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AN ANALOG COMPUTER DEMONSTRATION OF DOUBLE-PEAKED INSTANTANEOUS UNIT HYDROGRAPHS[1]

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ABSTRACT. The properties of an instantaneous unit hydrograph model consisting of two cascades of linear reservoirs in parallel were explored with the aid of an analog computer. By proper choice of the model parameters it is possible to produce two-peaked instantaneous unit hydrographs. The relative magnitudes and locations of the two peaks can be varied by changing the values of the parameters. An example of the use of the analog computer to select the parameters of the model giving the best fit to an observed runoff hydrograph is also included. The analog computer used in the study was the ASTRAC II developed at the University of Arizona.
(KEY TERMS: analog computer; conceptual model; hydrology; instantaneous unit hydrograph; linear reservoirs cascade; rainfall-runoff; simulation)

INTRODUCTION

The use of an analog computer for the study of rainfall-runoff relationships is not a new idea. Paynter [1952], reporting on work done at M.I.T. on unsteady flow, outlines a scheme for representing the watershed by a system of admittance elements each of which is analogous to a section of the major river network in the watershed. Other examples for the use of analog computers can be found in the works of Ishihara [1961], Shen [1963], and Prasad [1967]. A review of the types of analogs and their uses in surface hydrology is given by Chow [1964].

One of the advantages of analog computers is their great flexibility. The possibility of continuously varying their parameters or changing the inputs and immediately observing the outputs obtained is very useful in cases where exploratory studies are made. In such studies there is often no significance to intermediate results and no need to keep record of them. Once the desired solution has been found, it can be studied more carefully either by the analog computer or by other means.

This paper is a demonstration of such an exploratory study on the properties of a conceptual model proposed [Diskin, 1964] for representing rainfall-runoff relationships. The model, consisting of two chains of linear reservoirs in parallel, has five free parameters. While the model does not present any theoretical or computational difficulties, the fact that there are five parameters, each varying over a wide range, leads to a very large number of possible

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combinations. The study of such a model on an analog model was very convenient and brought out the characteristics of the model. The use of the analog computer was found to be useful also for fitting a set of parameters to a model describing a given rainfall-runoff event on a semiarid watershed in southeast Arizona.

THE HYDROLOGIC MODEL

Instantaneous unit hydrographs are often depicted as single-peaked, positively skewed curves. The shape of the curve has been described by a number of mathematical models and by empirical dimensionless curves derived from field observations for particular regions. The best known mathematical or conceptual model for single-peaked unit hydrographs is probably that represented by a cascade of N equal linear reservoirs each having a time constant K (Fig. 1). The model has been proposed by Nash [1957, 1959] who derived the equation of the instantaneous unit hydrograph corresponding to the model. The equation has two forms. One is for an integer number of reservoirs.

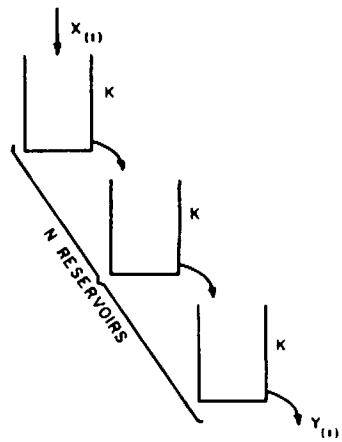

Fig. 1. Cascade of N equal linear reservoirs.

$$U_{(t)} = \frac{1}{K(N-1)!}\left(\frac{t}{K}\right)^{N-1} e^{-t/K} \tag{1}$$

The second form is an extension of the first equation for a noninteger number of reservoirs

$$U_{(t)} = \frac{1}{K\Gamma(N)}\left(\frac{t}{K}\right)^{N-1} e^{-t/K} \tag{2}$$

The difference between the two equations is in the use of the continuous gamma function in Equation (2) to replace the discrete factorial term in Equation (1). An equation of the same form as Equation (2), but using different notation and based on a different approach, has also been proposed by Edson [1951]. The equation proposed by Edson was given as suitable for both the instantaneous and the finite duration unit hydrographs.

The single-peaked instantaneous unit hydrograph gives good results in many cases, and the linear reservoir model represented by Equations (1) and (2) is widely used in hydrology. There are, however, cases where the simple single-peaked hydrograph does not give a good representation of the rainfall-runoff relationship. Poor agreement is obtained in these cases between the observed hydrographs of surface runoff and those computed by convolution of rainfall excess and the single-peaked instantaneous unit hydrograph. By varying the parameters of the unit hydrograph equation about their optimal values, it is possible to get good agreement in one or another part of the runoff hydrograph, but the entire curve cannot adequately be reproduced.

There are many cases where the runoff hydrograph resulting from a rainfall of short duration and uniform areal distribution is in the form of a double-peaked curve or a curve with an extended flat portion on its recession limb. These hydrographs are obtained at gaging stations located downstream from the confluence of main stream tributaries or in cases where the time

of travel for one part of the watershed is significantly different from that for the other parts because of local topographic or hydraulic conditions. The instantaneous unit hydrograph for such cases should also display a double peak if it is to give a close reproduction of the measured hydrograph.

A conceptual model which produces, under certain combination of its parameters, double-peaked instantaneous unit hydrographs is shown in Figure 2. The model proposed by Diskin [1964] is a generalization of the cascade model shown in Figure 1. It is composed of a parallel combination of two cascades of equal linear reservoirs. Each cascade is composed of identical reservoirs, but the number of reservoirs and their time constants are different for the two cascades. The number of reservoirs in each cascade is an integer number. The input to the model is split into two parts which serve as inputs to the two cascades. The outputs of the two cascades are combined to form the output of the model.

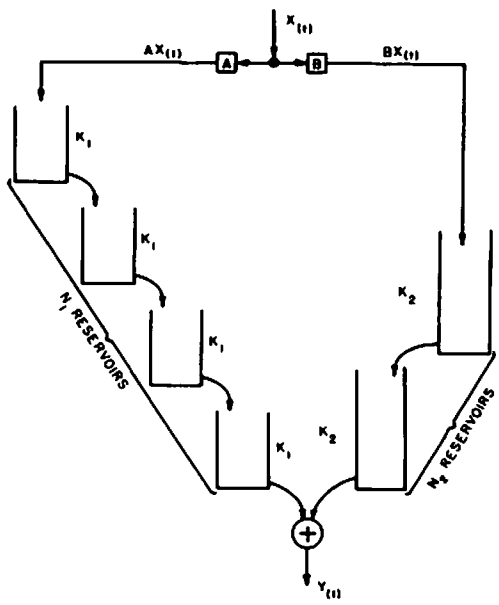

Fig. 2. Parallel combination of two cascades of linear reservoirs.

The equation of the instantaneous unit hydrograph for this model is

$$U_{(t)} = \frac{A}{K_1(N_1-1)!}\left(\frac{t}{K_1}\right)^{N_1-1} e^{-t/K_1} + \frac{B}{K_2(N_2-1)!}\left(\frac{t}{K_2}\right)^{N_2-1} e^{-t/K_2} \tag{3}$$

The equation contains six parameters (A, B, N_1, N_2, K_1, and K_2), five of which are independent. Two of the constants, A and B, describing the division of the input between the two cascades, are subject to the condition

$$A + B = 1.0 \tag{4}$$

Equation (1), for a single cascade of reservoirs, can be obtained as a special case of Equation (3) by taking either

$$N_1 = N_2 \text{ and } K_1 = K_2 \tag{5a}$$

or

$$A = 1.0 \quad \text{and } B = 0.0 \tag{5b}$$

Using the Laplace transform of Equation (3)

$$\overline{U}_{(p)} = \frac{A}{(pK_1+1)^{N_1}} + \frac{B}{(pK_2+1)^{N_2}} \tag{6}$$

It can be shown easily [Diskin, 1967] that the first two moments of the instantaneous unit hydrograph about the origin are

$$M_{u,o}^{(1)} = AN_1K_1 + BN_2K_2 \tag{7}$$

$$M_{u,o}^{(2)} = AN_1(N_1 + 1)K_1^{\,2} + BN_2(N_2 + 1)K_2^{\,2} \tag{8}$$

The shape of the curve defined by Equation (3) is obtained as a result of adding two simple single-peaked unit hydrographs having their vertical scales modified according to the relative values of the constants A and B. The location of the peaks of the two individual curves depends on the values of N and K of each cascade. The times to the peaks of the individual curves is obtained by differentiating Equation (1) with respect to t and setting the result equal to zero.

$$T_1 = (N_1 - 1)K_1 \tag{9}$$

$$T_2 = (N_2 - 1)K_2 \tag{10}$$

If these times are sufficiently separated, the peaks will remain as two distinct peaks when the two curves are added. The locations of the peaks will, however, be shifted with respect to the locations given by Equations (9) and (10) as a result of the addition of the two curves.

If the values of K_1, K_2, N_1, and N_2 are such that the peaks of the two individual hydrographs are not widely separated, adding the two curves will result in a single-peaked unit hydrograph. The shape of the curve will be different from that given by Equations (1) or (2), and in some cases it may give a better fit than that obtainable with the simpler equation for a single-peaked unit hydrograph.

A special case of the model shown in Figure 2 is obtained when the time constants of all reservoirs are made equal. Assuming, without loss of generality, that $N_1 > N_2$ and denoting the common time constant by K, the equation of the instantaneous unit hydrograph becomes

$$U_{(t)} = \left[\frac{A}{N_2(N_2 + 1)\ldots(N_1 - 1)}\left(\frac{t}{K}\right)^{N_1 - N_2} + B\right]\frac{1}{K(N_2 - 1)!}\left(\frac{t}{K}\right)^{N_2 - 1} e^{-t/K} \tag{11}$$

Another special case is obtained if, in addition to a common time constant K, it is also assumed that the numbers of reservoirs in the two cascades differ only by one

$$N_2 = N_o \tag{12}$$

$$N_1 = N_o + 1 \tag{13}$$

The equation of the instantaneous unit hydrograph for this case is

$$U_{(t)} = \left(\frac{At}{N_oK} + B\right)\frac{1}{K(N_o - 1)!}\left(\frac{t}{K}\right)^{N_o - 1} e^{-t/K} \tag{14}$$

It may be noted at this point that the shapes of unit hydrographs computed by Equation (14), with integer values of N_o and properly chosen values of the parameter A, can be made to approximate very closely the shapes of unit hydrographs computed by Equation (2) with noninteger values of N. This observation is not very important except that it may offer some relief to those who have difficulty accepting the idea of a model composed of a noninteger number of reservoirs.

USE OF THE ANALOG COMPUTER

The equation of the instantaneous unit hydrograph [Equation (3)] can be evaluated simply, for a given set of parameter values, on a digital computer. If a plotter is available, the computer output can be used as input to the plotter to obtain a graph of the curve. However, in an exploratory study, in which the parameters are varied over wide ranges to determine their effect on the shape of the instantaneous unit hydrograph, an analog computer offers a distinct advantage. Analog computers are also useful when the rainfall excess hyetograph and the direct surface runoff hydrograph are provided and the parameters of the best instantaneous unit hydrograph are to be found. The best instantaneous unit hydrograph is defined in this case as one that will give the best fit to the observed hydrograph when convoluted with the given hyetograph.

Instantaneous unit hydrographs associated with several specific configurations of reservoirs and a range of values of the parameters of the generalized conceptual model were closely approximated in this study by analog computation. The analog computer used was the ASTRAC II, which is a high-speed iterative differential analyzer, or analog/hybrid computer, developed in the Computer Science Research Laboratory, Electrical Engineering Department of the University of Arizona. The development and use of this computer has been well documented in student dissertations and other papers. Korn and Korn [1964] provide a valuable description and bibliography related to the ASTRAC II.

An iterative differential analyzer is an analog computer controlled by patched digital logic and actuated by a master time element or clock. The computer solves the set of differential equations represented by the wiring of the patchboard and repeats the solution after setting the initial conditions to specified levels. Solution repetition rates from 10 to 1,000 runs per second are available on the computer. The speed of 100 solutions per second selected in this study provided a convenient oscilloscope display of the hydrographs. The time occupied by each repetition was 10 milliseconds, divided by the computer into a 9.5-millisecond solution time and a 0.05-millisecond time interval for resetting the initial conditions.

The 10-millisecond repetition time for each solution is divided by the computer clock pulse generator into 1,000 pulses that could actuate any of three preset counters. The counters can operate analog switches through the digital control patchbay, which, in turn, can create input pulses and simple rainfall hyetographs. The switches were used in the first part of the study to generate input pulses for the two parallel cascades. The input pulses generated had an actual duration of 1/100 of the 10-millisecond computation repetition time. This means that the duration of the pulses was 0.1 millisecond rather than being really instantaneous. The pulses thus generated were routed through the two cascades, and the outputs of the cascades were added to produce the instantaneous unit hydrographs.

The wiring used to obtain the computer analog of the basic one-reservoir unit is shown in Figure 3, in which R is the input flow rate to the simulated reservoir and Q is the outflow value. The wiring shown represents the analog computer solution of the differential equation of the linear reservoir for which the outflow Q is proportional to the storage S in the reservoir. Defining the storage constant K by

$$S = KQ \tag{15}$$

the differential equation for the single linear reservoir is

$$R - Q = \frac{dS}{dt} = K\frac{dQ}{dt} \tag{16}$$

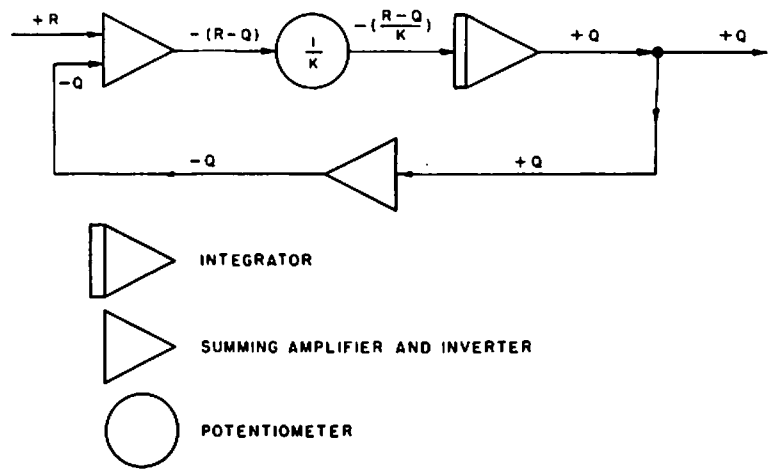

Fig. 3. Analog computer simulation of a single linear reservoir.

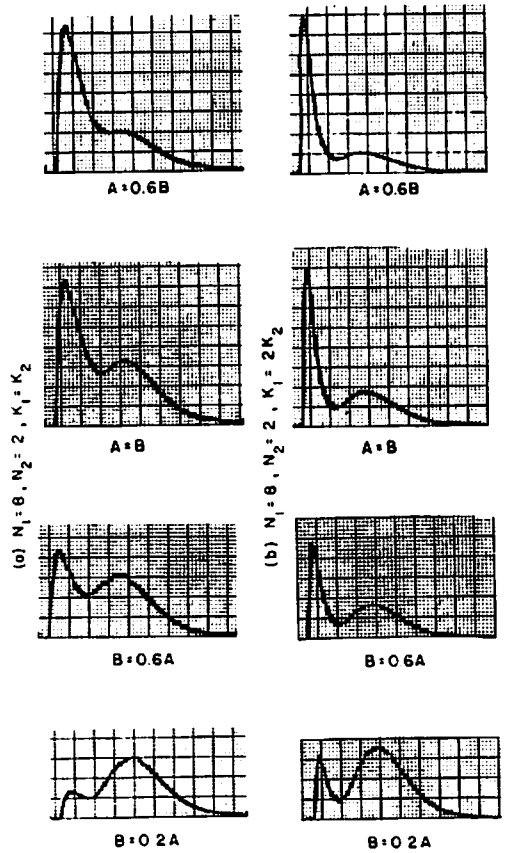

Fig. 4. Analog computer results (N_1 = 8).

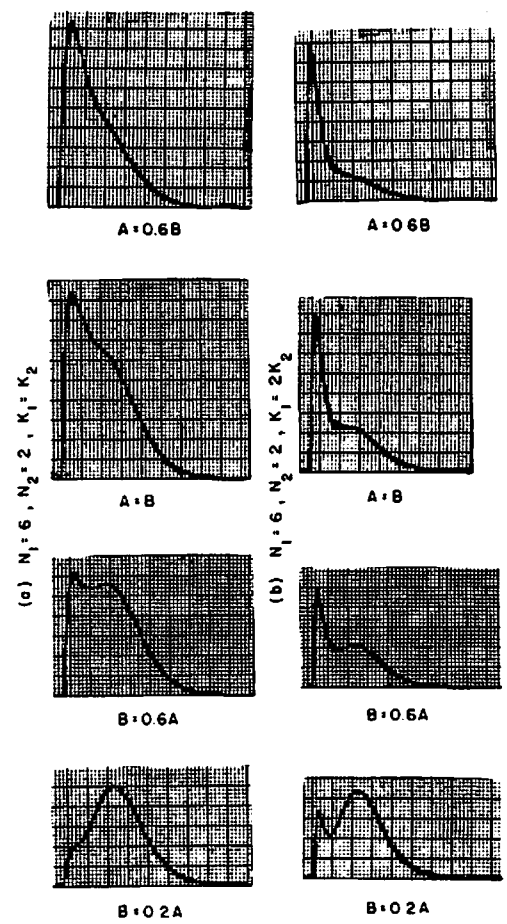

Fig. 5. Analog computer results (N_1 = 6).

or

$$\frac{R-Q}{K}=\frac{dQ}{dt} \qquad (17)$$

which is the form used in the analog computer solution in Figure 3. The reservoir time constant K is obtained by adjusting the potentiometer setting E and the integrator multiplying factor G. The value of $1/K$ is proportional to the product of these two factors

$$\frac{1}{K}=EG \qquad (18)$$

Single reservoir elements similar to that shown in Figure 3 were connected in series in each of two cascades according to an analog computer program that was determined by the flow pattern shown in Figure 2. The input was divided between the two branches using two potentiometers that were set according to Equation (4). Output of the two branches was added through a summing amplifier. Flow rates were represented by voltages throughout the computing system.

VARIATIONS IN SHAPES OF INSTANTANEOUS UNIT HYDROGRAPHS

Some of the instantaneous unit hydrographs obtained with the analog computer are shown in Figures 4, 5, and 6. The results demonstrate the wide range of shapes of instantaneous unit hydrographs obtainable with the parallel cascades model. The curves shown in the figures are reproduced from the chart recorder used. Because of various scale factors used in the analog and recorder, the curves shown need some scale adjustments before they can be compared with each other. The shapes of the curves remain unchanged in these scale changes, but the relative sizes of the horizontal and vertical axes change. Values of the reservoir constants used in the various tests are listed in Table 1.

The charts for Figures 4, 5, and 6 were obtained with a Sanborn[3] 150 recorder. The

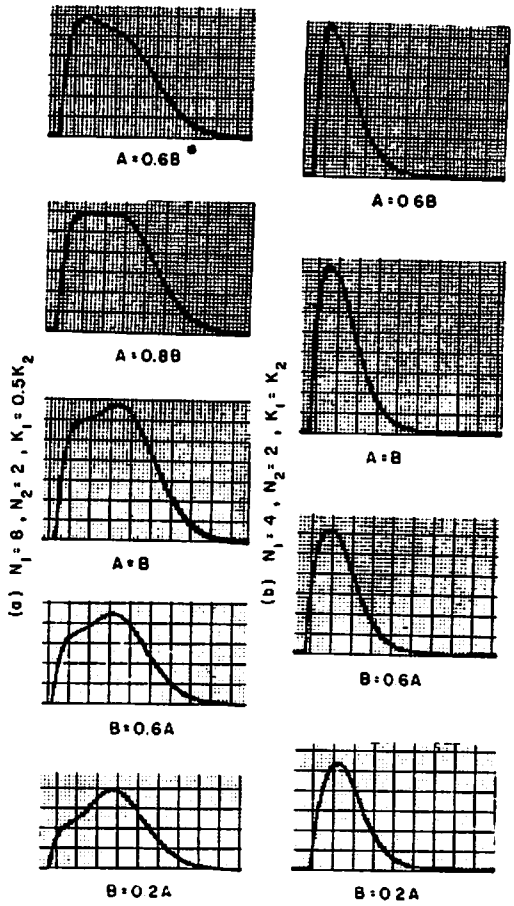

Fig. 6. Analog computer results (N_1 = 8, 4).

TABLE 1. Reservoir Constants Used for Instantaneous Unit Hydrographs

Figure	K_1	K_2
	(millisecond)	(millisecond)
4a	0.5	0.5
4b	0.5	0.25
5a	0.5	0.5
5b	0.5	0.25
6a	0.5	1.0
6b	0.5	0.5

[3]Use of a trade name does not imply endorsement by the U.S. Department of Agriculture; it is included only for the convenience of the readers.

analog computer is equipped with an automatic solution scan that, when used with a sample hold circuit, effectively increases the solution time by a factor of 1000 for use with recorders. Since the actual repetition time for each solution was 10 milliseconds, the corresponding recorder repetition time was 10 seconds. The recorder time available for each solution was thus ten seconds minus the 0.5-second reset interval. The chart speed used in the recorder was 5 mm/sec. It follows that the heavy vertical lines in the figures (5-millimeter intervals) represent 1-second intervals of recorder time which is 1 millisecond of real or analog computer time.

A convenient procedure for comparing the various curves obtained by the analog computer is first to adjust their vertical scale to make the area under the curve unity (dimensionless) and then to express the time base and the ordinate, which has a unit of (1/time), as dimensionless quantities.

The area under the curve is given by

$$\int_0^\infty U_{(t)}\, dt = 1.0 \tag{19}$$

Adopting the time to the centroid of the curve (L) as a basis for the dimensionless quantities, the time base would be expressed as the ratio t/L and the ordinate of the curve by the product $U{\cdot}L$. The area under the curve remains unity

$$\int_0^\infty (U{\cdot}L)\, d(t/L) = 1.0 \tag{20}$$

The time to centroid is also equal to the time lag between the centroids of the rainfall excess hyetograph and the direct surface runoff hydrograph. Since the area of the instantaneous unit hydrograph is 1.0, the time to the centroid is also equal to the first moment of the curve about the origin as given by Equation (7).

$$L = AN_1K_1 + BN_2K_2 \tag{21}$$

Values of the time to centroid L for some instantaneous unit hydrographs studied, including those shown in Figures 4, 5, and 6 are listed in Table 2. The values listed were computed by Equation (21) from values of the reservoir constants given in Table 1 and values of the other parameters used in the various tests.

Some comparable instantaneous unit hydrographs plotted with the dimensionless quantities given above are shown in Figure 7. The curves shown correspond to the curves shown in Figure 4a before the scale adjustments. They were, however, recomputed with a digital computer to avoid errors resulting from the small scales used in the recorder output.

The systematic changes in the magnitudes and locations of the two peaks and the saddle of the instantaneous unit hydrograph are shown in Figures 8 and 9. The model considered in these two figures is the same used in Figure 4a and in Figure 7. It consists of eight reservoirs in parallel with two reservoirs all having the same time constant. The free variable was the parameter A which was

TABLE 2. Computed Values of the Time to Centroid

Parameters	Values of L in milliseconds					
A = 1, B = 0	4.00	4.00	3.00	3.00	4.00	2.00
A = 0, B = 1	1.00	0.50	1.00	0.50	2.00	1.00
A = .6B	2.12	1.81	1.75	1.43	2.76	1.37
A = B	2.50	2.25	2.00	1.75	3.00	1.50
B = .6A	2.88	2.69	2.17	1.99	3.24	1.62
B = .2A	3.46	3.38	2.68	2.58	3.53	1.83
Figure	4a	4b	5a	5b	6a	6b

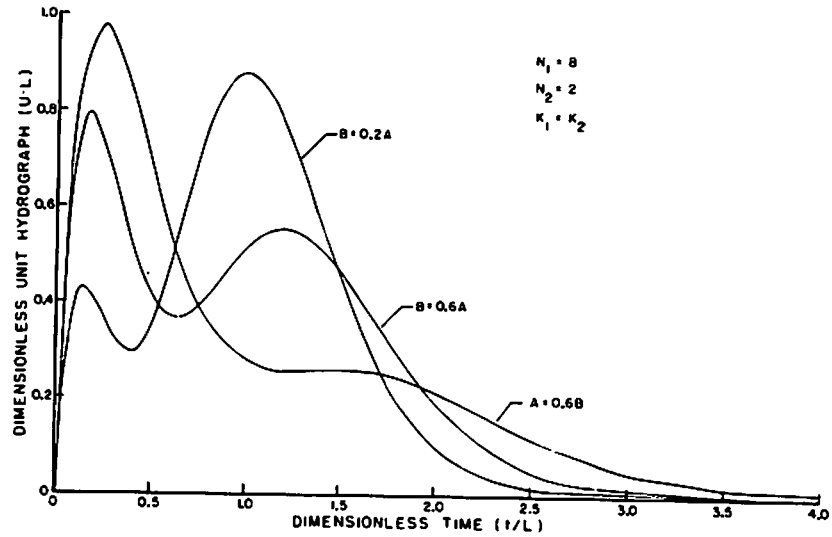

Fig. 7. Comparable dimensionless instantaneous unit hydrographs.

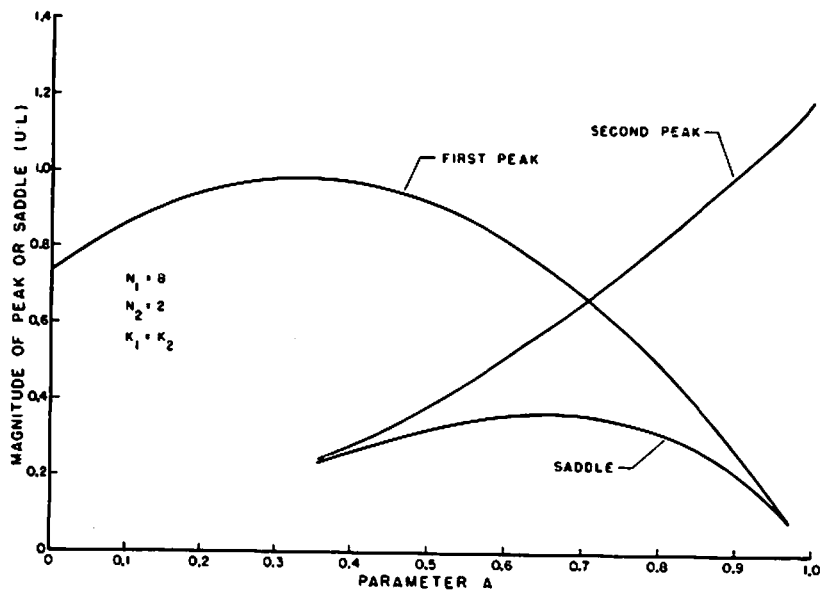

Fig. 8. Variations in magnitudes of peaks.

changed in the range

$$0.0 \leqslant A \leqslant 1.0 \tag{22}$$

The results shown in Figures 8 and 9 indicate that the instantaneous unit hydrograph for this model has two distinct peaks only for values of the parameter A in the approximate range of

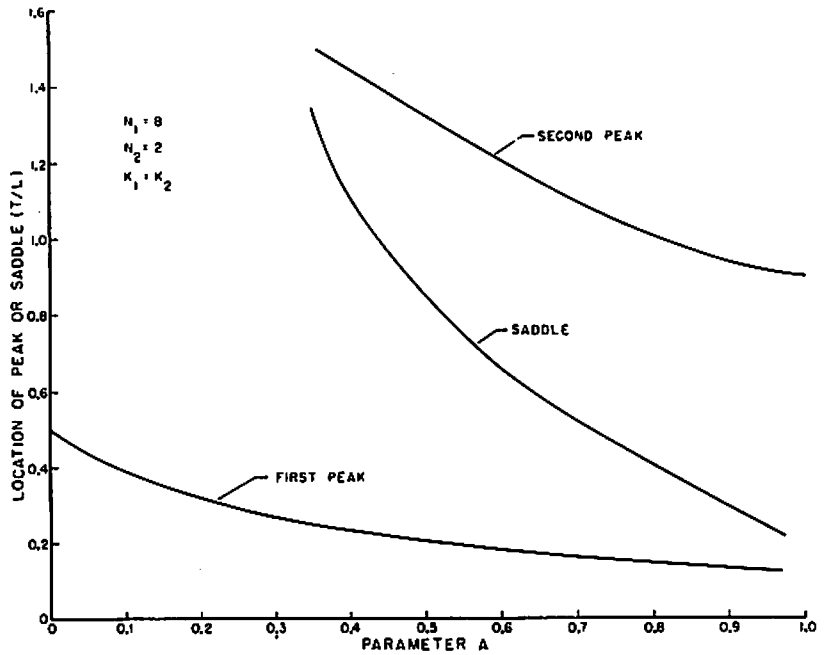

Fig. 9. Variations in locations of peaks.

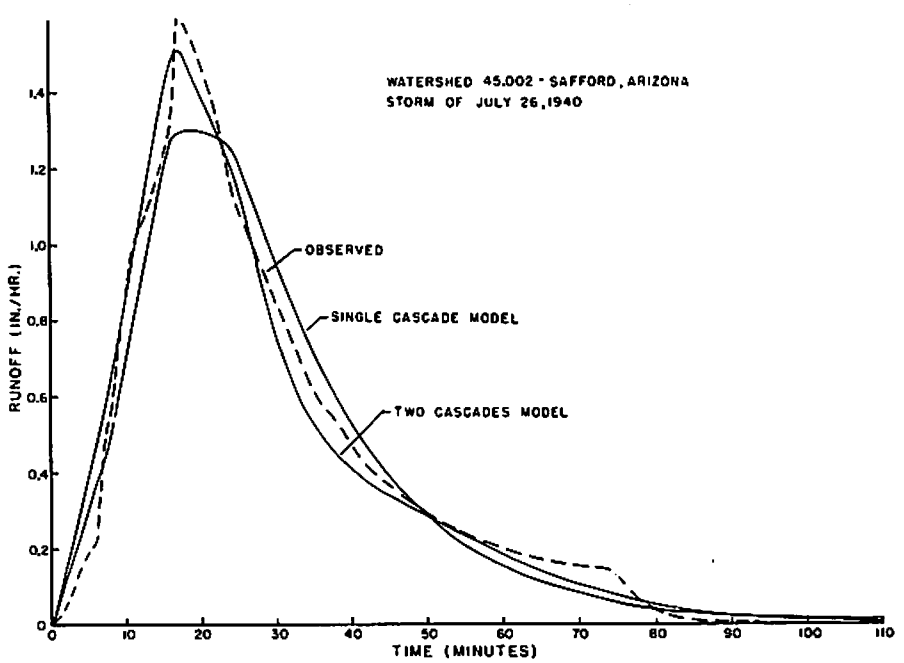

Fig. 10. Comparison of runoff hydrographs.

$$0.35 < A < 0.98 \tag{23}$$

The times to the two peaks, relative to the time to the centroid of the hydrograph, decrease with the increase in the values of the parameter A. The rates of the decrease are not the same for the two peaks and for the saddle of the curve. It appears that the saddle is nearer to the second peak for low values of the parameter A and nearer to the first peak for high values of the parameter, in the range where two distinct peaks are present.

APPLICATION TO HYDROGRAPH FITTING

The use of the analog computer for finding the parameters of an instantaneous unit hydrograph providing a good fit to a rainfall runoff event was demonstrated for a storm on one of the experimental watersheds operated by the Southwest Watershed Research Center of the USDA, Agricultural Research Service.

The watershed, identified as Watershed No. 45.002, is located approximately 28 miles northwest of Safford, Arizona. The area of the watershed is 682 acres (1.07 square miles).

The storm, selected at random from the available records, was recorded on July 26, 1940. The runoff hydrograph for the storm (Fig. 10) has only one peak, but the recession curve is rather irregular, indicating that a two-cascade model may give a better fit than a model composed of only one cascade of linear reservoirs. The rainfall hyetograph for the storm was derived from the chart of a recording rain-gaging station on the watershed. The actual hyetograph was simplified, and the rainfall excess hyetograph was assumed to be composed of three 8-minute blocks with rainfall intensities of 1.54, 2.78, and 0.99 inch/hour.

The initial choice of the instantaneous unit hydrograph was obtained by inspection of the set of instantaneous hyetographs obtained in the first portion of the study. The numerical values of the parameters for the instantaneous unit hydrograph selected initially were adjusted to satisfy Equation (21) with the value of L equal to the time interval between the centroids of the rainfall excess hyetograph and the runoff hydrograph which was L = 17.8 minutes. The model defined by this set of parameters was set up on the computer patchboard, and the various potentiometers were adjusted to their estimated values. Input values of rainfall excess data, representing the above hyetograph, were supplied to the analog computer

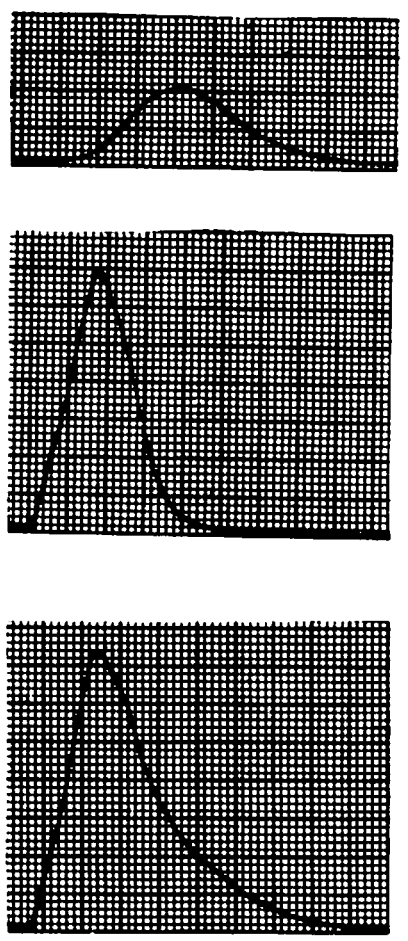

Fig. 11. Analog computer output for storm event of July 26, 1940.

through a hybrid interface by a standard digital computer program. The outflow hydrograph obtained by routing the input hyetograph through the model was observed on the cathode ray oscilloscope of the analog computer and compared to the observed hydrograph.

Values of the model parameters were then changed, by trial and error, to obtain a hydrograph that closely followed the observed curve. The curve fitting was done by eye, and no attempt was made to satisfy a best fit criterion such as the least sum of squares. The final result is shown in Figure 11 and is compared with the observed hydrograph in Figure 10. The model consisted of a cascade of five reservoirs with storage constant K_1 = 7.5 min., in parallel with a cascade of two reservoirs with K_2 = 4.5 min. The value of the parameter A was 0.30. The recorder tracings in Figure 11 show the output due to the given rainfall input of (a) the five-reservoir cascade of the model, (b) the two-reservoir cascade, and (c) the complete model.

Figure 10 shows also the runoff hydrograph obtained with a single cascade model. The parameters of the model, N = 1.27 and K = 14.05 min., were obtained by the simultaneous solution of the equations for the first two moments of the instantaneous unit hydrograph. The comparison between the observed hydrograph and hydrographs produced by the two models indicates that a better fit was obtained with the more flexible two parallel cascades model.

SUMMARY AND CONCLUSIONS

An analog computer was used to demonstrate the range of instantaneous unit-hydrograph shapes obtainable with a generalized conceptual model composed of two parallel cascades of linear reservoirs. Using specific examples of instantaneous unit hydrographs obtained with various parameters, the flexibility and wide applicability of the generalized conceptual model was more readily appreciated than through earlier analytic descriptions.

The model is capable of producing either single- or double-peaked instantaneous unit hydrographs depending on the structure of the model and the values of the parameters describing it. When matching actual data, the two-cascade model was better than the one-cascade model even in cases where single-peaked instantaneous unit hydrographs were needed. This results from the model's ability to obtain unit hydrographs displaying combinations of a sharp peak and unusual recession curves that cannot be approximated by a model composed of a single chain of reservoirs.

The unique feature of the two-cascade model is its ability to produce double-peaked instantaneous unit hydrographs. For some combinations of the parameters of the model, the hydrographs produced have two peaks separated by a saddle. The relative magnitudes and locations of these peaks and saddle point are functions of the values of the parameters. The variations in these quantities with the input ratio of one cascade to the total input are given for one of the models composed of a cascade of eight reservoirs in parallel with a cascade of two reservoirs.

The analog computer, which is in fact a high-speed differential analyzer, was found to be a very valuable tool for determining the various shapes of instantaneous unit hydrographs that could be produced by varying the parameters of the model. The analog computer was also found to be useful in the trial-and-error solution for the parameters of a two-parallel cascade model that gives a good fit to a set of observed data for a rainfall-runoff event.

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NOTATION

A parameter of model expressing relative input to first cascade of reservoirs;
B parameter of model expressing relative input to second cascade of reservoirs;
E potentiometer setting used to obtain value of time constant on analog computer;
e base of natural logarithms;
G multiplying factor of integrator in analog computer;
K time constant of a single linear reservoir in conceptual models;
L time to the centroid of instantaneous unit hydrograph;
$M_{u,o}^{(2)}$ moment of curve;
N second moment of U curve about origin;
p number of linear reservoirs in a cascade;
Q the Laplace transform variable;
R outflow from a single reservoir;
S inflow into a single reservoir;
T time to peak of an instantaneous unit hydrograph;
t time variable;
U instantaneous unit hydrograph ordinate;
$\bar{U}$ Laplace transform of instantaneous unit hydrograph;
Γ the gamma function;

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