Sediment Budgets (Proceedings of the Porto Alegre Symposium, December 1988). IAHS Publ. no. 174, 1988.

Modelling sedimentation processes in small watersheds

V. L. LOPES & L. J. LANE

US Department of Agriculture/Agricultural Research Service, Aridland Watershed Management Research Unit, 2000 E. Allen Road, Tucson, Arizona 85719, USA

Abstract A physically-based, event-oriented, mathematical model of small watershed sedimentation is described. The unsteady and spatially varying sedimentation processes on overland flow areas and in small concentrated flow systems are described dynamically using simultaneous rates of sediment entrainment and deposition rather than the conventional approach using steady state sediment transport functions. The model is tested using sediment yield data from rainfall simulator plots and from two small experimental watersheds. The agreement between the simulated watershed response and the observed watershed response indicates that the governing equations, initial and boundary conditions, and structural framework of the model can satisfactorily describe the physical processes of sediment generation, transport, and deposition in small watersheds.

Mise en modèle des processus de sédimentation sur petits bassins versants

Résumé La sédimentation sur petits bassins versants peut être reproduite par un modèle mathématique simulant les processus physiques associés à une pluie isolée. Les processus de sédimentation, variables dans le temps et l'espace, sont décrits de façon dynamique, aussi bien dans le cas d'écoulement en nappe que dans celui des petits écoulements concentrés, en faisant appel à des taux d'érosion et de dépôts concomitants, plutôt qu'à celui de formules de transport en régime permanent comme il est d'usage. Les productions de sédiments de parcelles sous pluie simulée et de deux petits bassins versants ont été utilisées pour tester le modèle. La bonne coïncidence entre les réponses du modèle et des bassins montre que les équations de base, les conditions initiales— et aux limites, ainsi que la structure du modèle peuvent décrire de façon satisfaisante les processus physiques de production, transport et dépôt de sédiments sur petits bassins versants.

INTRODUCTION

Mathematical models can provide an approach to improved understanding of the fundamental sedimentation processes and to improved sediment assessment and control technology (Agricultural Research Service, 1983). However, any attempt to model sedimentation processes as they naturally occur in small watersheds is seriously constrained by the complexity of an open system with component processes and state variables that may change rapidly in space and time. Therefore, simplified representations must be used to model the complex processes of sediment generation, transport, and deposition.

Problem statement

The sedimentation process begins on overland flow areas with detachment of soil particles through raindrop impact and subsequent entrainment by overland flow. As overland flow concentrates in small channels, entrainment, transport, and deposition occur. Entrainment and transport of noncohesive sediment particles by running water is controlled by slope length, slope steepness, particle size and weight distribution, and the forces exerted on the sediment particles by the flow. Detachment and transport of fine, cohesive sediment particles will also be controlled by electrochemical inter-particle forces which make the entrainment and deposition processes even more complicated. When the external forces are diminished below a threshold value, the settling process predominates and net deposition occurs.

The objectives of this study were to accomplish the following two objectives: (1) develop a mathematical model of erosion and deposition in small watersheds based on fundamental erosion and deposition mechanics, and (2) test the model using actual data from rainfall simulator plots and small experimental watersheds.

MATHEMATICAL MODEL

Overland flow sedimentation component

To develop a mathematical model of erosion and deposition on overland flow areas, expressions are needed to describe the rate at which each separate source and sink contributes to sediment concentration in the overland flow. Because of imperfect knowledge concerning the phenomenon and rate of rilling, this study assumes that sediment flux can be represented approximately without explicit description of rill features when those are present. The definition of the overland flow erosion/deposition system under consideration is shown in Fig. 1. A control volume of the overland flow system is receiving sediment inflow at rates e_I and e_R and losing sediment at a rate d.

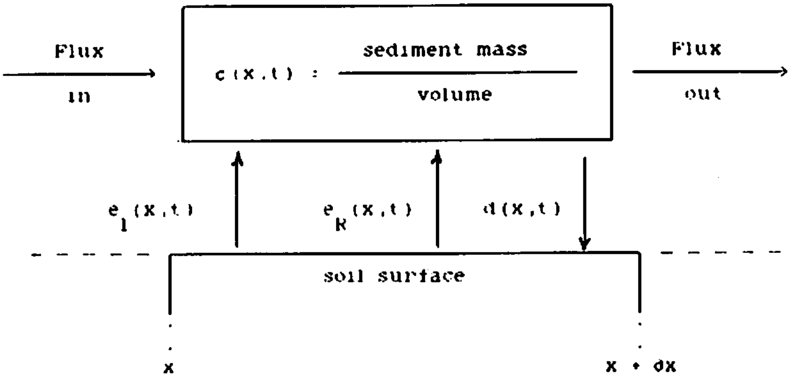

Fig. 1 Definition sketch of overland flow erosion/deposition system.

Sediment continuity equation

The continuity equation for sediment transport used for one-dimensional overland flow system (Bennett, 1974; Foster, 1982) is:

$$\frac{\partial(ch)}{\partial t} + \frac{\partial(cq)}{\partial x} = \Phi \tag{1}$$

Dispersion terms have been neglected in equation (1). The term $\partial(cq)/\partial x$ is the gain or loss of the sediment load with distance; $\partial(ch)\partial t$ is the storage rate of sediment within the flow depth; t is the time in seconds; x is the distance in the direction of flow (m); c is the local mean sediment concentration (kg m^{-3}); h is the local depth of flow (m); q is the local discharge per unit width (m^2 s^{-1}); and Φ is the sediment flux to the flow (kg m^{-2} s^{-1}).

Sediment entrainment in overland flow areas is accomplished by two different processes during a storm event: (a) sediment entrainment by rainfall, and (b) sediment entrainment by shear stress. Entrainment and deposition may occur simultaneously at different rates. The resultant sediment concentration is determined by the relative magnitude of these processes. Entrainment increases sediment concentration by upward flux of sediment from the soil surface into the overland flow. Deposition (settling) reduces sediment concentration by downward flux of sediment from the overland flow. The rate of each of these processes depends on different factors, and thus they should be separately represented in a mathematical model.

Therefore, the sediment flux to the flow, Φ, for a single overland-flow area might be written as

$$\Phi = e_R - d + e_I \tag{2}$$

in which e_R is the rate of sediment entrainment by shear stress (kg m^{-2} s^{-1}), d is the rate of sediment deposition (kg m^{-2} s^{-1}), and e_I is the rate of sediment entrainment by rainfall impact (kg m^{-2} s^{-1}).

Sediment entrainment by shear stress

Entrainment and transport of sediment occur when the forces tending to entrain and transport sediment exceed those tending to resist removal. Water flowing over the soil surface exerts forces on the soil particles that tend to move or entrain them. On bare soil and stream beds, the forces that resist the entraining action of the flowing water differ according to the particle size and particle size distribution of the sediment. For coarse sediment, the forces resisting entrainment are caused mainly by the weight of the particles. Finer sediments that contain appreciable fractions of clay, resist entrainment mainly by cohesion rather than by the weight of the individual particles. Also, in fine sediment, groups of particles (aggregates) are entrained as units whereas coarse noncohesive sediments are moved as invidual particles.

Sediment entrainment by shear stress can be represented by a relationship expressing the entrainment rate as proportional to a power of the average shear stress acting on the soil surface (Croley, 1982; Foster, 1982):

$$e_R = K_R \tau_e^b \tag{3}$$

where K_R is a soil detachability factor for shear stress (kg m $N^{-1.5}s^{-1}$), τ_e is the average "effective" shear stress assuming broad shallow flow (N m^{-2}), and b is an exponent in the range 1.0 to 2.0. In this model b was set to 1.5 (Foster, 1982).

Sediment deposition rate

The mass rate of sediment deposition (downward flux) is proportional to the local mean sediment concentration (Mehta, 1983). Thus:

$$d = \epsilon c V_s \tag{4}$$

where ϵ is a coefficient depending on the soil and fluid properties (dimensionless), V_s is the particle fall velocity (m s^{-1}) and c is the local mean sediment concentration (kg m^{-3}).

Sediment entrainment rate by rainfall

Sediment entrainment rate by rainfall, e_I is a function of the rate of detachment by raindrop impact and the rate of transport of sediment particles by shallow flow (Foster & Meyer, 1972).

A simple functional form of detachment by raindrop impact incorporates rainfall intensity, i as a measure of the erosivity of raindrop impact (Foster, 1982). If the rainfall intensity is uniform over the region of interest then:

$$e_I = ai^2 \tag{5}$$

where a is a coefficient to be determined experimentally. Lane & Shirley (1985) included rainfall excess in equation (5) to reflect rate of sediment transport by shallow flow on overland flow areas. They assumed a simple equation for sediment entrainment rate as:

$$e_I = K_I i^2(r/i) = K_I ir \tag{6}$$

where K_I is a coefficient to measure soil detachability by rainfall impact (kg s m^{-4}). The ratio of rainfall excess rate, r to rainfall intensity, i can be interpreted as a measure of normalized runoff intensity for sediment transport by shallow flow. Notice that when r = 0 (pre-rainfall excess phase), or when i = 0 (post-rainfall phase) there is no sediment entrainment by rainfall, and when $r = i$, (impermeable surface), sediment entrainment rate by rainfall is not limited by rainfall excess.

Numerical solution

In general, for most of the formulations of the functions in equation (2), there is no analytical solution for equation (1). Therefore, numerical techniques are required.

The relevant upper boundary and initial conditions of equation (1) (Lopes, 1987) are:

$$c(0,t) = K_I i(t) r(t)/\epsilon V_s + r(t)) \qquad \text{for } t \geqslant t_p \tag{7}$$

and,

$$c(x,t_p) = K_I i(t_p) r(t_p)/(\epsilon V_s + r(t_p)) \quad \text{for } x \geqslant 0 \tag{8}$$

where t_p is the time of ponding.

The numerical procedure for solving equation (1), subject to the above upper boundary and initial conditions, is:

$$\begin{aligned}
&\frac{\phi}{\Delta t}\left[(ch)_{j+1}^{i+1} - (ch)_{j+1}^{i}\right] + \frac{(1-\phi)}{\Delta t}\left[(ch)_{j}^{i+1} - (ch)_{j}^{i}\right] + \\
&\frac{\omega}{\Delta x}\left[(cq)_{j+1}^{i+1} - (cq)_{j}^{i+1}\right] + \frac{(1-\omega)}{\Delta x}\left[(ch)_{j+1}^{i} - (cq)_{j}^{i}\right] = \\
&\omega\left[\phi e_{R_{j+1}}^{i+1} + (1-\phi) e_{R_j}^{i+1}\right] + (1-\omega)\left[\phi e_{R_{j+1}}^{i} + (1-\phi) e_{R_j}^{i}\right] -
\end{aligned} \tag{9}$$

$$\omega\left[\phi d_{j+1}^{i+1} + (1-\phi)d_j^{i+1}\right] - (1-\omega)\left[\phi d_{j+1}^{i} + (1-\phi)d_j^{i}\right] +$$

$$\omega e_I^{i+1} + (1-\omega)e_I^{i}$$

where ϕ is the weighting factor for distance, and ω is the weighting factor for time. The scheme can be either explicit or implicit, depending on the values of the weighting factors ϕ and ω. If $\phi = 1$ and $\omega = 0$, the numerical scheme becomes an explicit scheme, and is subject to the Courant condition to maintain stability. If $\phi = 0.5$ and $\omega > 0.5$, the scheme is unconditionally stable; however, for accuracy the Courant condition must be maintained. For a definition sketch of the notation in equation (9), see Fig. 2.

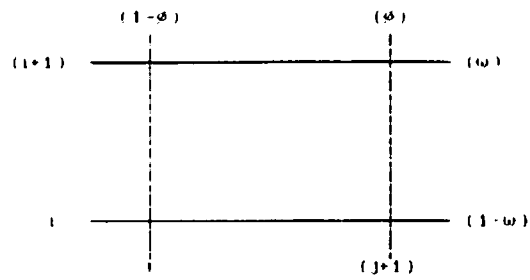

Fig. 2 Definition sketch of finite difference scheme.

Concentrated flow sedimentation component: sediment continuity equation

The continuity equation for sediment transport in one-dimensional flow in a single small channel element can be written as (Bennett, 1974):

$$\frac{\partial(CA)}{\partial t} + \frac{\partial(CQ)}{\partial x} = e_r - d + q_s \tag{10}$$

where e_r is the source term for bed-material load (km m^{-1} s^{-1}), d is the rate of sediment deposition (kg m^{-1} s^{-1}) and q_s is the source term for fine-material load (represented as the lateral sediment inflow from adjacent overland flow areas, in kg m^{-1} s^{-1}. C and Q are concentration of sediment and flow of water in a channel with cross section area A; the other variables are as defined earlier. Equation (10) is subject to the following boundary and initial conditions:

$$C(0,t) = C_0(t) \qquad \text{for } t \geqslant 0 \tag{11}$$

and,

$$C(x,0) = 0 \qquad \text{for } x \geqslant 0 \tag{12}$$

where $C_0(t)$ is the incoming sediment concentration from the upstream channel.

Sediment entrainment by concentrated flow

A general equation, initially developed for bed-load transport capacity, has been used to model entrainment (e_r) by concentrated flow (Foster, 1982):

$$e_r = \begin{cases} a(\tau - \tau_c)^n & \text{for } \tau \geqslant \tau_c \\ 0 & \text{for } \tau < \tau_c \end{cases} \tag{13}$$

in which,

$$\tau = \gamma R_H S_f \tag{14}$$

and,

$$\tau_c = \partial\,(\gamma_s - \gamma) d_s \tag{15}$$

where S_f is the slope of the energy line; d_s is characteristic particle size; a is a coefficient for sediment entrainment (kg m^2 N$^{-1.5}$ s^{-1}), τ is the average shear stress (N m^{-2}), τ_c is the average critical shear stress for the representative particle size (N m^{-2}), n is an exponent in the range 1.0 to 2.0, and ∂ is a coefficient depending on the sediment and fluid properties (dimensionless). The other variables are as defined earlier.

Sediment deposition rate

The rate of sediment deposition (downward sediment flux) is proportional to the local mean sediment concentration and to an effective particle fall velocity (Mehta, 1983). The deposition rate in kilograms per second per metre of channel width can be computed as:

$$d = \epsilon T_w V_s C \tag{16}$$

in which T_w is the flow top width (m), and the other variables are as defined earlier.

Numerical solution

There are no analytical solutions to equation (10) and therefore numerical solutions must be used. Using the same scheme and procedure used for solving equation (1), equation (10) subject to (11) and (12) can be written in terms of finite differences as:

$$\frac{\phi}{\Delta t}\left[(CA)_{j+1}^{i+1}-(CA)_{j+1}^{i}\right]+\frac{(1-\phi)}{\Delta t}\left[(CA)_{j}^{i+1}-(CA)_{j}^{i}\right]+$$
$$\frac{\omega}{\Delta x}\left[(CQ)_{j+1}^{i+1}-(CQ)_{j}^{i+1}\right]+\frac{(1-\omega)}{\Delta x}\left[(CQ)_{j+1}^{i}-(CQ)_{j}^{i}\right]=$$
$$\omega\left[\phi e_{r\,j+1}^{i+1}+(1-\phi)e_{r\,j}^{i+1}\right]+(1-\omega)\left[\phi e_{r\,j+1}^{i}+(1-\phi)e_{r\,j}^{i}\right]-\qquad(17)$$
$$\omega\left[\phi d_{j+1}^{i+1}+(1-\phi)d_{j}^{i+1}\right]-(1-\omega)\left[\phi d_{j+1}^{i}+(1-\phi)d_{j}^{i}\right]+$$
$$\omega q_s^{i+1}+(1-\omega)q_s^{i}$$

where ϕ is the weighting factor for distance step, and ω is the weighting factor for time step.

MODEL TESTING

To test the model, two sets of data were used: (a) data from rainfall simulator plots, and (b) data from two small experimental watersheds. The rainfall simulator plots and the two small watersheds are located on the US Department of Agriculture, Agriculture Research Service (USDA-ARS) Walnut Gulch Experimental Watershed near Tombstone, in southeastern Arizona.

Parameter estimation

The model parameters were estimated in two stages. In the first stage rainfall simulator plot data were used to estimate the soil erodibility parameters for raindrop impact (K_I) and runoff (K_R). In the second stage, sediment yield data from two small experimental watersheds were used to: (a) verify the applicability of the erosion parameters estimated from rainfall simulator plots to overland flow on a small watershed scale, and (b) estimate channel erodibility parameters (a in equation (13)) for concentrated flow. The sediment settling parameter, ϵ, was assumed 0.5 for overland flow (Davis, 1978) and 1.0 for channel flow (Einstein, 1968). Given estimates of K_I and K_R obtained from plot data (natural plot in Table 1) and varying these parameter values according to wetness, the concentrated flow erodibility parameter, a, was optimized to fit the estimated total sediment yield for each event. A starting estimate for a was assumed to be always the same as K_R. The final estimates of a are

*Table 1 Erosion parameters from rainfall simulator plots**

Run no.	*Treatment*	*Slope*	K_I *(kg* $s^{-1}m^{-4}$*)* *(x*10^7*)*	K_R *(kg m* $N^{-1.5}s^{-1}$*)*	*Sediment yield (kg): OBS*	*SIM****
*13204(D)***	*bare*	*0.122*	*16.50*	*0.0525*	*21.713*	*21.712*
13604(W)	*bare*	*0.122*	*15.00*	*0.0200*	*14.185*	*14.187*
13704(VW)	*bare*	*0.122*	*11.70*	*0.0211*	*13.970*	*13.943*
12605(D)	*clip*	*0.108*	*0.95*	*0.0027*	*0.991*	*0.979*
13005(W)	*clip*	*0.108*	*1.15*	*0.0022*	*0.384*	*0.391*
13105(VW)	*clip*	*0.108*	*0.90*	*0.0012*	*0.347*	*0.334*
12707(D)	*nat*	*0.115*	*2.56*	*0.0057*	*0.128*	*0.128*
12807(W)	*nat*	*0.115*	*1.34*	*0.0025*	*0.093*	*0.093*
12907(VW)	*nat*	*0.115*	*1.30*	*0.0008*	*0.161*	*0.159*

**Selected plots from all 1982 simulator trials on the Walnut Gulch Experimental Watershed.*
***Antecedent soil moisture class (D = dry, W = wet, VW = very wet).*
****OBS = observed; SIM = simulated.*

shown in Table 2. The parameter for critical shear stress, ∂, and the characteristic particle size, ds, were assumed to be 0.047 and 0.120 mm, respectively, for all simulations. The comparisons of the simulated and the measured sedigraphs are shown for one plot event (Fig. 3) and for one watershed event (Fig. 4).

CONCLUSIONS

The following conclusions can be drawn based on the model development and testing results:

(a) The agreement between the simulated and the observed watershed responses indicates that the governing equations and structural framework of the model can satisfactorily describe the sedimentation processes occurring in small watersheds.
(b) The source (entrainment) and sink (settlement) terms for the equations describing conservation of sediment mass in overland flow areas and in concentrated flow systems are mathematically consistent and incorporate appropriate initial and boundary sediment concentrations.
(c) The fact that the entrainment by rainfall term (e_I) has a behaviour similar to the entrainment by shear stress term (e_R) makes it evident that unique parameter identification may not be possible with the rainfall simulator data used. Perhaps, a better way to estimate these parameters is using runoff/sediment data from small plots (with detachment by raindrop impact only) to estimate K_I and using data from large plots with preformed rills to estimate K_R.

Table 2 *Erosion parameters for small watersheds*

Event date (year, month, day)	Watershed	Antecent moisture condition	Hillslope erosion parameters: K_I (kg s m^{-4}) ($\times 10^7$)	K_R (kg m $N^{-1.5}s^{-1}$)	Channel erosion parameter a (kg $m^2 N^{-1.5} s^{-1}$)	Sediment yield: estimated (kg)	simulated (kg)
750705	63.105	wet	1.340	0.0025	0.0018	138	142
750717	63.105	very wet	1.300	0.0008	0.0012	1500	1533
750913	63.105	very wet	1.300	0.0008	0.0005	36	35
750712	63.103	wet	1.340	0.0025	0.0026	6920	7003
750907	63.103	dry	2.560	0.0057	0.0081	1795	1792
750913	63.103	wet	1.340	0.0025	0.0082	3392	3373
760906	63.103	wet	1.340	0.0025	0.0046	7410	7474
760910	63.103	wet	1.340	0.0025	0.0145	1414	1412
770901*	63.103	wet	1.340	0.0025	0.0066	3074	3056
780725*	63.103	dry	2.560	0.0057	0.0068	4523	4523

*Events in which total sediment concentration was measured with the total load sampler.

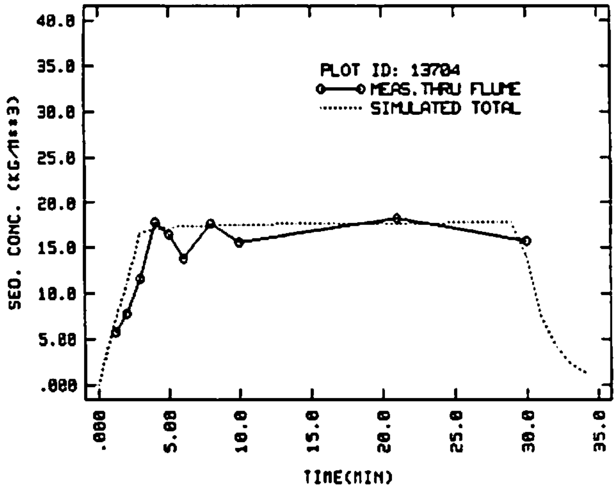

Fig 3 Sedigraph for very wet run on bare plot.

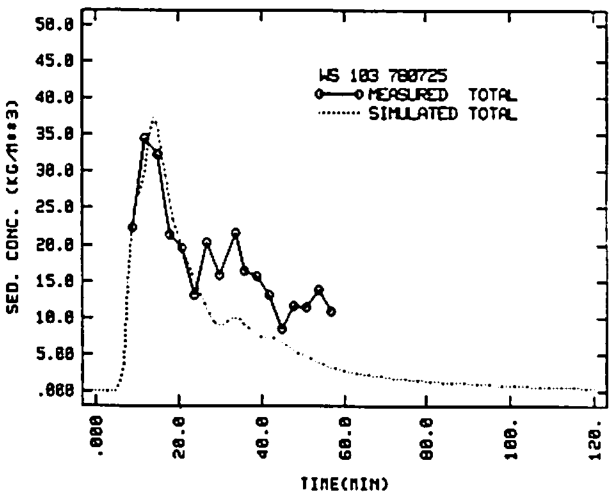

Fig. 4 Sedigraph for the storm event of 07/25/78 on WS 103.

Acknowledgements Technical support for this project was provided by the Aridland Watershed Management Research Unit of the US Department of Agriculture, Agricultural Research Service (USDA-ARS), in Tucson Arizona. Discussions with David A. Woolhiser and Edward D. Shirley are gratefully acknowledged, as are comments received from Kenneth G. Renard, Roger Simanton and Jeffry Stone. Our thanks to the reviewers for some helpful suggestions.

REFERENCES

Agricultural Research Service (1983) *Agricultural Research Service Program Plan.* USDA,

ARS.
Bennett, J. P. (1974) Concepts of mathematical modeling of sediment yield. *Wat. Resour. Res.* **10** (3), 485-492.
Croley, T. E., II (1982) Unsteady overland sedimentation. *J. Hydrol.* 56, 325-346.
Davis, S. S. (1978) Deposition of nonuniform sediment by overland flow on concave slopes. MS Thesis, Purdue Univ., West Lafayette, Indiana.
Einstein, H. A. (1968) Deposition of suspended particles in a gravel bed. *J. Hydraul. Div. ASCE* 94 (HY5), 1197-1205.
Foster, G. R. (1982) Modeling the erosion process. In: *Hydrologic Modeling of Small Watersheds* (ed. by C. T. Haan, H. P. Johnson & D. C. Brakensiek), 259-380. Am. Soc. Agric. Engrs Monograph no. 5.
Foster, G. R. & Meyer, L. D. (1972) A closed-form soil erosion equation for upland areas. In: *Sedimentation: Symposium to Honor Prof. H. A. Einstein* (ed. by H. W. Shen), chapter 12.12.1-12.19. Fort Collins, Colorado.
Lane, L. J. & Shirley, E. D. (1985) Erosion and sediment yield equations: solutions for overland flow. Paper presented at the Workshop on USLE Replacement, Nat. Soil Erosion Lab., West Lafayette, Indiana.
Lopes, V. L. (1987) A numerical model of watershed erosion and sediment yield. PhD Thesis, Univ. of Arizona, Tucson, Arizona.
Mehta, A. J. (1983) Characterization tests for cohesive sediments. In: *Proc. of Conf. on Frontiers in Hydraul. Engineering* (ASCE/MIT, Cambridge, Massachusetts) (ed. by H. T. Shen), 79-84.