93

Calculation of Pond Inflow Hydrographs by a Chord Slope Method and a Volume Increment Method

D. L. Chery, Jr., and L. J. Lane

THE Field Manual for Research in Agricultural Hydrology* presents a method by which inflow into a pond or small reservoir is calculated using the slope of chord segments along the curve of recorded water level elevations to approximate its derivative. The chord slope approximation of the derivative is then multiplied by values from a "Pondage Correction" table to obtain the amounts of flow going into or coming out of storage in the pond. This method approximates the differential relation describing the changes of storage in the pond, and requires calculation of a "Pondage Correction" table. The "Pondage Correction" table lists values of volume per unit time that are stored at fixed pond depth intervals for a uniform change of water surface elevation (usually 1 ft per min). An alternate method (referred to in this paper as the volume increment method) calculates increments of storage ($\Delta S/\Delta t$) directly and then uses a statement of continuity ($I = \Delta S/\Delta t + Q$) to determine the inflow hydrograph, where I is the inflow and Q is the outflow.

This paper explains in detail the two methods of calculating the pond inflow hydrographs and discusses the accuracy of each method.

Chord Slope Method

The chord method is based on the following argument:

The surface area (A) of a pond can be expressed as some function of elevation (z).

$$A = A(z) \quad [1]$$

A differential change of volume (dV) at any pond elevation can then be expressed as:

$$dV = A\,dz \quad [2]$$

Then, to express a time rate change of this differential volume, the expression becomes

$$\frac{dV}{dt} = A\frac{dz}{dt} \quad [3]$$

Substituting the functional relation of pond surface area with respect to elevation (equation [1] into equation [3]), the following relation is obtained:

$$\frac{dV}{dt} = A(z)\frac{dz}{dt} \quad [4]$$

In this method, a constant rate of elevation change (stage) is assumed, and it is usually assumed to be 1 ft per min. Since the units of runoff rates are usually expressed in cu ft per sec (cfs), the constant is expressed as:

1 ft per min = 1 ft per 60 sec.

When this constant rate of elevation change is used, equation [4] becomes:

$$q_p = \frac{dV}{dt} = A(z)\frac{1}{60} \quad [5]$$

and with this relation, a table of "pondages q_p" for uniform elevation increments in a pond is prepared by evaluating equation [5] (measuring the area within the contour line) at each of the elevation increments and dividing by 60, i.e.,

$$(q_p)_i = \left.\frac{dV}{dt}\right|_{z=z_i} = A(z_i)\frac{1}{60} \quad [6]$$

Procedural steps for the chord slope method are:

(a) Prepare pond surface area versus pond elevation (gage height) relation by planimetering successive contours of a topographic map of the pond.

(b) Divide the area at each increment of elevation by 60 to obtain pondage (q_p) for the constant rate of stage change.

(c) Plot the q_p values against pond elevation (z) and draw a smooth curve through the points. Pick additional values from the curve for whatever increments of pond elevation are desired. (Usually increments of 0.1 ft are used.)

(d) If desired, further refinement of the area versus elevation relation may be made by interpolation. This is done to prepare tables of q_p versus elevation in 0.01-ft increments. (It is suggested that the method of increasing first differences† be used to make the interpolation.)

(e) Tabulate from recordings of pond water surface elevation (gage height) a series of discrete values of elevation versus time in chronological order.

(f) Calculate the rate of change of stage. The rate of change of stage is approximated by the slope of the chord between points *equal* time intervals preceding and succeeding the particular point under consideration.

$$\frac{\Delta z_i}{\Delta t_i} = \frac{z_{i+1} - z_{i-1}}{t_{i+1} - t_{i-1}} \quad [7]$$

If the times between stage intervals are sufficiently small, the chord is approximately parallel to the tangent.

(g) Calculate the instantaneous flow rate into the pond (I_t) at time t_i by multiplying the "pondage" (q_p) value of z_i times the rate of change of stage ($\Delta z_i/\Delta t_i$) [understood to be in units of ft per min because unit rate was 1 ft per min.]

$$I_{t_i} = q_{pz_i}\frac{\Delta z_i}{\Delta t_i} \quad [8]$$

The storage change associated with an actual pond level record is the product of the measured rate of elevation change (approximated by the slope of a chord) and the time rate change of storage per a unit rate of elevation change.

Volume Increment Method

The volume increment method is based on the following argument:

The surface area (A) of a pond can be expressed as a function of elevation (z).

$$A = A(z) \quad [1]$$

A differential change in volume (dV) at any pond surface elevation (z) can then be expressed as:

$$dV = A\,dz \quad [2]$$

An approximation for this differential volume element is:

$$\Delta V_i = 1/2(A_{i-1} + A_i)\Delta z = \bar{A}_i\,\Delta z \quad [9]$$

where $\bar{A}_i$ is the average surface area for this incremental change in elevation located at the midpoint of the interval, and i is an index variable. The incremental time rate of volume change is then

$$\frac{\Delta V_i}{\Delta t} = \bar{A}_i\frac{\Delta z}{\Delta t} \quad [10]$$

Procedural steps for the volume increment method are:

Article was submitted expressly for publication in TRANSACTIONS of the ASAE. Contribution of the Southwest Watershed Research Center, SWCRD, ARS, USDA, in cooperation with the Arizona Agricultural Experiment Station, Tucson.

The authors are: D. L. CHERY, JR., Research Hydraulic Engineer, and L. J. LANE, Hydrologist, Southwest Watershed Research Center, Tucson, Ariz.

* Field Manual for Research in Agricultural Hydrology. Agricultural Handbook No. 224, Agricultural Research Service, U.S. Dept. of Agriculture, June 1962.

† A numerical method of nonlinear interpolation that maintains an increasing first difference and a nondecreasing second difference throughout the interval of estimation.

This article is reprinted from the TRANSACTIONS of the ASAE (Vol. 15, No. 3, pp. 433, 434, 435 and 439, 1972)
Published by the American Society of Agricultural Engineers, St. Joseph, Michigan

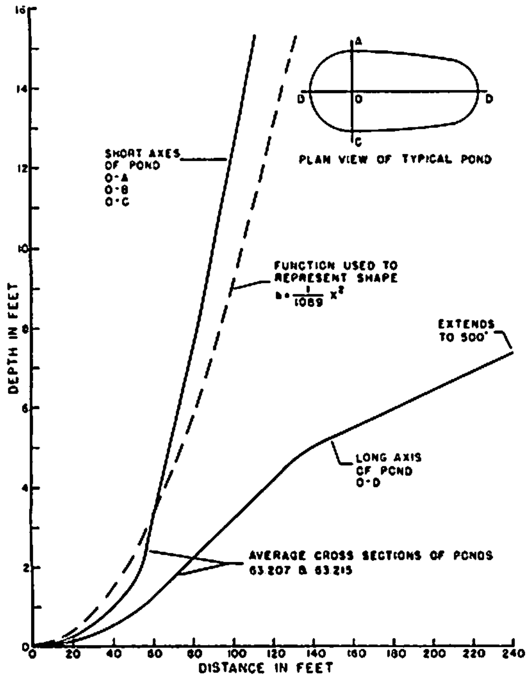

FIG. 1 Comparison of assumed pond cross section with actual pond cross section on the Walnut Gulch Watershed.

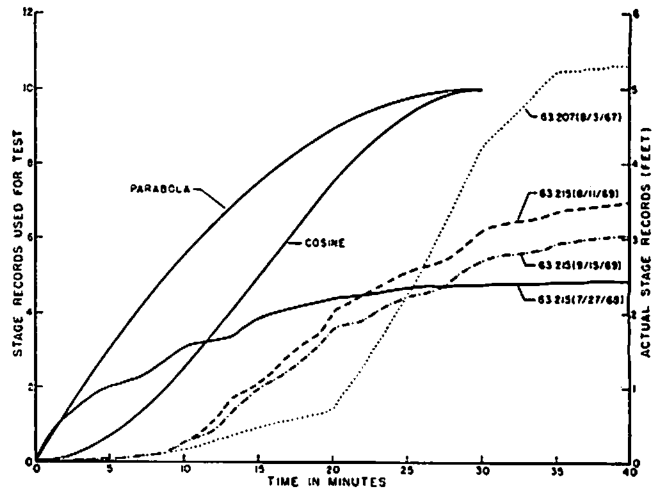

FIG. 2 Stage functions and actual stage records.

(a) Prepare pond surface area (A_i) versus pond surface elevation (z_i) relation by planimetering, or otherwise calculating, the area enclosed within successive contours of a topographic map of the pond. By this relation V_i is determined for each gage height z_i, using equation [9]

$$\Delta V_i = 1/2(A_{i-1} + A_i)\Delta z = \bar{A}_i \Delta z$$

(b) Prepare volume versus elevation table using the method of increasing first differences.

(c) From a tabulation of gage height versus time, read from the analog trace on the recorder chart, determine z_i, and $\Delta z_i/\Delta t_i$ (equation [7]).

(d) Calculate $\Delta V_i/\Delta t$ using equation [10] ($\Delta V_i/\Delta t = \bar{A}_i\Delta z/\Delta t$).

(e) The term $\Delta s/\Delta t$ in the statement of continuity ($I = \Delta s \Delta t + Q$) is equivalent to the term $\Delta V_i/\Delta t$ in equation [10].

(f) The continuity equation is evaluated at each z_i associated with time t_i for the inflow rate I, located in time at the midpoint of the interval t_{i-1} to t_i.

Evaluation of Methods

To evaluate how well each method calculated the inflow rates to a pond, mathematical functions representing the time records of pond elevations were convoluted by both methods with a theoretical pond shape and the results compared.

The pond was assumed to be a parabolic cylinder. The calculations were made with respect to a unit length of the assumed parabolic cylinder. The pond cross section represented by the parabola is realistic when compared with the average cross sections of two ponds on the Walnut Gulch watershed (Fig. 1). From the mathematical representation of pond shape, usable mathematical expression for surface area and volume at any height were available.

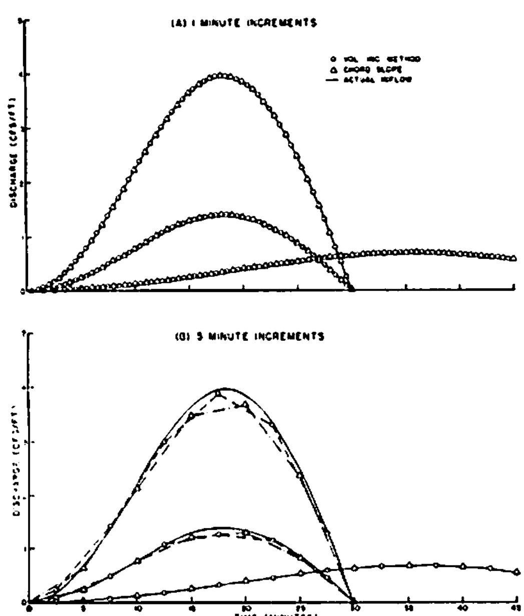

FIG. 3 Actual inflow rates with superimposed rates calculated by two different approximation methods, cosine input.

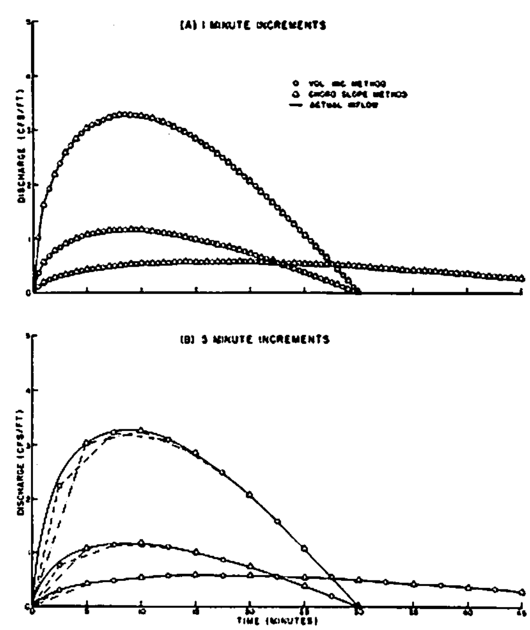

FIG. 4 Actual inflow rate with superimposed rates calculated by two different approximation methods, parabolic input.

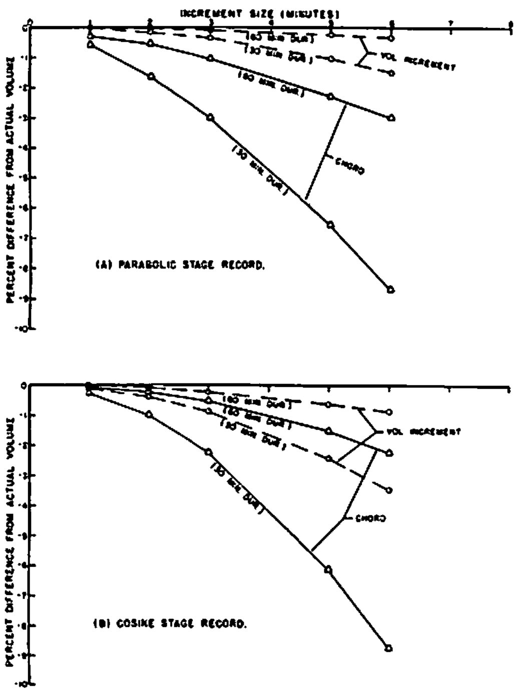

FIG. 5 Change in error with increasing increment size.

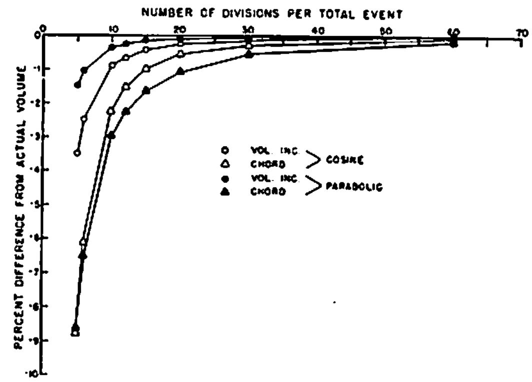

FIG. 6 Error with respect to total number of divisions into which the total duration of the event is divided.

Mathematical expressions for the stage records of pond inflow were assumed. The functions used for the evaluation are plotted in Fig. 2. The upper curve is the function:

$$h = a_1 - a_3(t - a_2)^2 \quad . \; . \; . \; . \quad [11]$$

which is a parabola with the origin at $h = a_1$ and $t = a_2$ (h is gage height, and t is time). The lower curve is the function:

$$h = a\,[1 - \cos(b)] \quad . \; . \; . \; . \; . \quad [12]$$

which is a cosine curve where the argument is: $b = 2\pi t/T$.

Both curves are reasonable representations of actual pond stage records, several of which are also illustrated in Fig. 2. Maximum rates of stage change vary between 0.2 and 1.0 ft per min. No rates greater than 1.0 ft per min have been measured in the records accessible to the authors. The maximum rates of change in stage used as tests in this study ranged from 0.26 to 1.29 ft per min.

Figs. 3 and 4 show the resulting inflow hydrographs to the pond as a result of the assumed stage records shown in Fig. 2. Notice that while the cosine function yields a steeper and sharper peak, the parabola yields a steep-rising hydrograph with a more realistic skew.

The numerical integration of each method for several combinations of increments between readings, total stage increase, and total flow duration was used to evaluate how well each method represented the *total hydrograph*.

The actual inflow hydrographs are compared with inflow hydrographs calculated by each method at selected increments for an evaluation of how well *instantaneous inflow rates* were determined.

Discussion and Conclusions

The results of the tests using the assumed pond shape and two variations of stage input and two time increments are illustrated in Figs. 3 and 4.

The estimates by each method are nearly indistinguishable from the actual flow record when the increments were one minute. Obviously, the fidelity decreases when the increments are increased to five minutes (Figs. 3B and 4B). For the cosine input, rates at the beginning of the flow were increasingly overestimated, and those near the peak increasingly underestimated as the increment size increased.

It also must be realized that each method may exactly locate or be within one-half of an increment of locating the peak flow rate. Further, each method locates the peak value at a position one-half of an increment away from the other. This disparity becomes more noticeable as the size of the increment increases. One method is not better than the other in locating the peak. The location of the peak inflow with respect to the true peak is a function of the increment chosen. For instance, when three-unit increments were used, the chord-slope method located the peak more accurately whereas, when five-unit increments were used, the volume-increment method located the peak more accurately.

For the combination of the parabolic stage relation with the chord slope method (Figs. 4A and 4B), the inflow rates were calculated exactly, because the method calculates the tangent of a parabola exactly. This special relationship does not hold in general.

Differences between the actual volume and that determined by numerical integration of the hydrographs determined by each method are shown in Figs. 5A and 5B. For a given stage function, the percentage error curves did not vary with the amplitude of the input, but only with the time increment size. The percent error was the same for small and large amplitudes, because while the absolute errors are larger with the larger inputs, the ratio of error to amplitude remains constant. Thus the absolute error is a function of the rate of stage change and the increment size and the percent error is only a function of the increment size for a given period or the number of increments for any period.

We also noted that the errors of the 60-min duration were exactly the same as those for the 30-minute duration input when the number of divisions for the entire event were the same. Or in another way of viewing it, the 60-min input with 1-minute time increments could be a 30-min input with 1/2 min time increments. Errors calculated for a 30-min input with divisions of 0.5, 1, 1.5, 2.5, and 3 min would be the same as for the 60-min duration with divisions of 1, 2, 3, 5, and 6 min. As a consequence, the percent error for a given input of the calculated pond hydrograph shape is merely a function of the total number of divisions into which the stage record is divided, as is illustrated in Fig. 6.

These results show that at least 20 divisions per event should be taken to assure that the error will not be greater than −1 percent. It probably would be better to divide the event into 30 to 50 divisions and be well assured of a good representation of the hydrograph shape.

On the average, the calculated flow rates will be less than the actual flow rates, because the summed volume always comes out less than the actual volumes. This has been indicated by percentage differences less than zero.

Once a record has been divided into a sufficient number of divisions, both methods are essentially comparable, especially when other errors in obtaining pond inflow measurements and preparing the records are considered. In addition, some ponds may have a high infiltration rate through the sides and bottom and the inflow rates must be adjusted by some estimates of seepage rates. Thus the method used becomes a matter of preferences based on external reasons.