Experimental Investigation of Converging Overland Flow

D. A. Woolhiser, M. E. Holland, G. L. Smith and R. E. Smith

ASSOC. MEMBER ASAE

It has been suggested (Veal, 1966; Woolhiser, 1969) that a linearly converging surface (segment of a cone), along with overland flow planes and channels, is a basic element of a distributed model for surface runoff. Veal (1966) derived the continuity and momentum equations for spatially varied, unsteady flow on a converging surface and obtained numerical solutions for a limited number of subcritical situations. Woolhiser (1969) presented non-dimensional forms of the kinematic wave approximation to these equations and obtained numerical solutions for the rising side of the hydrograph and an analytical solution for recession from equilibrium.

The validity of the kinematic approximation in most overland flow situations has been demonstrated for flow over a plane (Woolhiser and Liggett, 1967), and experimental data have been used to estimate parameters in the kinematic model (Morgali, 1970; Schreiber, 1970).

The objective of this paper is to present experimental data from a converging overland flow section and to compare the properties of the experimental hydrographs with those predicted by kinematic theory.

Kinematic Wave Model

A definition sketch of flow over a converging surface is shown in Fig. 1. The kinematic wave equations for unsteady flow over this surface are:

\[ \frac{\partial h}{\partial t} + \frac{\partial uh}{\partial x} = q(x,t) + \frac{uh}{(L_0 - x)} \]  

[1] 

where \( h \) = the local depth, \( u \) = the local velocity, \( q(x,t) \) = the lateral inflow rate in volume per unit time per unit area, \( L_0 \) = the radius of the converging section and \( \alpha \) and \( n \) are parameters of the friction relationship. For the Darcy-Weisbach relationship, \( \alpha = \sqrt{\frac{8gS_0}{f}} \) and \( n = 3/2 \) where \( S_0 \) is the slope, \( f \) is a friction factor and \( g \) is the acceleration of gravity. For laminar flow on a smooth surface \( f = 24/R \) where \( R \) is the Reynolds number.

If we let the parameter \( r \) define the degree of convergence (see Fig. 1), equations [1] and [2] can be combined and written in a dimensionless form:

\[ \frac{\partial h^*}{\partial t^*} + \frac{\partial uh^*}{\partial x^*} = q^* + \frac{(1-r)h^*}{[(1-(1-r)x^*)]} \]  

[3] 

where the non-dimensional variables, designated with asterisks, are given by:

\[ h^* = \frac{h}{H_0^*}; \quad q^* = \frac{q}{q_0}; \quad t^* = \frac{t}{T_0}; \]  

[4] 

and the normalizing quantities are defined as follows:

\[ H_0^* = \text{normal depth for a discharge equal to the total steady-state outflow from the converging section divided by the mean width of the section.} \]  

\[ \begin{align*} 
X_0 &= L_0 (1-r) \quad \text{[5]} \\
T_0 &= \left( \frac{1}{q_{max}} \right) \left[ \frac{L_0 (1-r)}{\alpha} \right]^{1/n} \quad \text{[6]} 
\end{align*} \]

The normalizing lateral inflow is:

\[ q_0 = \frac{H_0^* V_0}{L_0(1-r)} \]

[7]

The normalizing time defined above is more convenient than that used by Woolhiser (1969) because it is not as sensitive to the parameter, \( r \).

Equation [3] can be written in the characteristic form:

\[ \frac{dh^*}{dt^*} = q^* + \frac{(1-r)h^*}{(1-(1-r)x^*)} \]  

[8] 

\[ \frac{dx^*}{dt^*} = \frac{n h^*-1}{(1-(1-r)x^*)} \]  

[9] 

The asterisks have been dropped in the remainder of the paper all variables will be dimensionless unless otherwise identified.

Equations [8] and [9] are sufficient if the flow is always laminar or turbulent. In this case, a single dimensionless hydrograph will describe the rise to equilibrium and the recession from equilibrium for each regime. If the flow begins in the laminar regime and becomes turbulent at some critical Reynolds number (which for overland flow at constant temperature is the same as a transitional depth, \( h_T \)) another parameter must be introduced. Let \( \beta \) be the dimensionless ratio \( h_T/H_0^* \) where \( H_0^* \) is determined by the turbulent friction relationship.

Equations [8] and [9] with \( n = 1.5 \) are valid when \( h > \beta \). For the laminar flow region, the following characteristic equations must be used:

\[ \frac{dh}{dt} = q + \frac{(1-r)h^3}{(1-(1-r)x)} \]  

[10] 

\[ \frac{dx}{dt} = \frac{3h^2}{(1-(1-r)x)} \]  

[11] 

when \( h < \beta \).

Rising hydrographs were obtained by numerical integration of equations [8] or [10] in the direction specified by equation [9] or [11]. Recession hydrographs were obtained numerically or from analytical solutions (Woolhiser, 1969). Solutions for the rising and falling equilibrium hydrographs are shown in Fig. 2 for mixed flows with four values of the parameter \( \beta \). As \( \beta \to 0 \) the flow becomes purely turbulent and when \( \beta = 13.1 \), (the dimensionless depth at the downstream boundary for \( r = 0.0106 \)) the flow is purely laminar. An examination of this figure suggests that if the kinematic wave approach is valid for the converging section, there should be a single dimensionless hydrograph.

Fig. 1 Definition sketch of flow on a converging surface

---

Article was submitted for publication on May 4, 1971; reviewed and approved for publication by the Soil and Water Division of ASAE on August 20, 1971.

The authors are D. A. WOOLHISER, Research Hydraulic Engineer, USDA, Fort Collins, Colo.; M. E. HOLLAND, Assistant Chief, Planning and Research Div., State Water Resource Control Board, Sacramento, Calif.; G. L. SMITH; Associate Professor of Civil Engineering, Colorado State University, Fort Collins; and R. E. SMITH, Research Hydraulic Engineer, USDA, Tucson, Ariz.

Contribution from the Northern Plains Branch, SWORD, ARS, USDA, in cooperation with the Colorado Agricultural Experiment Station. This research was supported in part by funds provided by the United States Department of Interior, Office of Water Resources Research under grant B-594-COLO.


Trade names and company names used in this paper are included for information only and do not constitute endorsement by the U.S. Department of Agriculture.
sionless equilibrium rising and falling hydrograph only if the flow remains laminar throughout or becomes turbulent at very low values of $\beta$. A schematic diagram of the solution domain is shown in Fig. 3. The laminar to turbulent transition zone is indicated with a heavy line. Rainfall begins at $T = 0$ and stops at $T = 1.0$. Experimental evidence (Kissel, 1971) indicates that raindrop impact increases the laminar flow parameter $k$ in the relationship

$$ f = k/R $$

where $f$ is the Darcy-Weisbach friction factor and $R$ is the Reynolds number. With raindrop impact the friction factor follows a laminar law until it intersects the turbulent line for undisturbed flow effectively causing a higher transition Reynolds number for disturbed flow. When rainfall stops, the friction factor will decrease all along the profile and the turbulent zone could expand.

From these observations it appears that the simplest friction law that can be used in the kinematic model to accurately predict rising and recession hydrographs must include laminar and turbulent flow and the retarding effect of rainfall. A possible model to include these effects is shown in Fig. 4. Without rainfall, the friction law would be

$$ f = k_o/R ; \quad R < R_T $$

$$ f = k_o/R_T ; \quad R \geq R_T $$

where $k_o$ is the friction parameter and $R_T$ is the transition Reynolds number without rainfall. With rainfall we would have

$$ f = k_o + k(q)/R ; \quad R < [k_o + k(q)]R_T/k_o $$

$$ f = k_o/R_T ; \quad R \geq [k_o + k(q)]R_T/k_o $$

where $k(q)$ expresses the relationship between the friction parameter and rainfall intensity. This model implies a Chezy relationship for turbulent flow although it could also be formulated to include the Manning equation. Either of these models requires the evaluation of three parameters if the increase in $k$ is linear in $q$ and four parameters if it is nonlinear. The rainfall impact effect becomes less important as $k_o$ increases.

An obvious manifestation of the laminar to turbulent transition is a rapid drop in discharge on the recession hydrograph. The position of this drop depends upon the length of the surface as well as the rainfall intensity and is most pronounced when transition occurs in the region $0.1 < x < 0.4$. For overland flow on a plane, this transition is propagated as a kinematic shock. This phenomenon has been studied experimentally and numerically by Iwagaki (1955) and has been observed in experimental data (Yu and McNown, 1963).

**Experimental Procedures**

The experimental data reported herein were obtained from the converging section of the Rainfall-Runoff Experimental Facility at Colorado State University (Fig. 5).

The basic element in the system is the sprinkler riser assembly shown in Fig. 6. The sprinkler nozzle used is the Rain Jet No. 78°. Water is supplied to the nozzles through standard 2-in. diameter aluminum irrigation pipe supported by the bipod supports clamped to the risers as shown. The lines are...
17.5 ft apart and the risers are spaced at 10-ft intervals along the lines. The basic pattern for the lowest rainfall rate is formed by spraying from nozzles that are at the corners of a 40-ft equilateral triangle. Additional patterns are superimposed by adding nozzles to increase the intensity. The system is controlled by hydraulic valves on the risers which in turn are connected to a manifold system that can be pressurized or vented to air through electrical solenoid valves (see Fig. 7). The 2-in. aluminum lines are attached to a 6-in. aluminum supply manifold supplied from a 10-in. buried main. Pressure regulators maintain the nozzle pressure at approximately 28 psi at all flow rates. Precipitation uniformity tests indicate a coefficient of variation of 18 percent at a rainfall rate of approximately 0.5 in. per hr. The coefficient of variation is halved as the rainfall rate doubles, resulting in a value of 2 to 3 percent at the maximum rate of approximately 4 in. per hr.

These experiments were conducted in the converging portion of the facility where $L_0 = 110$ ft, $S_0 = 5$ percent, $r = 0.0106$. The surface was covered with butyl rubber and roughened with 1%-in. diameter gravel on top of the butyl.

Rainfall rates were computed by measuring the steady-state outflow through a 1.5-ft flume equipped with a FW-1 recorder. The chart drum on the recorder is driven at 2 revolutions per hour by a synchronous electric motor.

Experimental runs were performed at each of five rainfall intensities. An equilibrium run was performed first to establish the steady-state input rate. This was followed by a varying number of partial equilibrium runs.

A normalizing time, $T_n$, was computed for each equilibrium run by dividing the steady-state storage, $V_n$, by the inflow rate, $q_n$. The steady-state discharge was used as the normalizing discharge.

Pertinent data for the experiments reported here are included in Table 1.

**TABLE 1. EXPERIMENTAL PARAMETERS**

<table>
<thead>
<tr>
<th>Run no.</th>
<th>Surface</th>
<th>$q_n$, in. per hr</th>
<th>$V_n$, in.</th>
<th>$T_n$, sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>115</td>
<td>Butyl</td>
<td>3.692</td>
<td>0.08206</td>
<td>80.02</td>
</tr>
<tr>
<td>125</td>
<td>Butyl</td>
<td>0.756</td>
<td>0.0284</td>
<td>135.34</td>
</tr>
<tr>
<td>126</td>
<td>Butyl</td>
<td>0.835</td>
<td>0.03384</td>
<td>145.94</td>
</tr>
<tr>
<td>127</td>
<td>Butyl</td>
<td>1.892</td>
<td>0.05547</td>
<td>105.56</td>
</tr>
<tr>
<td>128</td>
<td>Butyl</td>
<td>3.84</td>
<td>0.08733</td>
<td>81.87</td>
</tr>
<tr>
<td>114</td>
<td>Butyl</td>
<td>1.862</td>
<td>0.05355</td>
<td>103.54</td>
</tr>
<tr>
<td>177A</td>
<td>40 lb gravel</td>
<td>0.384</td>
<td>0.03872</td>
<td>363.14</td>
</tr>
<tr>
<td>178A</td>
<td>40 lb gravel</td>
<td>0.878</td>
<td>0.07567</td>
<td>310.29</td>
</tr>
<tr>
<td>179A</td>
<td>40 lb gravel</td>
<td>1.818</td>
<td>0.13950</td>
<td>276.19</td>
</tr>
<tr>
<td>180A</td>
<td>40 lb gravel</td>
<td>2.693</td>
<td>0.18138</td>
<td>242.47</td>
</tr>
<tr>
<td>181A</td>
<td>40 lb gravel</td>
<td>3.656</td>
<td>0.22666</td>
<td>223.19</td>
</tr>
</tbody>
</table>

**RESULTS**

Dimensionless experimental rising and recession hydrographs for a butyl rubber surface and for a butyl surface covered with 40 lb per sq yd of 1.5-in. gravel are shown in Figs. 8(a) and (b). The scatter of points on the rising hydrographs is very small and it appears that a single dimensionless rising hydrograph for each surface is an adequate representation. A single dimensionless recession hydrograph appears adequate for the butyl surface but there is a systematic variation with intensity in the shape of the recession hydrograph for the graveled surface. Careful examination of individual recession hydrographs reveals a rapid
The composite recession hydrographs shown in Fig. 10, show much greater variability than the rising hydrographs. The increase in the outflow rate after the end of rainfall for the butyl surface is indicative of the relatively greater effect that raindrop impact has on this smooth surface compared with the gravel-covered surfaces. Again, as the gravel density increases, the composite hydrographs approach the Chezy recession at high rates of flow but deviate at the lower rates after the transition to laminar flow occurs. The turbulent Chezy recession curve is very near the middle of the range of data for all T.

A plot of the normalizing time, $T_0$, versus the steady-state input rate, $q_0$, is shown in Fig. 11. The normalizing time decreases rapidly for all surfaces as the rate increases from 0.5 to 2.0 in. per hr, demonstrating the nonlinearity of the runoff process. The slope of the curve decreases at higher flow rates, indicating that the process is more nearly linear at high runoff rates. The dramatic increase in the response time as the surface roughness increases is also evident from this figure.

**Conclusions**

Experimental equilibrium hydrographs of overland flow from a converging surface with a 5 percent slope agree reasonably well with those predicted by kinematic theory if both laminar and turbulent flow regimes are included. If only one resistance law is used, errors during recession may be as great as ±20 percent for this simple configuration.

The simplest friction law that can be used in the kinematic model to predict rising and recession hydrographs from smooth surfaces must include the retarding effects of raindrop impact as well as defining regimes of laminar and turbulent flow. Such a law may require a minimum of three to four empirically determined parameters.

A rapid drop in discharge during the recession indicates that a laminar to turbulent transition occurred during the steady-state case.

The shape of the recession hydrographs predicted by kinematic theory is observed in experimental data and shows that there is no general basis for the commonly accepted negative exponential recession.

**References**

1. Iwasaki, Yuichi. 1955. Fundamental studies on the runoff analysis by characteristics. Bulletin No. 10, Disaster Prevention Research Institute, Kyoto University, Kyoto, Japan.