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AN APPROXIMATE OVERLAND FLOW ROUTING MODEL

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SUMMARY: The approximate overland flow routing method estimated peak runoff and duration of runoff accurately when disaggregated rainfall intensity was input to generate the rainfall excess distribution.

KEYWORDS:

Hydrology, rainfall, runoff, watersheds

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Abstract

The Water Erosion Prediction Project (WEPP), under the leadership of the Agricultural Research Service, is to develop a physical process model of soil loss and sediment yield for croplands and rangelands. Hydrologically, the model includes a rainfall disaggregation scheme, the Green and Ampt infiltration equation, and two methods of routing rainfall excess based on the kinematic wave equations and the approximate overland flow routing method. The purpose of this paper is to describe the second method to route the excess rainfall. The approximate overland flow routing method was developed based on the assumption that the routed overland flow hydrograph may be well approximated by the rainfall excess distribution. The method consists of a set of regression equations derived using the method of characteristic solution for the rising hydrograph. The method estimates peak runoff and duration of runoff based on plane characteristics and rainfall excess pattern. To assess the approximate routing method, data from three small watersheds were used to obtain peak runoff and duration of runoff using an analytical solution to the kinematic wave equations. Comparison between the approximate method and kinematic routing agreed within 5% error.

Introduction

The WEPP is developing new generation erosion prediction technology for dealing with soil conservation and environmental problems resulting from soil erosion by water. The model structure of WEPP is based on fundamental hydrologic and erosion processes. The model structure includes: climate, snow accumulation, infiltration, runoff, erosion, crop growth, plant residue and other soil disturbing activities. The WEPP Model uses the steady state sediment continuity equation as a basis for erosion computation. To solve the steady state sediment continuity equation one requires the peak runoff, duration of runoff and flow shear stress. The first two hydrologic variables are obtained by routing the rainfall excess along the overland flow plane. Overland flow has been modeled in the past using the one-dimensional kinematic wave approximation (Henderson and Wooding 1964, Liggett and Woolhiser 1967, Woolhiser and Liggett 1967, Eagleson 1970). Finite difference methods have been used to solve the one-dimensional kinematic wave equations with the resultant increase in computer time. The approximate routing method, which will be described in the next section, provides the peak runoff and duration of runoff without resorting to finite difference methods.

The kinematic wave equations for one-dimensional overland flow result when the momentum equation is approximated by assuming the land slope, So, is equal to the friction slope, Sf. The kinematic wave equations for runoff on a plane are

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = r - f = v \qquad (1)$$

and

$$q = \alpha h^{3/2} \tag{2}$$

where h is the local depth of flow (m), t is the time (s), q is the discharge per unit width (m^2/s), x is the distance down the plane (m), r is the rainfall intensity (m/s), f is the infiltration rate (m/s), v is the rainfall excess rate (m/s), and α is the depth-discharge coefficient ($m^{1/2}/s$). If v in equation (1) is constant, then equations (1) and (2) can be solved analytically by the method of characteristics (Eagleson 1970). Analytic solutions to these equations have been derived for the case where v is made up of a series of step functions in the rainfall intensity pattern, i.e. where intensity is constant within an arbitrary time interval but varies from interval to interval (Shirley 1987).

Development of the Approximate Flow Routing Method

Let the duration of rainfall excess, Dv, be defined as the time from the first time to ponding to the last time during the storm when rainfall rate is greater than the calculated infiltration capacity. Let the volume of rainfall excess be V. Therefore, we can define an average rainfall excess rate, σ as

$$\sigma = \frac{V}{Dv} \tag{3}$$

where σ is the average rate of rainfall excess (m/s), V is the volume of rainfall excess (m), and Dv duration of rainfall excess (s). If the time to equilibrium, te, for runoff on a plane of length x is the time to steady state runoff given an average rainfall excess rate, σ , for a long period, then the time to equilibrium is calculated as follows (Eagleson 1970).

$$te = \left[\frac{x}{\alpha} \right]^{2/3} \left[\frac{1}{\sigma} \right]^{1/3} \tag{4}$$

where x is the length of the plane (m), te is the time to equilibrium (s). Having characterized the rainfall excess pattern and the overland flow plane, it is now possible to define the dimensionless variables used to approximate the peak rate of runoff and the duration of runoff without doing the actual routing process. Let the normalized time to equilibrium, t*, be

$$t^* = \frac{te}{Dv} \tag{5}$$

and the normalized peak rate of runoff, q*, be

$$q* = \frac{qp}{vm} \tag{6}$$

where qp is the peak rate of runoff (m/s), and vm is the peak rate of rainfall excess (m/s). Let the normalized duration of runoff, D*, be

$$D* = \frac{Dq}{Dv} \tag{7}$$

and let the normalized rainfall excess rate, v*, be

$$v* = \frac{vm}{V} te \tag{8}$$

where Dq is the duration of runoff.

Analysis of equations (5), (6), (7), and (8) suggested a relationship between q* and t* of the form,

$$q* = \exp\left[-b1 \ (t*)^{b2} \right] \qquad \begin{array}{l} \text{for } t* < 1 \quad qp = vm \\ \text{for } t* > 1 \end{array} \tag{9}$$

and between D* and v* of the form,

$$D* = b3 + b4 \ (v*)^{b5} \tag{10}$$

Such relationships are depicted in figure 1. Where b1 to b5 are coefficients to be determined. Once the coefficients have been determined, the peak rate of flow and duration are obtained by solving equations (6) and (7) for qp and Dq, respectively.

Description of the Simulation Study

To determine the coefficients b1 through b5 in equations (9) and (10), equations (1) and (2) were solved for a range of rainfall intensities, soil textures, surface roughness, and slope lengths and gradients. The runoff hydrograph was obtained using the Infiltration Runoff Simulation (IRS) program (Lane et al. 1989). The program computes rainfall excess using the Green and Ampt equation for infiltration. The lateral inflow (rainfall excess) is routed as a positive step function up to a given time and is zero thereafter.

In table 1 "Triangular" refers to a triangular rainfall intensity pattern used for disaggregation, "Constant" refers to a constant intensity pattern, and "Double Exp." refers to the double exponential intensity pattern. Tp and ip are the ratio of time to peak to duration of precipitation and the ratio of maximum intensity to average intensity, respectively.

Soils data representative of 11 textural classes were selected, the soil texture varies from loamy sand to clay. Table 2 presents soil properties based on soil textural class.

A number of overland flow planes were selected to produce a wide range of te, t*, and D* values given the information in tables 1 and 2. Characteristics of these overland flow planes are listed in table 3.

Results of the Simulation Study

A nonlinear least squares curve fitting program, based on the maximum neighborhood method of Marquardt (1963), was used to evaluate the coefficients b1 to b5 in equations (9) and (10).

The coefficients were evaluated for each soil texture and rainfall distribution using 22 different flow planes described in table 3. Consequently, 330 values were obtained for each coefficient. The nonlinear least squares analysis on equations (9) and (10) indicates that values for the five coefficients vary as shown in table 4.

To check whether equations (9) and (10) can present a reasonable approximation to the data, the coefficient of determination between the observed and predicted values was calculated. Clearly, a mean value of 0.97 and 0.98 indicates an excellent fit to equations (9) and (10), respectively.

The next step in the simulation study was to obtain the coefficients b1 to b5 as a function of rainfall distribution and hydraulic conductivity. For this purpose, a linear model was proposed to represent the relation between bi values and rainfall distribution and hydraulic conductivity. Such model has the following form,

$$bi = c0 + c1\ Dr + c2\ tp + c3\ ip + c4\ ksj \qquad \text{for } i = 1,..,5 \qquad (11)$$

where bi, coefficients in equations (9) and (10); c0, c1, c2, c3, and c4 coefficients to be determined; Dr duration of precipitation (s); ksj, hydraulic conductivity (m/s); j, corresponds to soil classification index in table 2, and the other variables are as defined earlier.

The results from the multiple linear regression analysis show a poor relation between the bi values and the described independent variables. That is, the coefficient of determination for the five cases are low, as it can be seen in table 5.

Due to the low values of the coefficients of determination, the coefficients b1, b2, and b5 were determined using a mean value based only on soil texture and for all rainfall distributions, and equation (11) for b3 and b4. The computation of b3 was performed without the variable ksj. Similarly b4 was computed without the variables Dr, tp and ip. The regression analysis showed that such variables did not reduce the unexplained variance significantly. Three sets of eleven mean values were generated, see table 6.

Similarly, a mean value was computed for each coefficient for a given rainfall distribution and for all soil textures. Consequently, three sets of thirty mean values were produced, see table 7.

Further simplification was made to values of the coefficients in tables 6 and 7. The criterion for such simplification was based only on soil texture. Table 8 shows the values of b1, b2, and b5 for different soil textures.

In contrast, b3 was determined using equation (11) for each soil texture. Thus a set of eleven equations were obtained to calculate b3. For instance table 9 shows the values of the coefficients in equation (11) for all soil textures.

Similarly, b4 can be determined as

$$b4 = 0.47 + 5.55E\text{-}03 \; ksj \qquad (12)$$

Application and Discussion

Infiltration-based runoff estimation procedures require rainfall time-intensity data. However, often times only storm total or daily rainfall data are available. Therefore Nicks and Lane (1989) developed a disaggregation scheme to develop approximate rainfall intensity patterns using information on rainfall intensity amount, storm duration, time to peak rainfall intensity, and maximum rainfall intensity within the storm. In this section we evaluate the approximate method versus kinematic routing for observed rainfall intensity patterns and for approximate rainfall intensity patterns produced by the disaggregation scheme.

The approximate routing method was tested using data from three small watersheds. The following information was provided for the three watersheds: observed rainfall data and disaggregated rainfall data, see figures 2 to 6, slope and length of the plane, ground and canopy cover, initial saturation and Chezy roughness coefficient. Table 10 presents watershed characteristics at the time of the storms.

Results for the 5 events are shown in tables 11 and 12. The data in Table 11 show how well the approximate method agrees with kinematic routing when measured rainfall data are used as input. The data in Table 12 show the corresponding information when the disaggregated rainfall data are used as input.

Summary

The WEPP Model uses the approximate routing method to obtain peak runoff and duration of runoff since a rainfall disaggregation scheme (Nicks and Lane 1989) was developed in the model to calculate rainfall statistics such as amount, duration, time to peak intensity and maximum rainfall intensity. These statistics are used to fit a double exponential distribution to a normalized intensity pattern. The Green and Ampt equation computes infiltration and rainfall excess as a function of this normalized intensity pattern.

The approximate overland flow routing method estimated peak runoff and duration of runoff accurately when disaggregated rainfall intensity was input to generate the rainfall excess distribution. Tables 11 and 12 show values for peak runoff and duration of runoff for the three small watersheds. When observed data were input estimated values were unsatisfactory, see table 11. Notice that all storms except 7/19/68 in table 11 have an irregular intensity pattern, figures 2, 3, 5, and 6. Conversely, storm 7/19/68, figure 4, shows a more regular pattern. Consequently, difference between kinematic routing and approximate method is small. In addition, storm 7/19/68 shows a similar pattern to the disaggregated storm, figure 4. When disaggregated rainfall intensity data were input the approximate method agrees with the routing values. Notice that all disaggregated rainfall intensity distributions show no abrupt discontinuities, figures 2 - 6. The disaggregation process smooths out the observed rainfall intensity pattern.

The approximate routing method was developed based on the assumption that the routed overland flow hydrograph may be well approximated by the rainfall excess distribution. As a result, peak runoff and duration of runoff were calculated accurately when the rainfall excess distribution was generated from a disaggregated rainfall intensity pattern. For instance, figure 7 shows rainfall excess distributions obtained from storm 3/19/70. Clearly, the rainfall excess distribution from observed rainfall intensity shows more discontinuities than the one obtained from disaggregated intensity, figure 7.

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Table 1.
Summary of selected storms for the simulation study.

Storm Type	No.	Depth (mm)	Duration (min)	tp	ip
Triangular	1	29	30	0.0	2.0
	2	29	30	.25	2.0
	3	29	30	.50	2.0
	4	29	30	.75	2.0
	5	29	30	1.0	2.0
Constant Double Exp.	6	29	30	1.0	1.0
	7	29	30	.10	4.0
	8	29	30	.25	4.0
	9	29	30	.50	4.0
	10	29	30	.75	4.0
	11	29	30	.90	4.0
	12	40	90	.30	1.5
	13	40	90	.30	5.0
	14	40	90	.30	7.0
	15	40	90	.70	1.5
	16	40	90	.70	5.0
	17	40	90	.70	7.0
Disaggregated	18	64	305	.12	10.0
	19	51	71	.11	2.2
	20	89	633	.82	10.1
	21	69	1,131	.42	9.1
	22	17	88	.36	6.2
	23	16	63	.07	7.5
	24	39	40	.20	1.8
	25	25	180	.62	6.6
	26	67	45	.18	2.1
	27	85	98	.18	1.9
	28	29	160	.07	7.4
	29	64	519	.14	6.5
	30	44	380	.75	11.5

Table 2.
Summary of representative soils parameters by textural class.

Textural Class j	Effective Porosity n (%)	Matric Potential Sf (mm)	Hydraulic Conductivity Ks (mm/h)	Relative Saturation Se (%)
Loamy sand	40	63	30.0	22
Sandy loam	41	90	11.0	22
Loam	43	110	6.5	22
Silt loam	49	173	3.4	22
Silt	42	190	2.5	22
Sandy clay loam	35	214	1.5	22
Clay loam	31	210	1.0	22
Silty clay loam	43	253	0.9	22
Sandy clay	32	260	0.6	22
Silty clay	42	288	0.5	22
Clay	39	310	0.4	22

Table 3.
Summary of the overland flow planes.

Slope S (%)	Chezy C ($m^{1/2}/s$)	K ($m^{1/2}/s$)	Length x (m)
1	2.0	0.200	10
	5.0	.500	10
	10.0	1.000	10
	2.0	.200	50
	5.0	.500	50
	2.0	.200	75
	5.0	.500	100
5	2.0	.447	10
	5.0	1.118	10
	10.0	2.236	10
	2.0	.447	50
	5.0	1.118	50
	2.0	.447	100
	5.0	1.118	100
10	2.0	.632	1
	2.0	.632	10
	5.0	1.581	10
	10.0	3.162	10
	2.0	.632	50
	5.0	1.581	50
	2.0	.632	100
	5.0	1.581	100

Table 4.
Extreme values for b1 to b5.

Coefficient	Minimum	Maximum
b1	0.400	2.920
b2	0.819	7.156
b3	0.912	18.051
b4	0.109	1.069
b5	0.663	2.130

Table 5.
Coefficient of determination for b(i).

Coefficient	Coefficient of determination
b(1)	0.50
b(2)	0.16
b(3)	0.63
b(4)	0.55
b(5)	0.05

Table 6.
Coefficients as a function of soil texture.

Soil type	Coefficient b(1)	b(2)	b(5)
loamy sand	0.730	1.161	1.521
sandy loam	0.747	1.341	1.502
loam	0.757	1.404	1.495
silt loam	0.754	1.410	1.517
silt	0.820	1.455	1.497
sandy clay loam	0.910	1.506	1.524
clay loam	0.917	1.547	1.518
silty clay loam	0.890	1.507	1.507
sandy clay	0.996	1.524	1.538
silty clay	0.958	1.545	1.519
clay	1.031	1.518	1.531

Table 7.
Coefficients as a function of rainfall distribution.

Rainfall Distribution	Coefficient		
	b(1)	b(2)	b(5)
1	0.639	1.701	1.643
2	0.669	1.668	1.545
3	0.634	1.717	1.533
4	0.555	1.816	1.555
5	0.502	1.775	1.529
6	0.230	4.345	1.528
7	1.186	1.066	1.777
8	1.127	1.114	1.562
9	0.995	1.230	1.473
10	0.951	1.223	1.502
11	0.827	1.232	1.530
12	0.488	1.235	1.544
13	0.889	1.320	1.384
14	0.820	1.294	1.328
15	0.488	1.639	1.550
16	1.081	1.573	1.434
17	1.199	1.486	1.415
18	1.040	1.225	1.405
19	0.925	1.286	1.693
20	0.856	1.543	1.306
21	1.378	1.812	1.389
22	0.831	1.107	1.544
23	0.914	0.960	1.462
24	0.703	1.275	1.625
25	0.705	1.573	1.463
26	0.946	1.480	1.637
27	1.172	1.765	1.826
28	0.935	1.173	1.515
29	1.666	1.890	1.220
30	0.822	1.554	1.537

Table 8.
Values of b1, b2 and b5 as a function of soil texture.

Soil type	Coefficient b1	Coefficient b2	Coefficient b5
loamy sand sandy loam loam silt loam silt	0.70	1.26	1.51
sandy clay loam clay loam silty clay loam sandy clay silty clay clay	1.07	1.64	

Table 9.
Coefficients to obtain b3 as a function of Dr, tp and ip.

Soil texture	Coefficient			
	c0	c1	c2	c3
loamy sand	-0.31	6.31	0.12	1.58
sandy loam	-0.48	6.73	0.12	1.45
loam	-0.56	4.41	0.12	1.38
silt loam	-0.53	1.34	0.13	1.30
silt	-0.52	3.17	0.11	1.13
sandy clay loam	-0.39	2.62	0.09	0.79
clay loam	-0.40	2.80	0.09	0.78
silty clay loam	-0.37	2.42	0.09	0.77
sandy clay	-0.41	4.42	0.07	0.79
silty clay	-0.40	4.25	0.07	0.81
clay	-0.27	5.75	0.05	0.66

Table 10.
Watershed characteristics at of storms.

Location	Watershed (acres)	Area (ha)	Storm date	Land use & management.
Watkinsville, GA Sandy Loam, approx. 63% Sa, 21% Si, 16% Cl	19.2	7.77	3/19/70	Dormant costal bermuda grass, just beginning spring growth, excellent cover
Riesel, TX 70% Houston Black Clay 30% Heiden Clay	2.99	1.21	8/12/66	100% bermuda grass pasture 2-4" high, good cover, not grazed
			7/19/68	100% bermuda grass pasture 10" high
Hastings, NE 75% of area is Holdrege silt loam & 25% is Holdrege silty clay loam (severely eroded)	3.77	1.53	7/3/59	Sorghum about 6" high and in good condition. Weeds beginning to grow.
			5/21/65	No tillage during spring.

Source of Data: USDA-ARS 1963. Hydrologic data for experimental agricultural watersheds in the United States, 1956-59. USDA Misc. Publication No. 945, US Dept. of Agriculture, Agricultural Research Service, Washington, DC.

Table 11.
Comparison between kinematic routing and approximate method.

Rainfall	Measured Rain				
	Kinematic Routing			Approximate Method	
	Volume Runoff mm	Peak Runoff mm/h	Duration Runoff min	Peak Runoff mm/h	Duration Runoff min
03/19/70	19.9	7.2	1,215.0	29.3	2,626.7
08/12/66	41.8	40.7	309.0	77.7	600.0
07/19/68	12.5	21.1	121.0	20.9	151.7
07/03/59	59.7	163.8	55.0	185.6	63.5
05/21/65	57.9	79.8	117.0	82.6	140.0

Table 12.
Comparison between kinematic routing and approximate method.

Rainfall	Disaggregated Rain: Kinematic Routing: Volume Runoff mm	Kinematic Routing: Peak Runoff mm/h	Kinematic Routing: Duration Runoff min	Approximate Method: Peak Runoff mm/h	Approximate Method: Duration Runoff min
03/19/70	33.4	22.2	626.0	22.2	613.2
08/12/66	54.0	59.0	231.0	60.3	217.5
07/19/68	12.2	20.3	99.0	24.4	96.7
07/03/59	59.7	157.1	49.0	157.1	51.7
05/21/65	56.6	69.5	113.0	68.7	113.8