

COMPUTATION OF FLOW TRANSITIONS IN OPEN CHANNELS WITH STEADY UNIFORM LATERAL INFLOW

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ABSTRACT: A numerical procedure is developed for quickly and accurately finding the location and flow depth of a flow transition (critical control section) in open channels with discharge gradually varying along the channel length. General equations are presented for trapezoidal, triangular, and rectangular cross sections. A numerical analysis of the procedure shows that the geometry of the channel cross section is an important factor determining the existence of a unique solution to the flow-transition problem in open channel flow. A procedure is described to decide if solutions exist and, if they do, to compute them and select an acceptable solution as the starting point for water-surface profile calculations. Test runs on a computer are performed to verify the speed and accuracy of the proposed procedure. Examples involving the finding of location of flow transitions in open channels are used to illustrate the usefulness of the procedure. The findings from this study are beneficial to hydraulic engineers solving problems related to the design of lateral spillway channels and gutters for conveying stormwater runoff; to hydrologists using spatially varied flow equations to describe flow processes in natural channels; and to the enhancement of algorithms for spatially varied flow computations in hydrologic simulation models such as the CREAMS field-scale model and the WEPP watershed model.

INTRODUCTION

The purpose of this paper is to develop a numerical procedure for quickly and accurately finding the location and flow depth of a critical control section in spatially varied open-channel flow. In particular, a special effort is made to: (1) Present a procedure to decide if solutions exist and, if they do, to compute them and choose an acceptable one; (2) perform test runs on a computer to verify the speed and accuracy of the proposed procedure; and (3) validate the procedure using experimental data.

The determination of the water surface profile in an open channel under steady flow conditions is an important problem in hydraulic and hydrologic analysis and one that has received considerable attention (Humpidge and Moss 1971). There has been a rapid development of computer programs to compute water-surface profiles in open channels recently. One aspect of this work involves finding the location and flow depth of critical control sections, or flow transitions (a point in which a water-surface profile passes from subcritical to supercritical flow, or vice versa), and then computing the water-surface profile, both forward and backward, from the critical control section. A flow transition can be caused by a change either in the flow direction, slope, or cross section of the channel, or by lateral inflow or outflow that produces a change in the flow state. It is relatively simple to compute water-surface profiles when the location and/or flow depth of a critical control section are readily identifiable (e.g., by control structures

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such as sluice gates or weirs). A problem arises, however, when a flow transition develops at some unknown location along the channel reach. This problem is quite common in many constructed channels, such as side-channel spillways and gutters for conveying stormwater runoff, and natural channels, especially in channels with outlets restricted by ridges and heavy vegetation, and very flat terrace channels (Foster et al. 1980).

Previous work on determining the location and flow depth of critical control sections by graphical methods was presented by Chow (1959) using an "equivalent critical depth channel" method proposed by Hinds (1926), by Escoffier (1958) using the transition profile technique, and by Smith (1967) using a modification of the transition profile technique formulated by Escoffier (1958). Although the concepts behind the graphical procedures are valid, the rapid development of computer-aided design techniques has demanded computational approaches implemented on modern high-speed computers. Moss (1971) derived practical procedures for computing the location and flow depth of a critical control section, either by hand or computer, in a trapezoidal side-channel spillway but neglected friction slope and the effect of slope on pressure head in his development. Humpidge and Moss (1971) described a computational procedure for determining the location of potential critical control sections in open channels. According to their procedure, the location of a critical control section in a channel reach is found when the numerator of the dynamic equation of spatially varied flow is negative immediately downstream of the upper section and positive immediately upstream of the lower station. The exact location at which the critical control section occurs is found by interpolation. This procedure, a computerized version of the transition profile method described by Escoffier (1958), requires that the whole channel reach be examined, for a given discharge, to determine the potential location of a critical control section, a time consuming procedure considering that the channel length must be divided into small distance increments to find the location of the flow transition. Smith (1972) briefly discussed a computerized method for determining the location of a critical control section in an open channel following an approach quite similar to the one presented by Humpidge and Moss (1971).

Although there has been considerable progress during recent years in the development of computer programs to determine water-surface profiles in spatially varied open-channel flow, much less effort has been made in proposing computer algorithms for finding the location and flow depth of critical control sections. At the present time, there has not been any attempt to analyze the uniqueness of solution to the critical control section location problem as affected by channel cross-sectional geometry and the flow resistance equation. Although this problem has not been studied before, the recent implementation of spatially varied flow equations in computer simulation models imposes a need for developing procedures to decide if solutions to the problem exist and, if they do, to compute them before computations of the water-surface profile can be initiated.

STEADY, SPATIALLY VARIED FLOW

Basic Assumptions

The spatially varied flow, as defined in this paper, is the steady flow whose discharge varies gradually along the length of the channel. This definition indicates two major conditions: (1) That the flow is steady; that is, that the hydraulic characteristics of the flow remain constant for the time

interval under consideration; and (2) that the streamlines are practically parallel, that is, that the hydrostatic pressure distribution prevails over the channel section. In addition, the following basic assumptions are used in the subsequent developments: (1) The flow is one dimensional; (2) the slope of the channel segment is small and uniform; (3) the channel segment under study has constant alignment and geometry; (4) the momentum correction coefficient in the flow direction is constant; (5) the rate of lateral inflow is constant; (6) the flow at the upper end of the channel element is zero; and (7) the friction slope can be estimated from the Manning equation.

Spatially Varied Flow Equations

The net influx of momentum passing through the cross sections and perimeter of a control volume defined along an incremental length of channel may be equated to the sum of external forces acting on the control volume. This procedure has been used by many investigators to derive the spatially varied flow equation. By considering an incremental channel length, Δx , (Fig. 1), and equating the change in momentum flux across the the control volume of channel length Δx to the sum of the external forces acting upon it, the dynamic equation for spatially varied flow with uniform lateral inflow rate and uniform velocity distribution over the cross section can be derived and written in two forms as given by Chow (1959):

$$\frac{dy}{dx} = -\frac{1}{gA} \left[\frac{d(QV)}{dx} \right] + (S_o - S_f) \dots\dots\dots (1a)$$

and

$$\left(1 - \frac{Q^2}{gA^2D} \right) dy = \left[S_o - S_f - \frac{2Q}{gA^2} \left(\frac{dQ}{dx} \right) \right] dx \dots\dots\dots (1b)$$

where y = flow depth (L); x = longitudinal distance along the channel (L); Q = flow rate (L^3/T); V = average velocity over the cross section (L/T); S_o = channel bottom slope (dimensionless); S_f = friction slope (dimensionless); A = flow cross-sectional area (L^2); D = hydraulic depth (L); and g = acceleration due to gravity (L/T^2) (note: L represents length

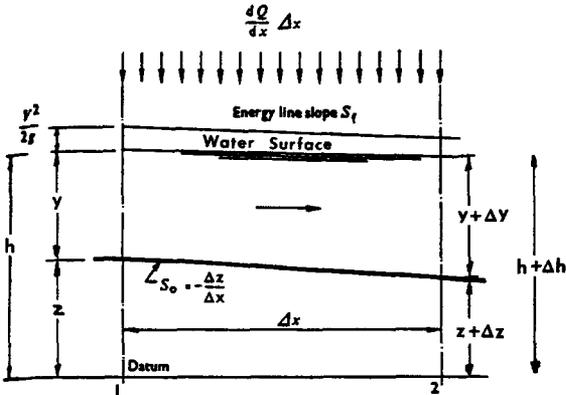


FIG. 1. Definition Sketch for Gradually Varied Flow Equation

and T represents time in the dimensions of variables). A velocity coefficient is sometimes used in (1) in an attempt to correct for nonuniform velocity distribution over the cross section due to friction or channel curvature.

The steady, spatially varied flow equations are used in the chemicals, runoff, and erosion from agricultural management systems (CREAMS) model (Knisel 1980) and the water erosion prediction project (WEPP) watershed model (Stone et al. 1990) to describe flow processes in small stream channels including terrace channels, diversions, grassed waterways, hillside ditches, surface drains, tail ditches, and other major flow concentrations (Foster et al. 1980). The approach used in CREAMS and WEPP to compute water-surface profiles in natural channels is based on a procedure that approximates the energy gradeline along the channel using a normalized version of (1) for a triangular channel cross section with a critical control section located at the channel outlet. Regression equations were fitted to the solutions of the normalized spatially varied flow equation for a range of channel side slopes, surface roughnesses, and flow depths and discharges at the end of the channel. The effects of assuming triangular channel cross section and critical control section at the channel outlet on the calculation of water-surface profiles in natural channels need to be investigated. Moreover, the occurrence of a critical control section within the channel reach will affect the water-surface profile calculations based on a preassumed control at the channel outlet. As a consequence there will be a change in the bottom hydraulic shear by some calculated amount that in turn will lead to more/ or less sediment transport, which will have consequences in the final channel erosion estimates.

Conditions for Developing Critical Control Section

For convenience, the derivation of the conditions for developing a critical control section is briefly restated here. A critical control section occurs where the specific energy, E , for a steady one-dimensional flow (the energy per unit weight relative to the bottom of the channel) is a minimum for a given discharge (Henderson 1966). The equation for specific energy in a channel of small slope with uniform cross-sectional velocity distribution can be written as:

$$E(y) = y + \frac{Q^2}{2gA^2} \dots\dots\dots (2)$$

Differentiating (2) with respect to y and noting that $dQ/dy = 0$ and $dA/dy = T =$ flow top width (L), gives:

$$\frac{dE}{dy} = 1 - \frac{Q^2T}{gA^3} \dots\dots\dots (3)$$

The second term of the right-hand side of (3) is the square of the Froude number, F . A critical control section occurs where $dE/dy = 0$, or:

$$F = Q \left(\frac{T}{gA^3} \right)^{1/2} = 1 \dots\dots\dots (4)$$

For spatially varied open-channel flow with distributed lateral inflow, the flow at a distance x from the upper end of the channel is defined by:

$$Q = q_Lx \dots\dots\dots (5)$$

where $q_L =$ the distributed lateral inflow rate (L^2T^{-1}). Substituting (5) into (4) and solving for x gives a relation between the location and depth at a critical control section:

$$x = \frac{1}{q_L} \left(\frac{gA^3}{T} \right)^{1/2} \dots\dots\dots (6)$$

At a critical control section, the left-hand side of (1b) is zero and thus so is the right-hand side. Then, at a critical control section:

$$S_o - S_f - \frac{2Qq_L}{gA^2} = 0 \dots\dots\dots (7)$$

Substituting (5) into (7) and eliminating x by using (6), gives:

$$S_o = S_f + \frac{2q_L}{(gAT)^{1/2}} \dots\dots\dots (8)$$

The procedure to be followed in finding a critical control section is to first express S_f in terms of y . Then solve (8) for $y = y_c$ (flow depth at the critical control section) and use y_c in (6) to find $x = x_c$ (location of the critical control section).

The calculations to determine the location and flow depth of a critical control section provides a mean for determining whether the occurrence of a flow transition is possible. If there is no solution to (8) or if x , as determined from (6) is greater than the actual length of the channel reach, a flow transition is not possible within the channel reach under consideration. In this situation, the flow will be subcritical along the entire channel reach and subject to a control at the downstream end. This control may, of course, take the form of an free-overfall beyond the region of lateral inflow. If a critical control section does exist within the channel reach, flow will be subcritical upstream of this section and supercritical downstream. If, in this situation, there is a control at the downstream end of the channel reach, a hydraulic jump will occur if the controlled depth is large enough and may even move upstream and drown out the critical control section.

Flow Resistance Equations

The Manning flow resistance equation is the most widely used equation to relate the flow rate to the friction slope. The Manning equation is:

$$V = \left(\frac{S_f^{1/2}}{n} \right) R^{2/3} \dots\dots\dots (9)$$

where $n =$ the Manning coefficient of roughness ($L^{-1/3}T$); and $R =$ the hydraulic radius (L). Using $R = A/P$ and $Q = AV$ in solving (9) for S_f gives:

$$S_f = \frac{n^2 Q^2 P^{4/3}}{A^{10/3}} \dots\dots\dots (10)$$

in which P is the wetted perimeter (L). Using (4) to eliminate Q in (10) and then substituting into (8) gives:

$$S_o = gn^2 \left(\frac{P}{T}\right) \left(\frac{P}{A}\right)^{1/3} + \frac{2q_L}{(gAT)^{1/2}} \dots \dots \dots (11)$$

Eq. (11) is solved for the critical depth of the critical control section.

Geometric Properties of Channel Section

For a fixed location in a general trapezoidal channel with bottom width $b(L)$ and side slopes z_1 and z_2 (Fig. 2), flow cross-sectional area, top width, and wetted perimeter, are related as:

$$A(y) = (b + c_1y)y \dots \dots \dots (12)$$

$$T(y) = b + 2c_1y \dots \dots \dots (13)$$

$$P(y) = b + c_2y \dots \dots \dots (14)$$

in which c_1 and $c_2 =$ constants given by:

$$c_1 = \frac{(z_1 + z_2)}{2} \dots \dots \dots (15)$$

$$c_2 = (1 + z_1^2)^{1/2} + (1 + z_2^2)^{1/2} \dots \dots \dots (16)$$

A general trapezoidal channel can be classified as triangular ($b = 0, c_1 > 0$), rectangular ($b > 0, c_1 = 0$), or trapezoidal ($b > 0, c_1 > 0$). Also note that $c_1 > 0$ just when $c_2 > 2$.

NUMERICAL ANALYSIS

In a spatially varied flow with uniform lateral inflow, there may be a location at which it is possible to satisfy the condition given by (7), i.e., there is a value of x , from (6), giving a solution to the two simultaneous (1b) and (7). A numerical procedure to find this location, if it exists, and the flow depth at that location is presented here. The numerical procedure is developed for three types of channel cross section: trapezoidal, triangular, and rectangular.

Trapezoidal Channel

Substituting (12)–(14) with $b > 0$ and $c_1 > 0$ into (11) gives:

$$S_o = \alpha \left[\left(\frac{u - k}{u}\right) \left(\frac{u - k}{u^2 - 1}\right) \right]^{1/3} + \beta[u(u^2 - 1)]^{-1/2} \dots \dots \dots (17)$$

where

$$\alpha = gn^2 \left(\frac{c_2}{2c_1}\right) \left(\frac{2c_2}{b}\right)^{1/3} \dots \dots \dots (18)$$

$$\beta = \frac{4q_L c_1^{1/2}}{(g^{1/2} b^{3/2})} \dots \dots \dots (19)$$

$$k = 1 - \frac{2c_1}{c_2} \dots \dots \dots (20)$$

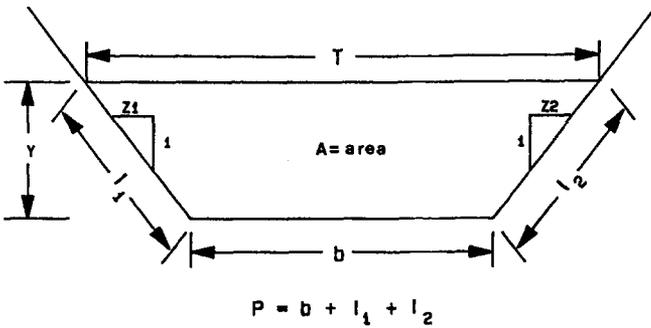


FIG. 2. Definition Sketch of Trapezoidal Channel Cross Section

$$u = \frac{T}{b} = 1 + \frac{2c_1 y}{b} \dots \dots \dots (21)$$

This form cannot be solved analytically, so a numerical procedure must be used. Define $h(u)$, $u > 1$, as:

$$h(u) = \alpha \left[\left(\frac{u - k}{u} \right) \left(\frac{u - k}{u^2 - 1} \right) \right]^{1/3} + \beta [u(u^2 - 1)]^{-1/2} - S_o \dots \dots (22)$$

We need to solve $h(u) = 0$ for $u > 1$. Since $h(1) = \infty$ and $h(\infty) = -S_o$, a solution exists. Differentiating (22) with respect to u produces:

$$h'(u) = \alpha f(u) - \beta G(u) \dots \dots \dots (23)$$

where

$$f(u) = \left(\frac{u - k}{u^2 - 1} \right)^{1/3} \left(\frac{-u^3 + 5ku^2 - u - 3k}{3u^2(u^2 - 1)} \right) \dots \dots \dots (24)$$

and

$$G(u) = \frac{[u(u^2 - 1)]^{-3/2}(3u^2 - 1)}{2} \dots \dots \dots (25)$$

Differentiating (23) with respect to u produces the second derivative of function $h(u)$:

$$h''(u) = \alpha M(u) + \beta N(u) \dots \dots \dots (26)$$

where

$$M(u) = 2 \left(\frac{u - k}{u^2 - 1} \right)^{1/3} \left[\frac{2u^6 - 20ku^5 + (7 + 20k^2)u^4 + 10ku^3 - (1 + 21k^2)u^2 - 6ku + 9k^2}{9(u - k)u^3(u^2 - 1)^2} \right] \dots \dots \dots (27)$$

$$N(u) = \frac{3}{4} [u(u^2 - 1)]^{-5/2} (5u^4 - 2u^2 + 1) \dots\dots\dots (28)$$

In general, there may not be a unique solution. In the case of $h''(y)$ positive, $h'(y)$ is strictly increasing and, since $h'(\infty) = 0$, $h'(y)$ is negative. This says that $h(u)$ is decreasing from $h(1) = \infty$ to $h(\infty) = -S_o$. Thus, in this case, there is a unique solution to this problem. Extensive numerical calculations showed that $h''(u)$ is positive when $k \leq 0.512$ (or $c_1/c_2 > 0.244$). This fact has been mathematically verified for $k \leq 0.398$. [This is done by showing that $M(u)$ is positive since $N(u)$ is always positive]. In the special case of a channel with equal side slopes, z , this corresponds to $z \geq 0.559$. In practice, this is frequently the case in most constructed and natural channels.

For computational purposes, when there are multiple solutions within the channel reach, the one closest to the lower end of the channel is chosen because it will likely be the control (Chow 1959). The following procedure aims to find the smallest solution.

A solution to $h(u) = 0$ can be obtained as follows: start iterations with $u > 0$ such that $h(u) > 0$ and $h'(u) < 0$. Use Newton's method (Press et al. 1987) of stepping as long as $h(y) > 0$ and $h'(y) < 0$. If the step is larger than a predetermined value, the step is limited to that value. If a location with $h(u) > 0$ and $h'(u) \geq 0$ is found, a predetermined amount can be added to u for the next step. If in stepping, a value for u is reached for which $h(u) \leq 0$, then the bisection method on the interval between this and the previous step will find a root of $h(u) = 0$. This stepping procedure will converge to a solution, but it cannot be guaranteed that the solution is the first root of $h(u) = 0$. In this case, it should be close to the first solution. Limiting the stepping size improves the chance of obtaining the first solution.

Triangular Channel

Substituting (12)–(14) with $b = 0$ and $c_1 > 0$ into (11) gives:

$$S_o = \alpha y^{-1/3} + \beta y^{-3/2} \dots\dots\dots (29)$$

where

$$\alpha = \left(\frac{gn^2}{2}\right) \left(\frac{c_2}{c_1}\right)^{4/3} \dots\dots\dots (30)$$

$$\beta = q_L \left(\frac{2}{g}\right)^{1/2} \frac{1}{c_1} \dots\dots\dots (31)$$

This form cannot be solved analytically, so a numerical approach must be undertaken. Define $h(y)$ as:

$$h(y) = \alpha y^{-1/3} + \beta y^{-3/2} - S_o \dots\dots\dots (32)$$

$h'(y)$ is negative so $h(y)$ is strictly decreasing from $h(0) = \infty$ to $h(\infty) = -S_o$. Thus $h(y) = 0$ has a unique solution. The second derivative $h''(y)$ is positive, so Newton's method will converge if iterations start at a $y > 0$ with $h(y) > 0$.

Rectangular Channel

Substituting (12)–(14) with $b > 0$ and $c_1 = 0$ into (11) gives:

$$S_o = \alpha(1 + \varphi y)^{4/3}y^{-1/3} + \beta y^{-1/2} \dots\dots\dots (33)$$

where $\alpha = gn^2$, $\varphi = 2/b$, and $\beta = 2q_L/bg^{1/2}$.

This form cannot be solved analytically, so a numerical approach must be used. Define $h(y)$ as:

$$h(y) = \alpha(1 + \varphi y)^{4/3}y^{-1/3} + \beta y^{-1/2} - S_o \dots\dots\dots (34)$$

$h'(y_o) = 0$ has a unique solution (minimum) satisfying:

$$2\alpha y_o^{1/6}(1 + \varphi y_o)^{1/3}(\varphi y_o - 1/3) = \beta \dots\dots\dots (35)$$

Thus $h(y)$ decreases from $h(0) = \infty$ to its minimum at $h(y_o)$ and then increases to $h(\infty) = \infty$. It is also true that $h''(y) > 0$. Thus $h(y) = 0$, $y > 0$ has no solution when $h(y_o) > 0$, one solution when $h(y_o) = 0$, and two solutions when $h(y_o) < 0$. It is not necessary to solve for y_o . Newton's method can be used as follows: Start iterations at $y > 0$ with $h(y) > 0$ and $h'(y) < 0$. If there is a solution, Newton's method with increasing estimates will converge to the smallest solution. If there is no solution, Newton's method will eventually produce a y value with $h'(y) > 0$.

PERFORMANCE AND EVALUATION TESTS

Computational Efficiency

Test runs of the numerical procedure were performed on a VAX 750 running under VAX/VMS to determine computational efficiency (speed and accuracy) of the proposed numerical procedure. Computational efficiency was tested using two problem sets. The first problem set consisted of all combinations of the values in the six columns of Table 1, except those with a zero bottom width and zero side slope. The second problem set used the critical flow depth values shown in the seventh column of Table 1 in place of the bottom slope values. Eq. (11) was then used to compute bottom slope from the critical depth values. Thus for the second problem set, the answer was known [unless the critical flow depth was not the first solution to (11)]. In both sets of problems, the testing program terminated a problem as soon as it identified that there was a solution or that the location of the critical control section exceeded the lower end of the channel reach by 1 km (in applications for which the routines were developed, channel lengths were considerably less than 1 km). In the second set of problems, those

TABLE 1. Parameter Values Used in Testing Proposed Procedure

Manning's <i>n</i> (1)	Bottom width (2)	First side slope (3)	Second side slope (4)	Lateral inflow (5)	Bottom slope (6)	Critical depth (7)
0.01000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00001
0.01167	3.33333	2.00000	2.00000	0.00100	0.00010	0.00100
0.01333	6.66666	4.00000	4.00000	0.10000	0.00100	0.10000
0.01500	10.0000	6.00000	6.00000	10.0000	0.01000	1.00000
					0.10000	10.0000

with bed slopes greater than 100 m/m were not solved. This resulted in 6,096 problems in the first set and 5,588 problems in the second set.

In the first set of problems, the computed error was measured by the relative error between the left- and right-hand sides of (11). This error measures how well the computed solutions satisfy (11). In the second set of problems, the computed error was measured by the relative error between the correct answer and the computed answer. This error measures how well the computed answer agrees with the correct answer. In both cases, $-\log(\text{error})$ is the approximated number of decimal places to which the two compared values agree.

Many of the problems tested (Table 1) may represent unrealistic situations. About 1.4% of the problems had no solution. About 62% had solutions (critical control sections) exceeding the real length of the channel. The possibility of multiple solution was verified with the second set of problems. In 16 problems of the second problem set, the "known" solution was not the first solution of (11). These problems were included in estimating speed of calculation and were excluded in accuracy evaluations.

In both problem sets, the internal tolerance used in testing for convergence was 0.0001. The first set required 0.0046 s/problem and the second set 0.0026 s/problem of CPU time. In the first problem set, the maximum computational error was 2.4×10^{-4} . Only four problems in the first problem set presented a computational error greater than 1.0×10^{-4} . These were cases for which flow transitions were found exceeding the lower end of the channel reach by 1 km. The maximum computational error was 6.0×10^{-5} for the second problem set.

Example of Method

The numerical procedure was used to determine the critical control section (flow transition) in a lateral spillway channel of uniform cross section (Chow 1959). To allow a comparison to be made, the problem solved was identical to that solved by Chow (1959) using the "equivalent critical depth channel" graphical procedure (Hinds 1926). To further test the proposed procedure, experimental data from a symmetrical V-shaped tilting flume were also used (Brutsaert 1971).

Chow's Example

Chow (1959, p. 342) presented a worked example for finding the location and depth of a critical control section in a spatially varied flow system. A trapezoidal lateral spillway channel 121.92 m (400 ft) long was designed to carry a lateral inflow rate of 3.716 m²/s (40 sq ft/s). The cross-sectional area had a bottom width of 3.048 m (10 ft) and side slopes of 0.5:1. The longitudinal slope of the channel was 0.1505. Manning's n is taken as 0.015, and a velocity distribution coefficient of 1.0 is assumed.

The location and flow depth of the critical control section were computed using the numerical procedure described in this paper. These are illustrated in Fig. 3 with the equilibrium water-surface profile computed using a finite difference version of (1) and the method presented by Chow. Distance x is measured from the upper end of the spillway channel. From Fig. 3 the critical control section is located at $x = 49.63$ m and at a depth of 5.37 m, showing full agreement with Hinds' graphical method used by Chow.

Brutsaert's Experiment

Brutsaert (1971) conducted a series of flume experiments to verify the applicability of a selected integration procedure for solving the Saint-Venant

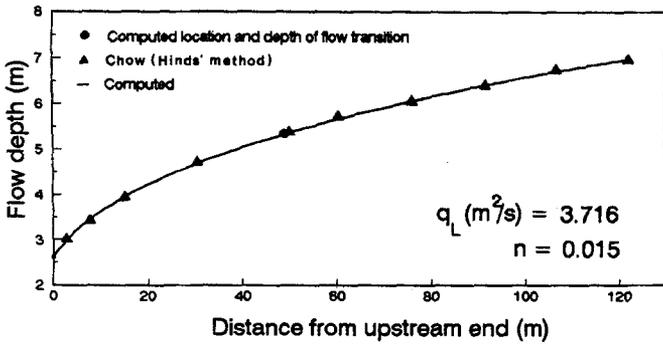


FIG. 3. Equilibrium Water-Surface Profile for 15% Slope and 121.92-m-Long Side-Spillway Trapezoidal Channel with $y_c = 5.37 \text{ m}$ at $x_c = 49.63 \text{ m}$ from Upstream End

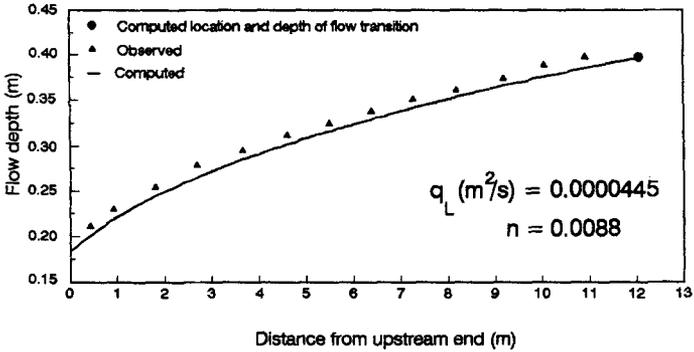


FIG. 4. Equilibrium Water-Surface Profile for 0.5% Slope and 12.04-m-Long Tilting V-Shaped flume with $y_c = 0.358 \text{ m}$ at Channel Outlet

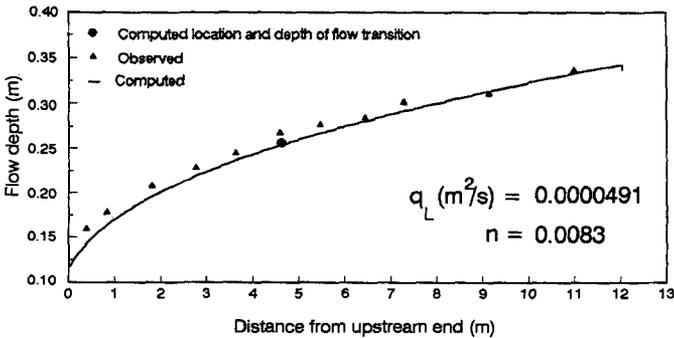


FIG. 5. Equilibrium Water-Surface Profile for 1% Slope and 12.04-m-Long Tilting V-Shaped Flume with $y_c = 0.256 \text{ m}$ at $x_c = 4.728 \text{ m}$ from Upstream End

equations. The experimental flow system consisted of a symmetrical V-shaped 12.04-m (39.5-ft) long tilting flume with side slopes of 1 on 1. No flow entered the upstream end and a velocity distribution coefficient of 1.0 was used. Inflow to the system was introduced laterally at a constant rate, regulated with an adjustable weirplate in a constant head tank.

The computed location and flow depth of the critical control sections using the procedure described herein are shown with the computed and measured equilibrium water-surface profiles for three flume experiments in Figs. 4–6. From Fig. 4, the critical control section was found at $x = 12.04$ m (channel outlet) and at a depth of 0.358 m. This result showed full agreement with Brutsaert’s experimental results. In this particular case, the critical control section was clearly influenced by an outlet structure (free overfall) rather than by lateral inflow. As a consequence, the predicted water-surface profile was slightly lower as compared to the actual profile. From Figs. 5 and 6, the critical control sections were found at $x = 4.728$ m and at a depth of 0.256 m (Fig. 5) and at $x = 0.845$ m at a depth of 0.124 m (Fig. 6), respectively, from the channel upstream end. From Fig. 5, the flow profile changed from subcritical to supercritical at about $2/5$ of the channel length. From Fig. 6, the flow transition occurred at a position very close to the upper end of the channel. Again, these results fully agreed with the experimental results of Brutsaert.

Nonzero Upstream Inflow

The proposed procedure can be expanded to address the more general cases of nonzero flow at the upstream boundary of the channel by introducing the concept of effective channel length. This concept is illustrated in Fig. 7. The effective channel length, L_e , is the length of a channel required to produce the outflow discharge, Q_e , given the lateral inflow rate q_L and upstream inflow rate Q_i . Applying the principle of conservation of mass:

$$Q_e = q_L L + Q_i \dots\dots\dots (36)$$

where L = the actual length of the channel (L). The outflow discharge can also be computed as:

$$Q_e = \frac{q_L L}{L_e} \dots\dots\dots (37)$$

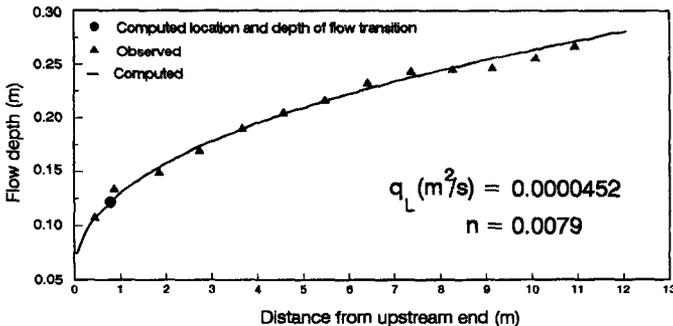


FIG. 6. Equilibrium Water-Surface Profile for 2% Slope and 12.04-m-Long Tilting V-Shaped Flume with $y_c = 0.124$ m at $x_c = 0.845$ m from Upstream End

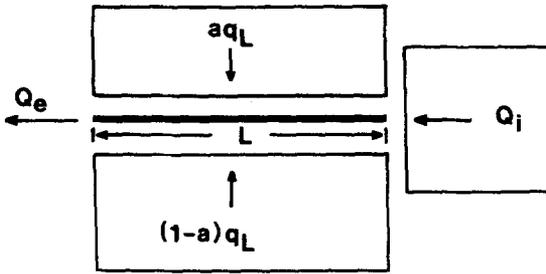


FIG. 7. Illustration of General Case for Concentrated Flow in Field-Sized Channel (length L) with Upstream Inflow (Q_e), Uniform Lateral Inflow (q_L), and Outflow (Q_i).

Substituting (37) into (36) and solving for L_e :

$$L_e = L + \frac{Q_i}{q_L} \dots\dots\dots (38)$$

The proposed procedure can be applied by solving the spatially varied flow equations [(1)] for a channel of length L_e given by (38).

CONCLUSIONS

A numerical procedure was described for quickly and accurately finding the location and flow depth of a critical control section in open channels with discharge gradually increasing along the length of the channel. It was found during this study that there are situations where multiple solutions exist.

A procedure was developed to decide if solutions exist and, if they do, to compute them and select an acceptable solution as the starting point for water-surface profile calculations. Test runs were carried out on a computer to verify the speed and accuracy of the procedure and experimental data were used to validate the procedure. A good agreement between the computed and measured location and flow depth of critical control sections was obtained for graphic example and a set of flume experiments.

The development of a critical control section within a channel reach subject to spatially varied flow with uniform lateral inflow depends on flow conditions and channel characteristics, including, channel width, channel slope, surface roughness, and cross-sectional geometry. Results from a wide range of flow conditions and channel characteristics examined in this study showed that the geometry of the channel cross section is an important factor determining the existence of a unique solution to the critical control section location problem in open channel flow. The study conducted here was limited to three channel cross-sectional geometries: trapezoidal, triangular, and rectangular. There were no problems in finding unique solutions, when they exist, for triangular and rectangular channel cross sections. However, for trapezoidal channel cross sections, there were cases with multiple solutions within the channel reach. Sometimes multiple solutions were very close such that considering one or another as the critical control section would not affect the reliability of the computed water-surface profiles. However, in some cases, the solutions were far enough apart to affect significantly the accuracy of the computed water-surface profiles depending on what solution was chosen as the starting point. Although the reasons for the

occurrence of multiple solutions in trapezoidal channel cross sections are not evident, these findings demonstrate a need for further developments toward a more comprehensive mathematical analysis of the conditions for developing a critical control section in complex channel sections. Concurrent flume experiments should be carried out to validate the findings of the mathematical analysis.

The numerical procedure described in this paper is beneficial to hydraulic engineers solving problems related to lateral spillway channel design and to hydrologists using spatially varied flow equations to describe flow processes in natural channels. The procedure can also be used to extend and enhance the application of hydrologic simulation models such as the CREAMS field-scale model and the WEPP watershed model by eliminating the limiting assumption of triangular cross section and preassumed critical control section at the channel outlet.

APPENDIX. REFERENCES

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