Topic 4 Summary: Status and Future of Modeling
Spatial and Temporal Variability of Infiltration
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Introduction

One important objective of research on the variability of infiltration is to be able to incorporate a measure of variability in hydrologic models to make them valid over a variety of time scales and a variety of spatial scales. Models can be thought of as symbolic representations of our knowledge, and it has long been recognized, as reflected in the motivation for this workshop, that our representation of infiltration variability is often crude in comparison with the variations seen in nature. Temporal variability may be considered as manifesting itself most dramatically in the effects that tillage may have on the intake properties of a soil. Temporal variability also can be observed, albeit to a much smaller extent, in seasonal changes that can be observed on natural watersheds. Spatial variability is the dominant problem in dealing with scale effects in simulating runoff on watersheds, and in extrapolating plot measurements to estimate watershed runoff. We will briefly summarize our interpretation of the state of current knowledge (i.e., models) and the major challenges facing us in these areas. We will assume a general point model and describe various treatments for applications to larger scales.

Definitions. In the following we will assume that at a point, and if current soil conditions are known, local infiltration behavior for a uniform simple soil can be described in relation to local soil hydraulic properties. Infiltration rate, $f$, equals the soil-limited infiltration capacity, $f_\infty$, when water is supplied at a rate $r$ exceeding that capacity. For lower values of $r$, $f = r$. In either case, infiltrated depth, $I$, is defined as

$$I = \int_0^t f dt$$

Two basic infiltration parameters describe infiltration capacity: First, effective, saturated hydraulic conductivity, $K_s$ constitutes the asymptotic value of $f$, if the profile is homogeneous. Second, capillary drive, $G$ (often termed $H_j$), is a basic soil parameter defined as

$$G = \int_0^h \frac{K(h)}{K_s} dh$$

in which $h$ is soil capillary head, $K(h)$ is the conductivity-capillary head relation. $G$ [units of L] is effectively a $K$-weighted value of $h$. Also, infiltration is sensitive to the soil water deficit, $\Delta \theta$, defined as $\theta_s - \theta_i$ where $\theta_s$ is the maximum soil water holding capacity, by volume, and $\theta_i$ is the soil water content at the beginning of rainfall. Given these parameters, a quite general, basic

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infiltration relation can be described by defining the following dimensionless variables:

\[ f^* = \frac{f_i - K}{K} \]

\[ I^* = \frac{I}{G\Delta\theta} \]

Then the infiltration equation can be given as

\[ f^* = \frac{\alpha}{\exp(\alpha I^*) - 1} \] (4)

For \( \alpha \) approaching 1, Eq. (4) is the Smith-Parlange (1978) relation, and in the limit as \( \alpha \) approaches 0, Eq. (4) becomes the Green-Ampt infiltration relation. Employing \( I^* \) rather than \( t \) as the independent variable eliminates need for a separate computation of ponding time.

Models of Temporal Variability

Agricultural soils undergo a variety of changes in time, caused by both mechanical actions and the natural actions of weather. Tillage can cause enormous changes in bulk porosity and create a dual porosity medium composed of soil "clods", and can at the same time create wheel track compaction of soil in a few furrows. The effect of a given type of tillage implement further will vary with the specific tillage history, the soil texture, and the soil water content at the time of tillage. Significant amounts remain to be learned about interrelations of all these factors before a robust tillage infiltration model can be proposed.

Rainfall energy and rewetting on loose soil will often cause particle dispersion and structural reformation at the surface which can create a crust layer. The factors influencing this crust development and its properties in relation to the parent soil are poorly understood. In the Opus model (Smith, 1992), this transition is modeled as a function of soil clay content and cumulative rainfall energy. Clay content is assumed to govern the ultimate reduction ratio for \( K \), which can be attained in a crust. This ratio is assumed largest for moderate clay amounts.

Further wetting can cause a slow reformation of tillage induced "clods" and a reduction to more natural bulk porosity. Soils high in clay may be subject to swelling during wetting, and subsequent cracking upon drying. There has been some modeling of this process, which creates and destroys a special kind of macroporosity and a two-dimensional infiltration opportunity. The WEPP model contains a simulation of shrinkage cracks for clay soils, assuming the fraction of area cracked is a function of clay content, swelling ratio, and water content. Cracks both act as macropores and induce two-dimensional water intake.

Frozen Soils: On any soil, especially in climates subject to annual freezing, there are seasonal changes that are complex and difficult to model. In many northern latitudes, rain or snowmelt on
Seasonally frozen soils is the single leading cause of severe runoff and erosive events. A separate paper by Fierchinger (this volume) discusses modeling infiltration in frozen soil.

Infiltration Models for Heterogeneous Soil Conditions

Layering

Soil heterogeneity as it affects infiltration may be viewed as two-dimensional: both vertical and horizontal variations. Infiltration into layered soils has received some attention in the past. Early work focused on the application of the Green-Ampt model to special cases of layering, where the soil $K_v$ varied monotonically (Bouwer, 1976). Often it was assumed that $K_v$ could be obtained by use of saturated flow computations. In fact, however, for an arbitrary layering it cannot be assumed that the soil is saturated above a wetting front in a layered soil. Moore (1981) and Smith (1990) have published models that rigorously treat infiltration into a two-layer soil. Any number of layers can be treated by assuming the general functional relation such as Smith-Parlange or Green-Ampt with a capillary parameter $G$ (whose effective value changes with wetting front position) and finding an effective asymptotic value of $K_v$ for the current wetting front by solving the steady flow equation through all wetted layers. Internal boundary conditions at each layer interface must be satisfied, and saturation of layers above the wetting front cannot be assumed. This is not a trivial exercise, but does provide a general infiltration model for layered profiles.

Infiltration modeling through layers includes the case of temporally changing crusts mentioned above. Where crusts are significant infiltration controls, the ideas of Mualem and Assouline (1989) concerning a gradation from surface properties to subsoil properties, rather than a distinct layer, deserve further study. Mualem and Assouline have only studied this type of crust under steady flow.

Spatial Soil Heterogeneity

The treatment of natural heterogeneity is one of the greatest challenge in hydrologic modeling at larger scales is (Smith et al., 1994; Bloschl and Sivapalan, 1995). For the purpose of this discussion, "large scale" implies the field or hillslope scale (characteristic lengths > 100m) and beyond. Meter and sub-meter scale soils and infiltration heterogeneity can be classified as random while larger scale variability due to changes in soil type can be classified as organized variability (Bloschl and Sivapalan, 1995). For purposes of large scale modeling, it is assumed that organized variability can be resolved with the use of geographic information systems and treated within a distributed hydrologic model via variation in infiltration parameters from one model element to another. Small scale (sub-grid or sub-model element) random infiltration variability is often be treated via a statistical or probability distribution (Smith and Hebbert, 1979; Woolhiser and Goodrich, 1988; Binley et al., 1989; Smith et al., 1990; Wood et al., 1990). It is crucial to understand infiltration phenomena at the field and small watershed scale as this is the typical size of land areas subject to management, and is the scale of our major source of field data.

Considerable research has been published on the treatment of flow, both saturated and
unsaturated, in heterogeneous porous media. Most of this work is not directly relevant to infiltration. The perturbation approach for stochastic differential equations, (Mantoglu and Gelhar, 1987) for example, is not applicable either near soil saturation or near a boundary. Further, most work on unsaturated flow near the soil boundary has focused on the areal mean wetting fronts and their moments, and not looked at areal infiltration heterogeneities (e.g. Bresler and Dagan, 1983; Chen et al., 1994).

Models for Soil Variability: There have been many studies to evaluate a statistical model for the random spatial heterogeneity of soil properties, which are directly related to the spatial properties of infiltration. $K_s$ as a spatially random variable has often been a subject of inquiry (e.g., Nielsen et al., 1973). The question is not only one of establishing the distribution function (log-normal is almost always found applicable) but also the spatial structure: correlation scale, semivariogram, or other measure of spatial relation. A further question for hydrology is the relation of spatial statistical structure of a soil property to the scale of hydrologic interest. There are more theoretical questions involved than can be discussed here. What is important to remember is that one cannot arbitrarily average over heterogeneous areas as one considers larger scales. Small scale variations may have a significant effect for larger scale behavior. To model infiltration over a heterogeneous area, one may either reproduce the small scale variations, or establish by detailed simulation or by theory a method of lumping that is valid for the purpose.

Infiltration Model to Account for Spatial Heterogeneity: It should be understood that the parameters for the net behavior of an ensemble of nonlinear processes cannot be obtained by the average of the parameters of each member of the ensemble. Moreover, there may be no effective stationary parameter value that will allow a single process realization to mimic the behavior of the ensemble. These two facts must be kept in mind in modeling infiltration for areas containing infiltration heterogeneity. A further complication is involved in spatial interactions. The important spatial interrelations for adjacent infiltrating areas are upstream/downstream relations, not simple spatial correlation lengths. If an area producing early runoff flows away from an adjacent area that is pre-ponding, the distance or correlation between those points is unimportant. On the other hand, runoff onto a preponding area will act as an increase in rainfall rate and cause accelerated ponding and runoff production.

An overall picture of the ensemble behavior of a heterogeneous watershed is diagramed in Fig. 1. The areal distribution is diagramed at two successive times, during which rainfall is assumed to have dropped. Parts of the catchment (where $r > K_s$) have ponded and infiltration is controlled by capacity, $T$. The overall area has an expected value for $K_s = \xi(K_s)$, and an effective value, $K_c(r)$, which depends on rainfall. Hawkins and Cundy (1987) were one of the earliest to
Fig. 1. Illustration of infiltration capacity and infiltration rate changes in space and time during a rainfall event of rate \( r \). Note that infiltration is always limited by \( r \), and \( f_c \) decreases in time when limited by soil rather than \( r \). The dotted line indicates \( f_c \) at the previous time.

Point out that areal effective \( K_e \) can be described as

\[
K_e = \left( 1 - P_k(r) \right) r = \int_0^r kp_k(k) dk \tag{5}
\]

given the probability distribution \( p_k \) and cumulative distribution \( P_k \) for \( K_e \). Using \( K_e \) in place of \( K_s \) in Eq. (3), the areal infiltration equation may be expressed (Smith, unpublished) as

\[
f^* = r^* \left\{ 1 + \left[ \frac{r^*}{\alpha} \left( e^{\alpha r^*} - 1 \right) \right]^c \right\}^{-1/c} \tag{6}
\]

Note that this is not an infiltration capacity equation, because with a randomly distributed \( K_s \), a ponding time is not defined, and for small values of \( I \) this equation properly depicts \( f \sim r \), as shown in Fig. 2. The parameter \( c \) in Eq. (6) is a function of \( r^* \) and the coefficient of variation of \( K_e \).
Smith et al. (1990) demonstrated three methods to estimate infiltration and runoff on a small watershed, including ensemble net infiltration (ignoring spatial interaction), stratified (Latin Hypercube) sampling simulation using parallel strips (as in Woolhiser and Goodrich, 1988), and two dimensional sampling over the watershed with simulated upstream/downstream interactions. These were shown to have different degrees of accuracy, but all simulated peak flows were significantly larger than for the uniform, average infiltration assumption. It was also demonstrated that the effect of areal heterogeneity on runoff is most significant for the common case where runoff is a small portion of total rainfall.

Latin Hypercube sampling with parallel strips can be applied at scales from plots to small catchment surfaces. The method is illustrated in Fig. 3. Each strip represents an equally likely value of $K_s$ taken from the cumulative distribution as shown. The strip arrangement illustrated will not simulate runon-runoff phenomena, but they increase significantly the ability of the model to treat the effects of heterogeneity on runoff.

Surface Microtopography: The interaction of runoff and soil infiltration should in all cases involve the actual surface shape. Microtopography has been shown (Woolhiser, et al., 1996) to
Fig. 3. Latin hypercube parallel strip method for simulating a distribution of infiltration parameters on a runoff surface. Each strip contains an equally likely value of $K_s$.

have dramatic effects on runoff and infiltration that cannot be ignored at larger scales. On the other hand, such interactions need not be treated by simulation at the microscale. Rather, it appears that a statistical model relating extent of soil covered with mean depth of surface flow can suffice to model many of the interactions that are important. These interactions concern the loss of runoff water during recession and the successful travel of runoff to the stream after rainfall excess has turned negative. In other cases, there may be a correlation of infiltration characteristics with local micro-elevation. Examples of this include the shortgrass rangeland microtopography, composed of hillocks of grass clumps interspersed with crusty bare areas, or the higher infiltration rates under rangeland shrubs. Modeling an interaction between water flow depth and infiltration rate is rarely undertaken but is feasible in current models (e.g., KINEROS, Smith et al. 1995).

Macropore Flow Models: Distinct cracks or channels through the soil which distinguish a real soil from an ideal porous media have received considerable attention lately, and have collectively become known as macropores. This topic is covered in a separate paper in this volume.

Rainfall Heterogeneity

Variations in rainfall intensity at the local scale can have a significant effect on infiltration heterogeneity and should not be ignored. Faurès et al. (1995) and Goodrich et al. (1995) observed rainfall gradients up to 2.5 mm/100m within a 4.4 ha watershed. This spatial rainfall variation resulted in modeled peak runoff rates which varied by a factor of almost three (8 to 23 mm/hr) when two different recording rain gauges in the proximity were used independently with
the uniform spatial rainfall assumption. While this issue is not one of infiltration modeling, it is important not to forget the role that such variability plays in our treatment of heterogeneities. It is also important to remember that for modeling a balance is necessary between the treatment of process complexity and the data (rainfall or soil) availability.

Research Challenges

There are several significant areas of ignorance that should be addressed before a robust model of soil infiltration can be formulated to deal with spatial and temporal variability. Some of these areas are:

1. Much remains to be learned before a model for the statistical character of soil areal heterogeneity can be used with confidence across major soil types. Ultimately, some measure of inherent randomness and spatial scaling, such as correlation length and coefficient of variability of major soil hydraulic characteristics, should be part of our description of a soil, just as we now classify soils (albeit qualitatively) in terms of drainability and texture.

2. Probably the largest area of uncertainty in infiltration modeling is the changes that the infiltration function undergoes as a result of mechanical modifications, as indicated above. While some progress has been made in modeling the changes that a swelling, cracking clay undergoes with time, we have very little confidence in our ability to predict the formation of surface crust for a given soil texture, and our ability to anticipate the change in infiltrability of a soil at a given state caused by a given mechanical treatment.

3. There is progress being made in modeling the aggregate behavior of an area containing internal infiltration variability, but there remain significant challenges in their application in “management” models, and in understanding the conditions under which a variety of possible simplifications are acceptable. Given the preponderance of daily rainfall data, much needs to be done to improve our knowledge of disaggregation statistics and rainfall intensity distributions so that infiltration models can be used to improve the simulation of daily runoff: a physically and statistically sound lumping, rather than empirical lumping.

4. At larger scales (e.g. 10 ha +), modeling areally variable infiltration should not be done independently of the surface runoff, itself with considerable organized and random heterogeneity, nor should it be modeled without consideration of small-scale rainfall rate heterogeneities. One promising approach for larger areas might be a joint statistical/deterministic representation of the probability of local rainfall exceeding local infiltration capacity, integrated over the area.

References


