Comment on 'Derivation of an Equation of Infiltration' by H. J. Morel-Seytoux and J. Khanji

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The Green-Ampt equation [*Morel-Seytoux and Khanji*, 1974, (1)] is simply Darcy's equation applied to a vertical bundle of noninterconnected capillary tubes saturated to depth z_f, at which a capillary pressure head H_f helps to drive the infiltration process. Morel-Seytoux and Khanji have made a significant contribution by modifying the equation for the capillary tube model to make it more realistic for a soil column model. This involved deriving effective capillary drive H_c and the viscous resistance correction factor β. Although this modification may increase the prediction accuracy of the Green-Ampt equation for the column infiltration system and aid in understanding the hydraulic behavior of this system, it does not necessarily follow that hydrologists will realize these benefits when they apply the improved equation to natural soil infiltration systems, since these systems exhibit initial and boundary conditions that are quite unlike those of the assumed system and often dominate the infiltration process [*Dixon*, 1972]. Natural soils are seldom (if ever) stable, homogeneous, unlimited in depth, uniformly moist, and exposed to a constant surface head.

Because of its unrealistic physical basis the Green-Ampt equation (even with the authors' refinements) seems less suited for use in applied hydrology than an even simpler equation often referred to as Kostiakov's equation [*Kostiakov*, 1932; *Lewis*, 1937]. Kostiakov's equation is more convenient to use than the Green-Ampt equation, since it expresses infiltration rate I as a function of time instead of as a function of wetting front depth. In natural soil systems containing macropores and nonuniform initial moisture distribution, both H_f and z_f are often undefinable and incapable of direct physical characterization. For these soil systems the parameters K, A, and B are commonly determined empirically from estimated wetting depth and measured infiltration rate for two times. When they are determined in this way, they lose their physical significance (preciseness), as is shown by the results of *Swartzendruber and Huberty* [1958]. They found that fitting the Green-Ampt equation to field data could result in negative values of the parameter theoretically identifiable with the hydraulic conductivity or the final infiltration rate (parameters K and A in the authors' notation). They attributed these negative values to the failure of the basin infiltration system to satisfy basic assumptions of the Green-Ampt derivation. They concluded that Kostiakov's equation was preferable to Green and Ampt's for this natural infiltration system. Generally, physics-based equations have been shown to fit experimental field data less accurately than the strictly empirically derived equations. *Skaggs et al.* [1969] found that Horton's and Holtan's equations fitted data better than Philip's and Green and Ampt's. Again it was felt that the infiltration system under study did not satisfy the derivation assumptions of the physics-based equations.

Kostiakov's equation has been accurately fitted to field data from unstable soils to which Philip's two-term equation could not be fitted [*Taylor and Ashcroft*, 1972]. Yet it compared favorably with this same two-term equation in the fitting of data from a numerical analysis of Philip's flow equation [*Philip*, 1957]. Following the authors' intuitive reasoning, Kostiakov's equation has been found to fit so well that one might wonder if it is not physically based. It is, in fact, the first term of Philip's equation and was derived empirically 25 years earlier [*Dooge*, 1973]. Kostiakov's equation $I_v = aT^b$ can be considered a general infiltration equation (even though the authors contend that there is no such thing), whereas Ostiachev's equation $I_v = AT^{1/2}$ [*Ostiachev*, 1936] and Darcy's equation $I_v = kiT$ [*Darcy*, 1856] apply to the special cases of capillary-induced flow into dry infiltration systems and gravity-induced flow into near-saturated systems, respectively. In these equations, I_v is the cumulative infiltration volume, T is the time elapsed after incipient ponding, and a, b, A, k, and i are constants. In Ostiachev's equation, $a = A$ and $b = \frac{1}{2}$, whereas in Darcy's equation $a = ki$ and $b = 1$. The infiltration conditions under which Kostiakov's equation can be applied include (1) small, intermediate, or large elapsed times, (2) one-, two-, and three-flow dimensionality, (3) uniform and nonuniform porous media, (4) open, partly open, and closed lower boundaries, (5) time invariant or variant moisture content, and (6) zero to many infiltration-related decay processes. This equation has been successfully fitted to data from flooded basins, single- and double-ring infiltrometers, sprinkling infiltrometers, long laboratory columns, and numerical analyses of Philip's flow equation. It has been precisely fitted to field infiltration data collected under a wide range of vegetal, edaphic, and climatic conditions [*Free et al.*, 1940; *Lewis*, 1937; *Tisdall*, 1950]. This success suggests that the general form of Kostiakov's equation is appropriate for the initial and boundary conditions of field infiltration systems.

The apparent reason for the better fit of empirical equations to field data as compared with physics-based equations is that the parameters of empirical equations can be more appropriately adjusted to fit the complexities of natural infiltration systems. Many of these complexities are necessarily neglected in deriving physics-based equations. Infiltration has long been recognized as a process reflecting the net effect of several concurrent decay processes [*Horton*, 1940], since the rate of the infiltration process is inversely related to the time elapsed after the onset of the process. Physics-based equations (including Green and Ampt's) usually only account for the decay in the capillary pressure gradient resulting from the ever-deepening wetting front in a simple column model which is assumed to be homogeneous, initially uniformly dry, hydrophilic, stable, and exposed to atmospheric pressure at the lower end. In natural soils under complex initial and boundary conditions the decay in capillary pressure gradient may be relatively unimportant as compared with other infiltration decay sources or processes. These include (1) capillary pressure head reduction at the wetting front (resulting from increasing moisture content with depth), (2) surface crusting or sealing, (3) soil subsidence or settling, (4) soil air pressure buildup and air entrapment, (5) clay mineral hydration, (6) eluviation and illuviation, (7) surface water head dissipation, (8) macroporosity extent and continuity reduction with depth in the profile, and (9) anaerobic slime formation. *Childs and Bybordi* [1969] have extended the Green-Ampt approach to include one additional decay process in the form of depth hydraulic conductivity reduction, whereas *Hillel and Gardner* [1970] have taken a similar approach for soil crusting. However, the parameters of Kostiakov's equation can be adjusted by simple curve-fitting techniques to account for the interacting effect of most (if not all) of these decay processes.

Kostiakov's equation is often thought to be of the wrong form, because it lacks a constant rate term [*Childs*, 1967]; however, apparently, there is no clear physical basis [*Russel*, 1946] for such a term (which appears in the Green-Ampt equation and in many other empirical infiltration equations) in the modeling of natural infiltration systems. In fact, the constant rate term may be largely an experimental artifact associated with the inability to detect small rate changes and the increasing dimensionality of flow (with time) beneath the typical infiltrometer. Such constant rate terms are also supported by the mistaken notion that soils are modeled adequately by columns (open at the bottom but bounded laterally) of stable soil material, for which a constant rate exists in both theory and fact. Kostiakov's equation does allow for a diminishing rate of infiltration decrease such that after sufficient time the rate becomes practically constant or undetectably variable.

If the infiltration rate is identified with the rate of water storage within the soil profile (as was suggested by *Childs* [1969]), then certainly, the need for a constant rate term would be greatly diminished and the appropriateness of the form of Kostiakov's equation would be enhanced. The equation correctly provides for high storage rates when available storage is large or time is small and for an asymptotic approach to zero storage rate upon complete exhaustion of the finite storage space at large times. In a nonleaky infiltration system, total infiltration and infiltrated water storage would be identical, whereas in a leaky system they would be the same until leakage began, after which time, total infiltration would be the sum of storage and leakage. For infiltrometers, leakage is in the form of lateral flow and profile drainage, whereas for watersheds under natural rainfall it would be in the form of return flow (seepage) and profile drainage. Although steady state leakage (a condition seldom found in nature) is identifiable with final infiltration rate, it is perhaps a preferable term, since a measure of vertical hydraulic conductivity of the soil profile is not suggested. According to *Philip* [1969] the final infiltration rate as measured with an infiltrometer is not directly related to (and not necessarily well correlated with) the final infiltration rate for one-dimensional infiltration over a large area. This is because the boundary conditions, imposed by an infiltrometer and the underlying soil profile, usually allow increasing dimensionality of flow with time. The greater the dimensionality of flow allowed by the infiltrometer (a function of infiltrometer design), the greater the final infiltration rate will be. Thus to a certain degree, final infiltration rates are an artifact of infiltrometer type and should not be identified with hydraulic conductivity at residual air saturation. Instead, they merely reflect the ease with which water can leak from the system. Steady state leakage from a naturally watered soil system would also be multidimensional, although natural infiltration seldom proceeds long enough to approximate a steady state condition closely. Thus the presence of a final infiltration rate term in an equation modeling natural infiltration systems is at best misleading if not essentially superfluous.

In Kostiakov's equation $I_v = aT^b$, parameter a is the infiltration volume I_v during the first unit of elapsed time T after the onset of ponding, ab is the infiltration rate at the end of this unit, and parameter b is the ratio of the two [*Swartzendruber and Huberty*, 1958]. Thus if time is in hours, parameter a is estimated by the first-hour infiltration volume, ab is estimated by the intake rate at the end of the first hour, and b is the ratio of the two reflecting intake abatements during the first hour.

If Kostiakov's equation is regarded as modeling infiltration storage rather than total infiltration, then I_v becomes the storage volume of infiltrated water, T is the time elapsed after incipient ponding (during which water is being stored), parameter a is the storage during the first hour, ab is the storage rate at the end of the first hour, and b is the ratio of the two which reflects the degree of storage abatement during the first hour, $b = 1$ indicating no abatement and $b = 0$ complete abatement. The magnitude of parameter b is inversely related to the number and intensity of active infiltration decay processes. Parameter a is simply the product of the first-hour time-weighted means for the hydraulic conductivity and the hydraulic gradient at the soil surface in accordance with Darcy's law and the view of surface infiltration presented by *Childs* [1969]. Parameters a and b are interdependent, since the mean conductivity and gradient are affected by all of the decay processes. Some of the decay processes affect (and are affected by) boundary conditions. Thus the magnitudes of parameters a and b are controlled by an extraordinarily complex interaction of numerous decay processes, some of which in turn depend on system boundary conditions. Since boundary conditions are partially a function of the infiltrometer type, the values of a and b will also be affected by infiltrometer type.

Thus contrary to the inference that might be drawn from the authors' concluding statement, a successful general equation with easily determinable parameters (that have physical significance for the simple systems described by Darcy's and Ostiachev's equations and that are capable of physical interpretation for complex natural systems) has been available for more than 40 years. Workers in applied hydrology might profitably rediscover Kostiakov's equation.

These comments are not intended to detract from the authors' significant contribution, since deepening the understanding of a simple system can lead to a better comprehension (if only qualitative) of complex natural systems. Instead, my intention has been to contrast the simple Green-Ampt model with the complex natural systems by stressing that the step from the capillary tube model to the column model (assumed by the authors) is but a small step in the direction of natural soil profiles. This fact is not always appreciated, since many authors of soil physics papers erroneously use the term 'soil type' synonymously with 'column model,' as do *Morel-Seytoux and Khanji* [1974, Table 1]. This leads some inexperienced readers to confuse column models and infiltration into these models with natural soil profiles and natural infiltration. Such confusion only hampers the appropriate application of infiltration theory to watersheds and the soil phases (types) of which they are composed.

In summary, the accuracy and practicality of the authors' derived formulas are limited to the simple column infiltration system upon which they based their derivations. Direct physical characterization of equation parameters and their very precise physical meaning is similarly limited. The extension of the authors' approach to systems with rising air pressures and to systems with heterogeneous media would also represent significant contributions. Although the isolated effect of a single decay process may be quite unlike the interaction effect, this knowledge is still fundamental to the complete understanding of infiltration phenomena.

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