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A Review of Rainfall-Runoff, Physical Models as Developed by Dimensional Analysis and Other Methods[1]

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Abstract. Limited similitude relations for the construction of physical models of the watershed hydrologic system have been developed, using dimensional analysis, by Mamisao and Chery. Tests on both constructed models indicate that the hoped for similarity did not materialize, and that further testing is necessary to develop empirically the needed similarity relations. Grace and Eagleson have, using differential equations, also developed similitude relations for hydrologic system modeling and discussed the limitations of such modeling. Other individuals are also contemplating modeling, but their endeavors are not concerned with similitude relations, because their devices are non-scaled representatives of parts of the natural hydrologic system. (Key words: Hydrologic systems; drainage basin characteristics; dimensional analysis)

INTRODUCTION

Development of the desired comprehensive models of the watershed hydrologic system has, so far, been frustrated by the complexity of the natural system. Many avenues to the solution of the problem are being explored. The use of physical models or laboratory catchments to study the hydrologic cycle has been an intriguing possibility for many years, because of the advantages customarily assumed to accrue from carefully controlled laboratory experiments.

Some laboratory physical models are unscaled representations of part of a natural system; others, hopefully, are scaled representations of the entire rainfall-runoff system. The design of those models simulating a rainfall-runoff system must be developed according to a logic or set of principles by which the behavior of the two systems (model and prototype) will be similar. Those principles that are used to design, construct, and interpret the test results of models constitute the theory of similitude. The similitude relations may be developed either by a consideration of the differential equations describing the system or by dimensional analysis. Dimensional analysis may also be used, independently, to deduce information about the natural phenomena from the premise that the system can be described by a dimensionally correct equation of all the variables relevant to the system.

Efforts to develop, by dimensional analyses, similarity relations for physical watershed models will be reviewed in some detail. An alternative method of determining similarity relations and other 'model catchment' projects are reviewed briefly for comparative purposes. These reviews, it is hoped, should give one a summary perspective on the development and present state of physically modeling the watershed hydrologic system.

REVIEW OF PHYSICAL MODELING EFFORTS

Similitude Relations by Dimensional Analysis

Erzen. The earliest mention of dimensional analysis of rainfall-runoff phenomena with which I am familiar is that of Erzen, as presented in a book by *Langhaar* [1951]. Erzen's dimensional analysis of watershed discharge was made assuming a functional relation, using seven pertinent variables as given in Equation 1.

$$f(Q, t, A, H, g, \rho, \nu) = 0 \qquad (1)$$

[1] Contribution of the Soil and Water Conservation Research Division, Agricultural Research Service, USDA, in cooperation with the New Mexico Agricultural Experiment Station, University Park, New Mexico, and Utah State University, Logan, Utah.

in which

		Dimensions
Q	= discharge	L^3T^{-1}
t	= time	T
A	= drainage area	L^2
H	= total depth of rain	L
g	= acceleration due to gravity	LT^{-2}
ρ	= mass density	ML^{-3}
ν	= kinematic viscosity	L^2T^{-1}

The mass density ρ was eliminated as a variable by saying that it was the only variable containing a dimension of mass and thus could not be combined with any other variable to form a dimensionless product. Equation 1 was then reduced to the following function of four dimensionless terms:

$$\frac{Q}{g^{1/2}H^{5/2}} = f(tg^{1/2}/A^{1/4},\ A/H^2,\ \nu^2/gA^{3/2}) \quad (2)$$

With the assumption that the discharge Q is approximately proportional to the total depth of rain H (no further explanation of justification given), Erzen removed the dimensionless term A/H^2 from the right-hand side of Equation 2, raised it to the three-quarter (3/4) power, and combined it with the dimensionless term on the left-hand side of the equation. These manipulations then gave him the functional relation with three dimensionless terms of

$$Q/(g^{1/2}A^{3/4}H) = f(tg^{1/2}/A^{1/4},\ \nu^2/gA^{3/2}) \quad (3)$$

He made this development with the following assumptions and conditions:

1. 'Q is approximately proportional to H' [*Langhaar*, 1951, p. 112].
2. The effect of the duration of the rainstorm on the discharge is neglected, which means the same as saying that all the rain falls at the initial instant $t = 0$.
3. Evaporation is not accounted for in the analysis.
4. The functional relation as expressed in Equation 3, '. . . should be expected to be approximately the same for all watersheds that are geologically similar and that have roughly the same shape.' [*Langhaar*, 1951, p. 113].
5. The influence of the viscosity parameter ($k = \nu^2/gA^{3/2}$) was explained as:

'Although different curves should be expected for different values of the parameter 'k', the data are not extensive enough to exhibit this effect.' [*Langhaar*, 1951, p. 113]. Erzen plotted the discharge term versus the time term for three watersheds in Illinois, ranging in size from 334 square miles to 550 square miles, to obtain the plot of Figure 1.

The relationship proposed by Erzen (Figure

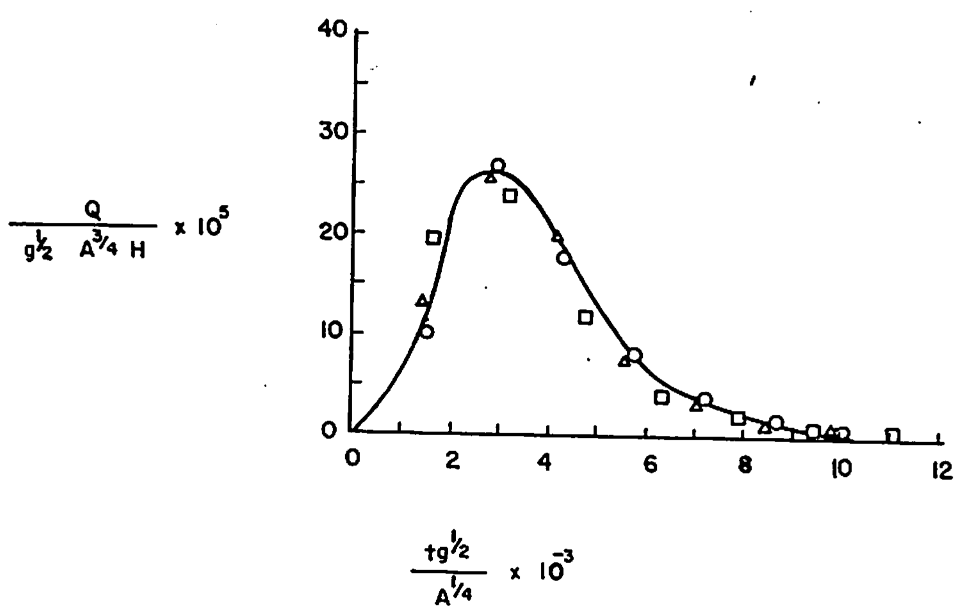

Fig. 1. Dimensional graph of runoffs from three watersheds in Illinois.

1) cannot be considered general. First, the development was restricted to physically similar watersheds. Second, it is questionable that the relation is constant for watersheds of different size, because of the more transient storage relations and greater temporal and spatial influence of rainfall as the area reduces relative to the 300- to 600-square-mile watersheds used as examples. Under further scrutiny, the Figure 1 relationship is, essentially, superimposed normalized discharge versus time plots for runoff from three similar size basins. As Erzen noted, the total discharge ($Q_T = \int_0^\infty Q\,dt$) is equal to the product of total rainfall depth and watershed area (AH) if there are no losses. Thus, in the ordinate term, the effect of ($A^{3/4}H$) will be nearly the same as (AH), and the term is essentially (K_1Q/Q_T), which is plotted against ($K_2t/A^{1/4}$), where K_1 and K_2 represent the accumulations of the constant terms.

Mamisao. In 1951 Jesus *Mamisao* [1952] completed a Master's thesis which described his dimensional analysis of the rainfall-runoff phenomena. From this development, he prepared the similitude relations for a physical model of a 129-acre agricultural watershed. He reasoned that the rainfall-runoff phenomena could be represented by the twelve pertinent variables as expressed in the functional relation of Equation 4

$$Q = f(I, t, l, b, h, r, i, \rho, \mu, \sigma, g) \tag{4}$$

in which

	Dimensions
Q = runoff	L^3T^{-1}
I = rainfall intensity	LT^{-1}
t = time	T
l = length [of watershed]	L
b = width [of watershed]	L
h = height [of watershed]	L
r = roughness of surface and resistance of vegetation	...
i = infiltration capacity of the soil	LT^{-1}
ρ = density of water	ML^{-3}
μ = dynamic viscosity	$ML^{-1}T^{-1}$
σ = surface tension	MT^{-2}
g = gravity	LT^{-2}

By referring to the Buckingham Pi Theorem, Mamisao derived from Equation 4 the following functional relation, having nine dimensionless terms:

$$\underset{\Pi_1}{\frac{Q}{Il^2}} = f\left(\underset{\Pi_2}{\frac{It}{l}}, \underset{\Pi_3}{\frac{l}{gt^2}}, \underset{\Pi_4}{\frac{b}{l}}, \underset{\Pi_5}{\frac{h}{l}}, \underset{\Pi_6}{\frac{i}{I}}, \underset{\Pi_7}{\frac{\rho l^2}{\mu t}}, \underset{\Pi_8}{\frac{\rho l^3}{\sigma t^2}}, \underset{\Pi_9}{r}\right) \tag{5}$$

He thought that equality between model and prototype dimensionless terms (Π_2—Π_4) could easily be accomplished with, or without, distortions. The other five dimensionless terms (Π_5—Π_9) could not be equated readily between model and prototype, because the model would distort some variables, and others were difficult to evaluate. Thus, Mamisao introduced a linear 'distortion factor' to establish equality between the dimensionless terms of the model and prototype. Because of these several distortion factors, a linear 'prediction factor' would be required to establish equality between the model and prototype dimensionless term containing the dependent variable Q. If the rainfall could be simulated without distortion, then the prediction factor would be a function of the five distortion factors. Only one of the distortion factors could be readily evaluated; thus

> Since difficulty would be encountered not only in evaluating the values of the other distortion factors but also in establishing the relationship of [the prediction factor] to all these distortion factors, the roughness of the surface may be modified so as to compensate for the effects of these five distortions. This modification would result in making the value of the prediction factor unity, and the prediction equation would remain . . . [as an unaltered equality between the model and prototype dimensionless terms containing the dependent variable, Q]. [*Mamisao*, 1952, p. 28].

However, the solution would not remain so simple for Mamisao. He equated between model and prototype the dimensionless term containing the gravity variable. From this equality, he derived a time ratio. Having a time ratio between model and prototype, he then investigated the range of model intensities that should be applied. 'The prototype rainfall intensity varied from less than one inch to about five inches per hour, which [meant] that the simulated rainfall intensity should, if undistorted, be about 0.05 to 0.24 inches per hour. . . .' [*Mamisao*, 1952, p. 44]. He thought that it would be most difficult to produce such small intensities in the laboratory. As a consequence, Mamisao thought it most necessary to have greater intensities in the model, or, that is, to distort them, which would require a distortion factor to equate the model and prototype dimensionless terms containing the intensity variable I. He used the rational formula $Q = CIA$ to determine the relation between the distortion factor and the prediction factor. From this analysis, he developed the similitude criteria by which he designed and built his model.

The model, which occupied an area of about 5 feet by 8 feet, had two parts, a rainfall simulator and a watershed model. The rainfall simulator was a very shallow rectangular tank with glass capillaries placed 2½ inches on center and projecting from its underside. Water flow to the tank was regulated by valves and could be controlled to produce actual intensities of 1.27 to 15.95 inches per hour. The rainfall simulator was suspended above the watershed model, which was built up from a contour map. The watershed model, which was made to a 1:450 horizontal scale and a 1:240 vertical scale, had a mortar surface. Runoff from the model was measured by collecting the outflow in 1-gallon cans in 7.1-second intervals, which represented 2.5 minutes of prototype time.

Three prototype rainfall-runoff events, having different intensities and falling on different soil cover, were simulated in the model. Mamisao concluded

> . . . that there was a close similarity between the two hydrographs [model and prototype] in each of the three rainfalls. These results strongly indicate the possibility of using the scale-model method in making hydrologic studies of watersheds. [*Mamisao*, 1952, pp. 100–101].

This pioneering effort has some questionable aspects. Two aspects, which may cause the most concern, are the use of the rational formula to develop relations between the dimensionless terms of model and prototype, and the assumption that the nonequality of several model-prototype dimensionless terms could be collectively compensated for by modifying the roughness of the model surface. With respect to the idea of collective compensation, it is questionable that loss or abstraction mechanisms and time delay or storage mechanisms of the hydrologic system can be adequately represented by a single 'roughness factor.' In an effort to improve both the volumetric and time relations of his model, Mamisao repeated some tests with the surface of the model covered with burlap. He was not completely successful in detaining the excess model runoff and commented that '. . . its removal cannot be accomplished by a simple roughening with the use of burlap' [*Mamisao*, 1952, p. 83]. He also recognized that the time and discharge rates of the model did not correspond exactly with those of the prototype. Of this situation, he said, 'One explanation that may be cited for this variability is the fact that while model A had uniform surface roughness (and no percolation), the roughness in the field (including infiltration) is not by any means uniform throughout the area.' [*Mamisao*, 1952, p. 84]. Nonetheless, from the results he did obtain, he optimistically recommended '. . . the continuance of this work in order to obtain the adequate model . . .' [*Mamisao*, 1952, p. 101].

Chery. Mamisao's encouraging report suggested a course of experimental study to some persons in the Agricultural Research Service. As a result, I was guided into a graduate program, which began in 1960, that had as its objective the construction of an improved physical watershed model.

Since there is no a priori knowledge of the pertinent or relevant rainfall-runoff variables, I attempted, first, to establish the pertinent variables or at least to give some justification for the variables selected. Such a procedure is dichotomous in its very nature, for it requires one to know what one is attempting to know. With several limitations and assumptions, the variables listed in the general functional relation (Equation 6) were deemed relevant:

$$f(q, t, I, i, r, x_1, x_2, x_3, \mu, \rho, g) = 0 \qquad (6)$$

in which

		Dimensions
q =	outflow rate (per unit catchment area)	LT^{-1}
t =	time	T
I =	application rate per unit area	LT^{-1}
i =	flow abstraction rate per unit area	LT^{-1}

r =	'resistance' to flow in system	...
x_1, x_2, x_3 =	space coordinates of the points of the watershed surface	L
μ =	dynamic viscosity of the liquid in the system	$ML^{-1}T^{-1}$
ρ =	density of the liquid in the system	ML^{-3}
g =	acceleration due to gravity	LT^{-2}

The limitations and assumptions involved in the analysis were:

1. The outflow at the basin outlet was derived entirely from surface runoff;
2. The source of the runoff was a rainstorm, more particularly a high-intensity thunderstorm (precipitation in the form of snow was not considered);
3. The topographic model would be constructed initially with an impervious surface, allowing for no losses through the surface of the model;
4. The designed rainstorm simulator would be an adequate representation of the prototype rainstorm phenomena;
5. The topographic model would be an undistorted, faithful representation of the prototype geometry;
6. The rainfall momentum flux and impinging angle could be combined with the resistance term;
7. From the perspective of the prototype, the surface tension of the water could be neglected as a relevant variable, but in the model its influence could be included in the resistance term;
8. The influence of sediment transportation on prototype performance was insignificant;
9. A portion of the surface storage on the watershed surface could be considered as an element of the resistance.

As a consequence of the limitations and the unexpressed dimensions of the resistance term, Equation 6 could not be considered complete. Nevertheless, it was considered an adequate expression of the variables involved in the system under investigation, and it was used as the basis for what was called a '*quasi*' dimensional analysis. The Buckingham Pi Theorem was used to transform Equation 6 into a functional relation of eight dimensionless terms as expressed in Equation 7

$$\begin{matrix} \Pi_1 & \Pi_2 & \Pi_3 & \Pi_4 & \Pi_5 & \Pi_6 & \Pi_7 & \Pi_8 \end{matrix}$$

$$f\left(\frac{qt}{x_1}, \frac{It}{x_1}, \frac{\mu t}{\rho x_1^2}, \frac{it}{x_1}, \frac{x_2}{x_1}, \frac{x_3}{x_1}, \frac{gt^2}{x_1}, r\right) = 0 \qquad (7)$$

Exact similarity between the model and prototype would require the pairwise equality between the Π_2 through Π_8 terms. In the construction and operation of a physical model, the maintenance of the several pairwise equalities is a physical impossibility. Nevertheless, it was thought that approximate simulation could be obtained, and then more exacting simulation developed after a model was constructed and tests were made with it. The approximate similarity used to design the model was obtained from the following tentative reasoning:

1. Assuming that the topographic model would faithfully represent the prototype geometry, the model-prototype equality of the fifth and sixth Pi terms should be satisfied.
2. From like reasoning, the assumption of an adequate rainstorm simulator provided for model-prototype equality of the second Pi term.
3. In the initial design and construction, the surface of the topographic model was to be impervious, and thus no provision was made for abstraction of the flow through the model surface. Further, there was no accurate method of determining amount of input that would go into permanent storage on the surface of the model. Therefore, with the assumption that the volumetric distortion of the outflow would not unduly affect the time relationships in the performance of the model, provision for model-prototype equality of the Π_4 term was neglected for the time being. Thus, this term became one of the distortions that would eventually have to be manipulated to establish verification of the model.
4. The same consideration was given to the undefined resistance Π_8 term. As hypothesized, many items (e.g., surface roughness, rainfall momentum, the detention, etc.) were included in the general resistance term. Again, it was impossible to predetermine a relation that would equate this term between the model and prototype. Further, it was speculated that proper manipulation of liquid physical properties, model surface-liquid interaction, and model

surface textural characteristics would allow for a simulation of the net effect of the many prototype resistances to the flow. (Note: I also resorted to the same expediency as used by Mamisao; the same criticisms also apply.)

5. It would have been impossible to satisfy simultaneously the model-prototype equality of the two remaining Pi terms (Π_6 and Π_7). Thus, for construction and initial operation of the model, the hypothesis was made that the gravity parameter (Π_7) expressed the dominating influence, and the design was made according to the equivalence of this Pi term between the model and prototype.

A 97-acre semiarid watershed, located near Albuquerque, New Mexico, and operated by the Agricultural Research Service, was selected as a prototype. The just-stated analysis guided the construction of a physical model composed of a rainstorm simulator and a topographic model. The rainstorm simulator was divided into eleven modules. Each module had a pump pumping the input liquid through about 680 2-foot-long plastic capillary tubes. The discharging ends of the tubes were placed 2 inches apart in a grid, and the entire apparatus was suspended above the topographic model. The speed of the pumps was automatically controlled to provide various combinations of simulated rainfall intensities and areal application. The topographic model was made with fiber glass and epoxy resin. A length scale ratio of 1:175 was used with no vertical distortion. The model occupies an area of about 9 feet by 20 feet.

Assumptions 3, 4, and 5, used to develop the design similarity relations, are questionable, and not enough testing has been completed to discuss their validity or to determine how better similarity may be developed. The proposed procedure of a systematic series of tests, adjusting elements of the model until the model and prototype outputs compared properly, was appraised by Amorocho and Hart. They compared the procedure to the method of iterative approximation employed in mathematical synthesis of hydrologic systems. In one, mathematical functions are altered; in the other, some physical elements are altered to modify the outflow. Such a machine or simulator may provide a good prediction of the outflow, but it does not necessarily model the prototype faithfully. They remarked

> . . . that there is no assurance that within the practical limitations of possible physical changes in the surface of the model, a set of arbitrary components can be found to accomplish the desired results. It appears, however, very worthwhile to attempt this type of investigation, which might lead to the development of useful techniques for procedural standardization and to gaining some insight into the interaction between the gross geometrical parameters of a drainage basin and the storage, frictional and other dampening effects introduced in the testing process. [*Amorocho and Hart*, 1965, pp. 112–113].

Similitude Relations with Differential Equations

A second approach to the development of similitude relations is through a consideration of the differential equations describing the phenomena. This approach is also contingent upon a priori knowledge of the complete set of pertinent variables, and further, upon having developed differential equations that completely describe the system. If the differential equations are available, similarity relations may be developed by one of two alternative methods which are intrinsically the same.

By one method, a scale factor is established between a variable in the prototype and the same variable in the model. The scaled variables are substituted into the differential equation to obtain the differential equation for the model. The scale factors are collected before each term in the differential equation, and each of these combinations of scale factors should be equal to unity for there to be similarity between the model and prototype.

In the other method, a dimensionless variable is created by dividing each variable in the system by a 'characteristic' quantity which is the same dimension as the variable. The product of the dimensionless variable and the characteristic quantity is substituted into the differential equation and the characteristic quantities collected before each term. This procedure is performed for both the model and prototype. For similarity to exist, the like combinations of characteristic quantities in the model and prototype equation must be equal.

Grace and Eagleson used the latter method to make an analysis of similarity criteria in the surface runoff process [*Grace and Eagleson*, 1965]. They began their analysis by developing the partial differential equations for two-dimensional unsteady overland flow with vertical inflow and for two-dimensional unsteady channel flow with vertical and lateral inflow. Dimensionless variables (or parameters as they referred to them) were defined by using 'appropriate reference parameters.' After a justification of the selected reference parameters, they made an 'order of magnitude analysis.' This procedure meant an investigation of the relative magnitudes of the several components in the collection of reference parameters before each term in the differential equations. When the magnitude of one component was significantly less than the others, its contribution was neglected. Then the simplified coefficients for model and prototype were equated for the similarity criteria.

It was reasoned that such simplification could relax the stringency of the absolute similarity criteria to a degree that would make it feasible to construct physical models. However, these relaxed criteria, which may also be dependent upon the specific problem, are the limit beyond which no modeling may be expected.

After their analysis, Grace and Eagleson concluded that 'It is physically possible to obtain good dynamic similarity when modeling the overland flow portion of the rainfall-runoff phenomenon . . .' [*Grace and Eagleson*, 1965, p. i] when the prototype basins are small (on the order of an acre), impervious, and moderately sloped (1° to 12°). For the channel flow portion of the rainfall-runoff process, it would not be physically possible to obtain good dynamic similarity. One of their concluding recommendations was that

> Experimental studies should be carried out on models, for which corresponding prototype rainfall-runoff data are available, which violate, in some controlled manner, the strict similarity criteria, in order to investigate to what extent these modeling criteria can be violated and yet still give results which are valid to a prescribed accuracy. [*Grace and Eagleson*, 1965, p. 60].

Laboratory 'Prototypes'

A discussion of how *Amorocho and Hart* [1965] are using laboratory catchments illustrates a different concept of their place in the development of functional relations for the hydrologic system. They have divided laboratory physical models into two classes. One class encompasses model studies such as those that have been discussed here and are of the nature of 'general system synthesis.' This class is designated as 'model catchments.' The second class is designated as 'laboratory prototype systems.' The laboratory prototype systems may be of two types: 'hydromechanic prototypes,' which encompass the many devices used to make detailed studies of specific hydraulic mechanisms, and 'prediction analysis prototypes,' which are used to provide laboratory data for the test of methods for the mathematical analysis of systems. As Amorocho and Hart emphasized, none of their laboratory prototype systems can be considered scaled models.

Their interest in the use of prediction analysis prototypes is to provide experimental verification of the nonlinear mathematical analysis they are making of the hydrologic system. For this purpose they have constructed an apparatus, covering an area of about 6 square feet, which supplies pulse inputs of water to a small rectangular catchment surface. The surface of the catchment is covered with a granular material, which provides roughness and storage elements. The device can be operated as either an open or closed system. In the closed system, all the inflow emerges at the outlet of the system to be measured and recorded. In the open system, losses are allowed to occur as the flow passes through the system, and only a part of the input is measured at the outlet of the system.

To date, only qualitative experiments with the apparatus have been reported. Amorocho and Hart are satisfied that the prediction analysis prototype has proved its usefulness in the tests of methods and theories of nonlinear analysis of hydrologic systems, and that its usefulness will become more evident as more complex experiments are performed.

Ven Te Chow at the University of Illinois is directing an ambitious laboratory project. The purpose of the project is an investigation of the basic laws governing flow of surface water over drainage basins through controlled experimentation. Such a watershed hydraulic model is analogous to Amorocho and Hart's prediction analysis prototype. The experimental device is, at present, an unscaled catchment; however, the studies may eventually be expanded to investigations of similitude relations. A rainfall simulator that will cover an area 40 feet by 40 feet is being planned. The simulated rain will fall on a surface that can be modified in roughness, shape, slope, permeability, and other essential considerations.

Other Hydrologic Model Work

Through personal contact or correspondence, I know of a few other contemplated hydrologic models. Yevdjevich at Colorado State University is contemplating an experimental study in the prediction analysis prototype category. The last information was that some thought was being given to how water could be sprayed on the acre or more of land that they are designating as their 'outdoor hydrologic laboratory.' M. J. Hall, a postgraduate student at Imperial College of Science and Technology, London, is also making rainfall-runoff studies with laboratory catchments up to 600 square feet in area. Though I do not know exactly what use is being made of these catchments, I suspect that they are for the same purpose as the prediction analysis prototypes.

To my knowledge, only one other individual is planning the construction of a physical hydrologic model that will be in the class of a general system synthesis. Jaromir Němec, Prague College of Agriculture, Czechoslovakia, contemplates the eventual development of such a model. He is now investigating material to be used in the construction of the model and is developing his similarity relations.

CONCLUSIONS

Only three efforts to make a dimensional analysis of the total rainfall-runoff system have been reported in the literature. Often, the development of similitude relations has been inferred from discussions of dimensional analysis, because it has been used to such a great extent to develop similitude criteria. A dimensional analysis does not, necessarily, mean that a physical model must follow, nor is it the only method by which similarity relations can be developed. However, with the many variables involved, nothing really informative can be gained by just making a dimensional analysis of the rainfall-runoff system; consequently, models have been employed in an effort to develop the complete functional relation between the many variables.

Two of the reported dimensional analyses were made for the purpose of developing similarity relations by which physical watershed models could be constructed. A third such model is being contemplated, but what method will be used to develop the similarity relations is not known. In both developments, it was physically impossible to satisfy all the similarity criteria; thus, the strict similarity criteria had to be violated, and various assumptions were made that would permit the design and construction of a physical model.

Tests with both models are sufficient to show that similarity was not maintained between model and prototype. Insufficient testing has been conducted to indicate what alterations of the model will direct it toward better simulation, or just how much the theoretical similarity criteria may be feasibly violated. It is apparent, however, that the similarity relations will have to be developed empirically. Thus, further investigations with scaled physical models appear worthwhile, because of the information that will be gained while attempting to establish the similarity criteria.

A development of similarity criteria using differential equations indicated that modeling of a very limited prototype may be possible, or that empirical similarity criteria will have to be developed by building models with systematic violations of the absolute similarity criteria and testing until simulation is achieved.

Several other investigators are planning or building laboratory catchments. They are not concerned with the development of similarity relations, however, because, in essence, their apparatuses will be non-scaled representations of prototypes. Their objective is to have a replication of a natural system over which they may have control of the input. They will then make comparisons between their mathematical analyses and the performance of the laboratory system for controlled inputs.

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