^2 \text{ and 96 km}^2)\).

Maximum Peak Discharge

**Maximum Expected Peak Discharge, Walnut Gulch.**—An apt descriptive term was sought for the occasional rare flood that occurs, or could occur, and that is not likely to be much exceeded. Such terms as "probable" and "possible" are used, but these terms already have specific meaning to different groups and persons. Therefore, the phrase "maximum expected peak discharge" was chosen as being a more appropriate description of this practicable limit. When the flood magnitude is based on known rainfall and runoff records in southern Arizona, it is termed "maximum expected peak discharge (WG)," where "WG" stands for Walnut Gulch.

All the storms on Walnut Gulch in which more than 2.0 in. \((51 \text{ mm})\) of rain was recorded in 30 min were investigated in developing a model of the exceptional or rare event as estimated by the maximum 30-min core of runoff-producing rainfall. Five storms that appeared well-centered within the watershed were used in determining the shape of the 30-min core, and point rainfall measurements throughout Arizona and New Mexico were investigated to estimate the
maximum center depth of a maximum 30-min rainfall.

There was similarity in the average cross section for the five major Walnut Gulch storms used in developing the thunderstorm rainfall model (Fig. 3). From these five storms, an elliptical rainfall model was developed with major and minor axis 1-1/2 to 1 (Fig. 4).

The maximum recorded 30-min rainfall in Arizona was 2.65 in. (67 mm) on Walnut Gulch on August 17, 1957. Over 2.5 in. (64 mm) of rainfall in 30 min has been recorded during two other events on Walnut Gulch in 15 yr of record.
Although the network of recording rain gages on Walnut Gulch is relatively dense, maximum recorded storm depths may be less than actual storm depths. However, comparisons of depths for the closest spaced gages on Walnut Gulch indicate that actual depths could only be slightly higher, with a maximum of about 15% higher. Therefore, it was assumed that 3.0 in. (76 mm) which is 15% higher than anything recorded to date, is the maximum 30-min depth of rainfall for a rare event in southeastern Arizona. As further indication, a search of U.S. Weather Bureau and other records and releases failed to yield any other record of more than 2.5 in. (64 mm) in 30 min in Arizona.

The maximum expected peak discharges (WG) for Walnut Gulch and subwatershed 5 were determined from: (1) A combination of known rainfall records in Arizona; (2) the model for maximum expected rainfall which was developed from Walnut Gulch data; and (3) the rainfall-runoff relationships also derived from Walnut Gulch records. The storm was centered on the watersheds to give the maximum effect. The equations for Walnut Gulch and subwatershed 5 were the same as Eqs. 3 and 5, respectively, except that a term for $V_{30}$ was added and $R$ was set equal to one. There are meteorological and probabilistic arguments why very intense, relatively long lasting thunderstorms occur following relatively hot dry periods, suggesting that for the extremely rare event, $R$ would more likely equal one, or nearly one, than a significantly higher value. The equations were:

$$Q_a = 0.02 + 0.01V_{10} + 0.01V_{15} + 0.03V_{20} + 0.15V_{25} + 0.30V_{30} \ldots \ldots \ldots \ldots (6)$$

and

$$Q_a = 0.05 + 0.10V_{10} + 0.15V_{15} + 0.40V_{20} + 0.60V_{25} + 0.80V_{30} \ldots \ldots \ldots (7)$$

for Walnut Gulch and subwatershed 5, respectively.

From these equations, the maximum expected peak discharge (WG) for the 58-sq mile and 8.5-sq mile (150-km$^2$ and 22-km$^2$) watersheds were 23,000 cfs (650 m$^3$/s) and 17,000 cfs (480 m$^3$/s), respectively.

The surprisingly small differences between the floods for these two watersheds deserves comment. Although Walnut Gulch is seven times the area of subwatershed 5 and the total storm covered most of Walnut Gulch, the expected maximum flood peak was only 35% higher. This is because the most intense portion of the core of runoff-producing rainfall was not much larger than the subwatershed, and because channel abstractions were greater in the larger watershed.

**Maximum Expected Peak Discharge, Southwest.**—The maximum expected peak discharge (SW) is defined as that discharge which could occur on Walnut Gulch from a greater 30-min rainfall, as determined from records from New Mexico as well, than would be expected based on Arizona records alone. The greater rainfall would be unlikely but could not be considered impossible.

The maximum known 30-min rainfall recorded on a recording rain gage in the southwest was 3.5 in. (89 mm) on the USDA Alamogordo Creek watershed in northeastern New Mexico. The storm was a frontal-convective type which tends to be more intense east of the Continental Divide (15). No storm with more than 3.0 in. (76 mm) of rainfall in 30 min has occurred at U.S. Weather Bureau recording rain gages in New Mexico. Several rainfall events of greater magnitude than those on Walnut Gulch, including four storms of greater than 3.0 in. (76 mm) in 30 min, were recorded on Alamogordo Creek during the
same period of record (18). Therefore, 3.5 in. (89 mm) in 30 min was chosen as the maximum depth for the storm that could produce the maximum expected peak discharge (SW). The maximum 30-min rainfall models for 3.0 in. and 3.5 in. (76 mm and 89 mm) are shown in Fig. 5.

**FIG. 5.—Comparison of 3-in. and 3.5-in. Rains for Walnut Gulch Model**

Using Eqs. 6 and 7 and allowing for a 25% reduction in peak discharge for appreciable overbank flooding above 20,000 cfs (570 m³/s) for Walnut Gulch and about 14,000 cfs (400 m³/s) for subwatershed 5, the maximum expected
peak discharge (SW) was 40,000 cfs (1,100 m³/s) and 22,000 cfs (620 m³/s) for Walnut Gulch and subwatershed 5, respectively. These values are higher than those resulting from the maximum expected Walnut Gulch storm. Certainly the SW storm approaches Wilson's criterion that "this storm could happen here while there is no evidence that a much larger storm can occur" (21). For the southwest, possibly west of the Continental Divide, and as far west as central Arizona, the SW storm may be too large; east of the Continental Divide it is more likely because of the possibility of greater moisture aloft and greater frontal activity (11).

**Recurrence Intervals.**—For the 58-sq mile and 8.5-sq mile (150-km² and 22-km²) watersheds, maximum annual peak discharges versus recurrence intervals were plotted, and maximum expected peak discharges (WG) were indicated (Fig. 6). Smooth curves based on the plotted points and the limits indicated by the maximum expected peak discharges were then constructed by eye. The shape of these magnitude-frequency curves suggests that most engineering hydrology problems should fall in one of two categories. The first would be designs based on frequent floods—those with expectancies of about 10 yr or less—with a check on what might happen with a larger, rarer, flood. The second would be designs for which loss due to failure is relatively great (7)—unacceptable—in which case the maximum expected floods (SW and WG) could be used.

Grove (5) and Lewis (9) published reports on flood peaks in Arizona. They produced a family of curves based on the familiar relationship of $Q$ versus $\sqrt{A}$, in which $Q = C\sqrt{A}$, $A$ = area, $C$ = coefficient, and showed where various flood peaks in Arizona plotted within this relationship. In Fig. 7, these curves have been duplicated, with the maximum expected peak discharges and the estimated 10-yr, 20-yr, 50-yr, and 100-yr storms from Walnut Gulch drawn in. The estimate for the maximum expected peak discharge (WG) for watersheds
of less than 100 sq miles (260 km²), encompasses all but one of the "record" events for small watersheds in Arizona. The one higher peak was a miscellaneous indirect USGS flood observation near Yuma, Ariz., about 250 miles (400 km) west of Walnut Gulch and only 60 miles (96 km) from the Gulf of California. This indirect observation is within the maximum expected peak discharge (SW) curve and about 50% higher than the (WG) curve.

Peak discharge per unit area decreased more rapidly with increasing watershed size for the family of curves based on Walnut Gulch data than for the family of curves based on the Q versus √A relationship. This suggests that there may be two families of curves; one for the small watersheds where flood peaks result from air-mass thunderstorms, and the other for large watersheds where snowmelt or frontal convective storms produce the major flood peaks. The curves probably intersect between 100 sq miles and 1,000 sq miles (260 km² and 2,600 km²). For runoff design for intermediate sized watersheds [between 100 sq miles and 1,000 sq miles (260 km² and 2,600 km²)] two probability estimates may be needed—the probability of storms of lesser intensities falling on most of the watershed, and the probability of more intense storms developing over several tributaries of the watershed in such patterns as to produce "record" peak discharges.

**Risk, Uncertainty, and Error**

**Uncertainty of Recurrence Interval.**—There are essentially two independent kinds of risk involved in the factors of hydrologic design. In the first, the assumption is that the recurrence intervals for various magnitudes of flow have been correctly determined, but that the design storm may occur in any year. The engineer must decide what losses will occur if the design flood is exceeded by various amounts, and assess how many times these greater flows will probably occur during the life of the structure (8).

There is also an element of risk because of the uncertainty in the estimated recurrence interval for a given flood discharge (or in the estimated discharge for a given recurrence interval). As Bell (1) pointed out, the actual recurrence interval for a 100-yr event may range from well under 50 yr to several hundred years, depending on the length of record. For example, there were only 15 yr of Walnut Gulch data from which to make these analyses, so statistically, for the estimated 100-yr event, within 65% confidence limits, the real recurrence interval would be between 25 yr and 300 yr.

Finally, recurrence intervals are determined from data that may contain errors. For example, the maximum recorded peak discharge from Walnut Gulch for 15 yr of record was 11,500 cfs (330 m³/s). This peak was estimated to be within 15% of the true value, or the actual maximum peak discharge was estimated to be no greater than 13,000 cfs (370 m³/s) and no less than 10,000 cfs (280 m³/s). The possible range in the recurrence interval, from Fig. 5, would then be about 10 yr to 22 yr. The probable range for the 15-yr flood within 95% confidence limits assuming a normal distribution of error would be 9,500 cfs to 13,500 cfs (270 m³/s to 380 m³/s). Without the assumption of normality, the range would be 7,500 cfs to 15,500 cfs (220 m³/s to 440 m³/s).

Furthermore, as pointed out earlier, predictions of peak discharge for the 100-yr flood on Walnut Gulch range from 12,000 cfs to 35,000 cfs (340 m³/s...
to 990 m$^3$/s) depending upon the assumed frequency distribution (Table 1). Assuming the worst possible distribution of errors for each method within the 95% confidence limits (about a plus or minus 33%) the actual range would be 8,000 cfs to 47,000 cfs (230 m$^3$/s to 1,330 m$^3$/s). Thus, knowledge of the correct distribution would appear to be important because the range of the estimate is greater for different distributions than for the probable errors in the measured data.

If a limit can be set on the maximum peak discharge, the possible range of values for floods of given recurrence intervals is also constrained. More important, if the limit forces the curve of peak discharge versus recurrence interval to bend abruptly with an asymptotic approach to the limit, the need for estimates of say the 50-yr or 100-yr flood will be reduced—the cost of designing for the limit will generally be very little greater than designing for the 50-yr event.

Error of Estimate.—Since a major conclusion of this study includes estimates of maximum peak discharge, the possible range of error of these estimates must be considered. The errors are generally errors in the data or errors in the models. Accuracy of the regression equations depends largely on accuracy of estimates of rainfall volumes and peak discharge for a few extreme events, and on the accuracy with which the model represents the true physical processes.

Errors in measuring peak discharge, particularly the few major peaks, are directly transmitted to the rainfall-runoff equation. The effect of errors in estimating rainfall volumes is more subtle. The fraction of rainfall volume contributing to peak discharge increases with increased rainfall depth, but the volume of rainfall above each succeedingly higher depth decreases. Also, because the volumes are based on records from a dense network of rain gages, errors in rainfall volumes between and within each 0.5-in. (13-mm) level are not apt to be cumulative, and are more likely compensating.

Methods have been suggested (3,4) for determining variances for predicted values from regression equations of the form

$$Y = k + C_1X_1 + C_2X_2 + \ldots + C_nX_n$$

These methods employ least squares fitting of the sample data. The methods generally indicate the variance of the mean of the predicted value, since this statistic is usually desired by statisticians. Hydrologists and engineers, however, are often interested in the variance of an individual predicted value.

As an estimate of possible errors in the predicted maximum peak discharge for Walnut Gulch, the variance of the mean predicted value was estimated, using matrix methods described by David and Neyman (3); then the variance about the regression line, calculated as the sum of squares of residuals over the residual degrees-of-freedom, was used to convert the variance of the mean predicted value to the variance for a single predicted value (13). For the Walnut Gulch watershed, assuming normality and 95% confidence limits

$$Y = \frac{Q}{A} = 0.62 \pm 0.11 \text{ cfs/acre (4.3} \pm 0.77 \text{ m}^3/\text{s/km}^2), \text{ or } Q = 23,000 \pm 4,000 \text{ cfs (650} \pm 110 \text{ m}^3/\text{s)}$$
and \( Y = \frac{Q}{A} = 0.62 \pm 0.22 \text{ cfs/acre (4.3} \pm 1.54 \text{ m}^3/\text{s/km}^2) \) or \( Q = 23,000 \pm 8,000 \text{ cfs (650} \pm 230 \text{ m}^3/\text{s}) \) 

assuming the Chebyshev inequality.

Therefore, confidence limits at the 95% level for the predicted maximum expected peak discharge (WG) are at best 19,000 cfs and 27,000 cfs (540 m\(^3\)/s and 760 m\(^3\)/s), and at the worst 15,000 cfs and 31,000 cfs (420 m\(^3\)/s and 880 m\(^3\)/s). The first interval represents the best possible conditions with normally distributed errors and the second interval represents the worst possible conditions based on Chebyshev's inequality. The question of whether to use something other than 95% confidence limits is left to the individual designer; for example, 65% confidence limits would theoretically halve the range of uncertainty. The question of where to cut off the "tails" of the distribution is always difficult. The 95% limits are used here since this appears to be the most widely used confidence level in testing hydrologic models, and because statistical tables based on the 95% confidence limits are generally available.

The extrapolated 100-yr peak discharge (Fig. 6) would be less than 23,000 cfs (650 m\(^3\)/s) and, therefore, the possible error less than 8,000 cfs (230 m\(^3\)/s) as compared to the 12,000 cfs to 35,000 cfs (340 m\(^3\)/s to 990 m\(^3\)/s), plus possible error, range in 100 yr peak discharge assuming various standard distributions. If a particular project in southeastern Arizona can be justified with the best guess of the maximum expected peak discharge (WG), then the value of 23,000 cfs (650 m\(^3\)/s) would be a good figure. If the project demands a more conservative estimate based on the estimated risk, some higher value, possibly based on the 95% confidence limits, might be required.

### Acceptability and Use

Thunderstorm rainfall models developed from Walnut Gulch data possibly should be applicable elsewhere in the southwest, particularly in the semiarid and arid valleys of south-central and southeastern Arizona, southwestern and south central New Mexico, and the Chihuauan plateau of north central Mexico. Variation of the maximum expected point rainfall depth from west to east and with elevation in this general region might be a refinement to be considered (11). The Walnut Gulch rainfall models would be expected to apply in other similar semiarid and arid rainfall regions as long as air-mass thunderstorms dominate flood-producing rainfall in the region. However, use of rainfall-runoff relationships from Walnut Gulch on other watersheds may be less certain.

Rainfall-runoff models from Walnut Gulch can possibly be transferred to similar-sized rangeland watersheds if the stream slopes and channel cross sections are similar. The models may be applicable primarily because the variability of thunderstorm rainfall tends to overshadow differences in surface infiltration, soils, and geology of the rangeland watersheds in the runoff process. Also, the relatively coarse alluvial channels on Walnut Gulch are typical of rangeland watersheds in much of the southwest. Rainfall-runoff relations from Walnut Gulch probably would not apply to flat cultivated watersheds or watersheds with physical controls on runoff.
The possibility of extending the semiarid valley magnitude-frequency relations in Fig. 7 to more mountainous watersheds is suggested, e.g., by the experience of the Sabino Creek watershed in the Santa Catalina Mountains north of Tucson. The maximum peak discharge in 38 yr of record on this 36-sq mile watershed is 8,500 cfs (240 m³/s) occurring in September, 1970. The prediction from Fig. 7 would be that this was a 12-yr flood. More than one instance needs to be cited as evidence to be convincing, but it is easier to believe that the 38-yr flood has not occurred on Sabino Creek than to believe that mountain streams have smaller flood peaks than the neighboring valleys.

Conclusions

Air-mass thunderstorm rainfall produces the major floods on small [100-sq mile (260-km²) and less] arid and semiarid watersheds in the southwest. It is possible to quantify the portions of these intense storms that are most highly correlated to peak rates of runoff. This intense portion of the air-mass thunderstorm is referred to as the core of runoff-producing rainfall. Analysis of rainfall and runoff in such a framework improves the correlation between precipitation and runoff and thus improves the prediction of runoff. The core of runoff-producing rainfall was represented by the maximum 30-min depth of rainfall.

The acceptance of a limit to what may occur has implications in engineering runoff design, particularly when the limit appears to be relatively close to the 100-yr flood as indicated by Walnut Gulch data. If the designer can markedly increase the safety of his project by increasing its capacity or protection by say 30%, he probably would do so. Also, an increase of 30% in the design capacities usually represents much less than a 30% increase in construction cost. Finally, the question of the real recurrence interval for the design 100-yr flood is avoided. For example, the likelihood of recurring loss due to the 100-yr (or greater) flood for a project with a 50-yr life is 1/2. Such weaknesses in cost-benefit analyses are avoided by looking to the magnitude of the truly rare event.

In the particular case of flood runoff from small watersheds in the southwest, design curves based on Walnut Gulch data should be of value to the engineering profession. These curves are based on the best thunderstorm rainfall-runoff records available at this time in the southwest. The flood predictions seem to agree with the rare floods experienced on small watersheds in the area. If larger floods had occurred, even in the less populated regions of southern Arizona, they probably would have been noted.

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Appendix I.—References

5. Grove, G. T., “Rillito Creek Flood Plain Study,” Report to the City-County Planning Dept., Tucson-Pima County, Arizona, Apr., 1962.

Appendix II.—Notation

The following are symbols used in this paper:
A = area, in acres (square kilometers);
AR1 = antecedent channel index for regression;
C = runoff coefficient based on watershed characteristics for rational method;
Q = peak discharge, in cubic feet per second (cubic meters per second);
Q_a = peak discharge, in cubic feet per second per acre (cubic meters per second per square kilometer);
R = multicative antecedent channel index ≥ 1;
R^2 = coefficient of determination;
SEE = standard error of estimate;
V_{05} = volume of rainfall between 0.25 in. and 0.75 in. (6 mm and 19 mm);
V_{10} = volume of rainfall between 0.75 in. and 1.25 in. (19 mm and 32 mm);
V_{15} = volume of rainfall between 1.25 in. and 1.75 in. (32 mm and 35 mm);
V_{20} = volume of rainfall between 1.75 in. and 2.25 in. (45 mm and 57 mm);
V_{25} = volume of rainfall between 2.25 in. and 2.75 in. (57 mm and 70 mm);
V_{30} = volume of rainfall between 2.75 in. and 3.25 in. (70 mm and 83 mm); and
Y = predicted value.
9871 THUNDERSTORM RUNOFF IN SOUTHEASTERN ARIZONA

KEY WORDS: Arizona; Flood control; Hydraulics; Hydrology; Peak discharge; Rainfall; Runoff; Thunderstorms; Watersheds

ABSTRACT: Air-mass thunderstorm rainfall produces the major floods on small [100 sq mile (259 km² or less] rangeland watersheds in the southwest. The intense central volume of thunderstorm rainfall, referred to as the core of runoff-producing rainfall, is correlated to peak discharge. The core was represented by the maximum 30-min. depth of rainfall at each gage as estimated from a dense rain gage network on the ARS Walnut Gulch Experimental Watershed. Physical limits are considered for thunderstorm rainfall and runoff in the southwest. These limits are important because of the scarcity of records and the uncertainties involved in extrapolating short records to longer recurrence intervals. Some upper limits for peak discharges are suggested for different sized Walnut Gulch watersheds, and the variances of the individual predictions are estimated through least squares fitting of the sample data.