

MHD-1

OBJECTIVES AND TECHNIQUES OF WATERSHED MODELING ^{1/}

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INTRODUCTION

Watershed modeling is a procedure by which watersheds and processes taking place in them are represented by simplified systems which can be conveniently handled by digital or analog computers or, in some cases, by analog or physical small-scale models. The processes represented by the models are usually the parts of the hydrologic cycle that involve rainfall on the watershed and some transformations of the rainfall that take place within the boundaries of the watershed.

Watershed models are a fairly recent development in hydrology. While a number of works may be cited as marking the beginning of this development, there is no doubt that the 1962 report by Crawford and Linsley on the Stanford model was the most important of the early developments of models representing the relationship between total rainfall and total runoff. One of the notable models developed before the Stanford model is the linear reservoir model proposed by Nash (1957) for the relationship between rainfall excess and direct surface runoff.

Models are constructed for a variety of reasons. Two of the most important objectives of model construction are: (a) to gain a better understanding of the functioning of the watershed and the effects of changes in the watershed; and (b) to provide a design tool for generation of synthetic hydrologic data, either as an extension of existing

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data or as a substitution for nonexistent data. While the first of these objectives calls usually for detailed and complicated models that follow as closely as possible the physical phenomena, the second objective allows more freedom in simplification of the model and elimination or lumping of many intermediate steps in the transformation of input to output.

The development of a watershed model calls for two major steps. One involves a decision about the structure of the model and the second is the choice of the numerical values of the various parameters that are needed for the description or the use of the model. These two steps are usually interrelated; the structure chosen for a model influences the values of the parameters, and vice versa, the evaluation of model parameters may point out deficiencies or redundancies in a proposed structure and the need for changes in the structure of the model.

The purpose of the present paper is to discuss some of the models available or proposed for rainfall-runoff relationships and some of the techniques for the evaluation of the parameters of these models. The models considered will be mostly those in which the main purpose is to provide a tool for generation of synthetic data, and to a smaller extent, models designed to follow as closely as possible the physical processes that take place in the watershed. Stochastic models which generate synthetic data using only statistical parameters of the prototype data as input will not be discussed.

OBJECTIVES OF WATERSHED MODELING

As stated above, the two main objectives of modeling in general and watershed modeling in particular are: (a) to achieve a better understanding of the prototype system represented by the model; and (b) to provide means of extending available data and of generation of data for watersheds where measurements do not exist.

The idea behind the first type of objective is to test our concepts about the physical processes that take place in the watershed, to reaffirm those that are valid, and to reject the concepts that are contrary to reality. The natural processes involved in converting rainfall to runoff as well as in any other input-output conversion that

takes place in the watershed are highly complex and interdependent. Any set of differential equations and boundary conditions used as a mathematical statement of the behavior of the watershed or any breakdown of the complex processes into a set of simpler processes constitutes a statement of our concepts of the behavior of the watershed or of our concepts as to how the watershed behavior may be approximated.

When these concepts are formalized in the structure of a model and the rules of its operation, it is possible to test the validity of the concepts by subjecting the model to some sets of specified input data, which may be historical data observed on the watershed or special data prepared for testing the model. The behavior of the model is judged in terms of the output obtained from the model as a whole as well as the outputs at a number of intermediate points within the model.

If the output from the model, as well as the outputs at the intermediate points, agree with measured outputs at the prototype watersheds or with output expected by independent considerations, the model is assumed to be a true representation of the prototype, and the concepts used in constructing the model are considered to be validated. By varying the structure of the model, the values of its parameters or the values of the input data, it is possible to get an estimate of the sensitivity of the model to the various factors and an indication of the relative importance of the elements of model. It is also possible to study the results obtained with extreme or unusual combinations of input data such as would be obtained in nature only on rare occasions. It is these types of studies on watershed models that lead to a better understanding, or at least to more confidence in the formalized concepts, of the behavior of prototype watersheds.

The second type of objective (extrapolation and extension of existing data) requires less details in the internal structure of the model as it does not normally involve comparisons or verifications of output at intermediate points. It is, of course, true that a model constructed for the first type of study could be used also for prediction purposes, but in most cases it is not economical to do so. A model developed specifically to produce synthetic data of a specific nature, such as values of monthly runoff or values of maximum yearly discharge, will in general be more efficient in terms of computer time and in most cases will also produce more accurate results of the specific data for which it was designed.

Ideally, the structure of a predictive model and the values of its parameters should be the most efficient for the purpose for which the model is constructed. The term most efficient may be interpreted here in terms of a simplest structure of the model and the optimal set of parameters that will minimize some objective function related to the type of output for which the model is designed. The objective function may be defined in terms of the sum of squared deviations between model output and observations, the sum of absolute deviations, the magnitude of the maximum deviation, or some other function related to the deviations between observed and computed results.

If the model will be used to produce more than one output, for example a model designed to predict values of monthly runoff and monthly contributions to regional groundwater, the objective function must be formulated so that the deviations of the two outputs from their respective measurements are considered. The two types of deviations may be taken with equal weight, or one of the outputs may be given a larger weight if this is considered to be appropriate.

Predictive models that are expected to yield more than two or three types of output may become so complex that their structure will resemble that of models constructed for the first type of objective. It may well be more efficient in such cases to start with a model of the first type and simplify or modify it so that it will produce the types of outputs needed.

CHARACTERISTICS OF RAINFALL-RUNOFF WATERSHED MODELS

Examination of a number of models for the rainfall-runoff relationships in watersheds indicates that they have some common characteristics which are useful in the discussion of their properties. One basic characteristic of these models is that their inputs and outputs represent quantities of water -- the input being the rainfall over the watershed and the output the runoff at the outlet of the watershed. Internal inflows and outflows of the models also represent flows of water between the various elements of which the models are composed.

Another important characteristic of watershed models is the fact that their operation is based on some accounting procedure that keeps track of the quantities of water entering and leaving the various parts of the model. The equation that governs this accounting procedure for each element of the models, as well as for the whole model, is the equation of continuity. This is expressed either in terms of instantaneous values of flow

$$I - Q = ds/dt \quad (1)$$

or in a finite difference form in terms of volumes entering and leaving the element concerned in a finite time interval Δt

$$P - R = \Delta S \quad (2)$$

In the above equations, I represents the instantaneous rainfall intensity and P the precipitation during the time interval Δt

$$P = \int_0^{\Delta t} I dt \quad (3)$$

Similarly, Q represents the rate of discharge and R the volume of runoff in time Δt

$$R = \int_0^{\Delta t} Q dt \quad (4)$$

The units of the above quantities are assumed to be compatible so that no conversion factors are needed in the equations.

An additional characteristic of a great number of models is that they operate in terms of discrete finite time intervals which may be a day, an hour, a month, or any other convenient time unit. The unit chosen in each case is the one most significant for the problem being studied. The results produced by the model are expressed in terms of volumes involved as input or output of the various elements during each time increment. Alternatively, the results are expressed in terms of average rates of flow of the quantities considered during these time intervals, but practically the two forms are the same.

The various elements of the rainfall-runoff models perform, for each time interval, one of the following operations:

- (a) Addition of a number of inputs during the time interval and production of an output equal to the sum of inputs during the same time interval.
- (b) Proportioning of the input received during a time interval between a number of outputs during the same time interval
- (c) Storage of the input received at a given time interval and its redistribution in time so that output will be produced in the given time interval and also in a finite number of subsequent time intervals.

Some elements may be complex in the sense that they perform addition and proportioning as in Items (a) and (b) above, combined with a storage and time distribution with respect to one or more of the outputs.

Each of the above operations is carried out according to fixed operating rules for the element which may include reference to the state of the element as expressed by the storage accumulated in it. The state of an element may also be specified with reference to some external information such as, for example, the mean temperature for the time increment which, in turn, affects the evapotranspiration for the period concerned.

A typical example of the structure and functioning of a watershed model for rainfall-runoff relationships is shown in Figure 1. The model shown is that proposed by Dawdy and O'Donnel (1965). It is a simplified version of the Stanford Model developed by Crawford and Linsley (1962). The overall model is represented by the operator Φ and shown schematically by the dashed line box in Figure 1. It receives the daily precipitation P as input and produces as its main output the daily runoff R , a byproduct output is the daily evapotranspiration E .

The model may be considered as made up of six elements, each performing one of the types of operations listed above. The operators $\phi_1 \dots \phi_6$ of the elements are described briefly below, and they require nine parameters to fully specify their properties and rules of operation.

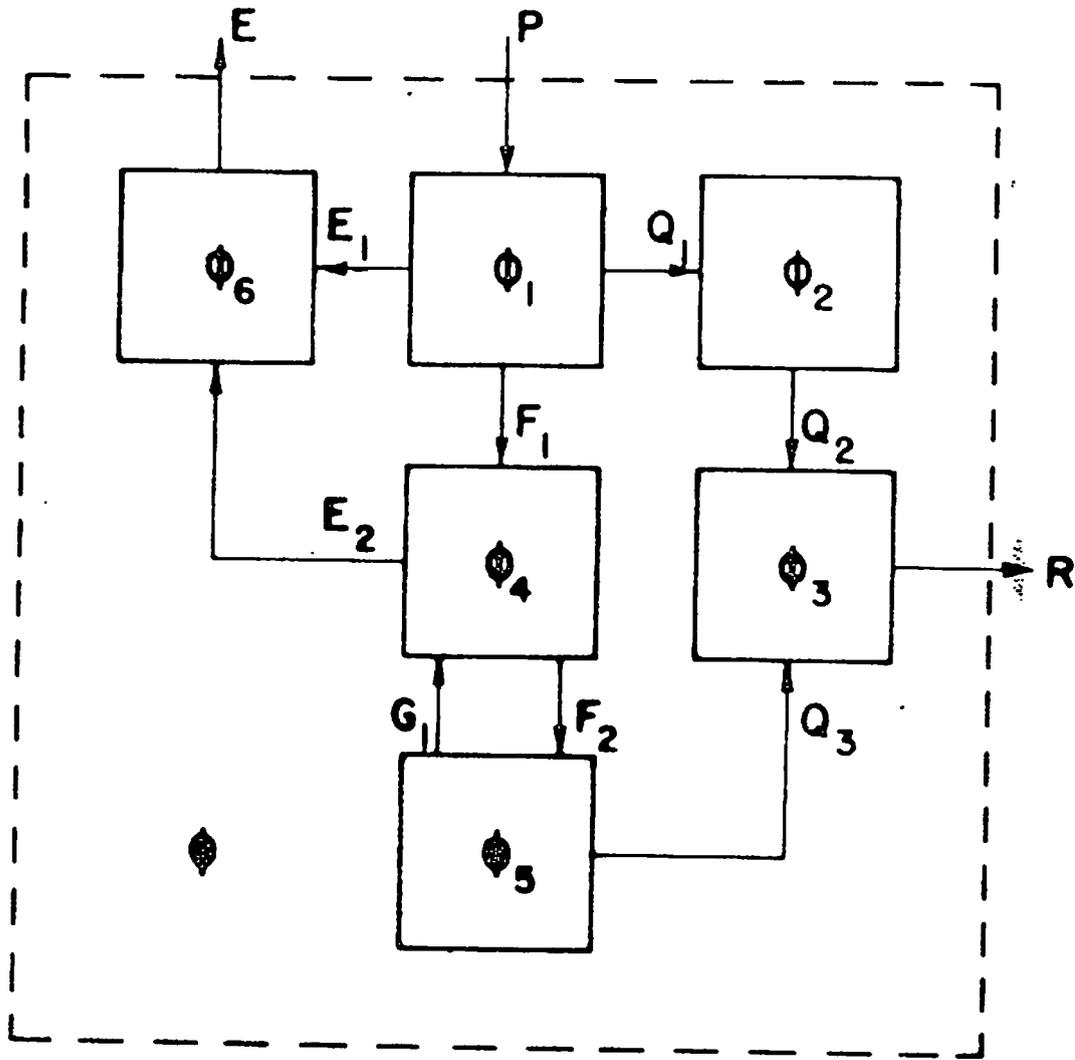


FIGURE 1. RAINFALL-RUNOFF WATERSHED MODEL (DAWDY AND O'DONNELL).

The operator ϕ_1 is a proportioning element receiving input P and producing 3 outputs: E, evaporation from surface storage; F_1 , infiltration to an intermediate zone; and Q_1 , surface runoff. Operator ϕ_2 is a distributing element receiving Q_1 as input and releasing it as surface runoff Q_2 distributed over a number of time intervals. Operator ϕ_3 is an additive element which receives two inputs, the modeled surface runoff Q_2 and the groundwater runoff Q_3 , adds the two, and produces the model output R, which is the total daily runoff. Operator ϕ_4 receives as inputs the infiltration F_1 and possible capillary rise of groundwater G_1 . It produces as outputs evapotranspiration E_2 and deep percolation F_2 . The operator ϕ_5 receives the deep percolation F_2 as input and produces two outputs, capillary rise G_1 and groundwater runoff Q_3 .

The number of parameters involved in the description of the various elements is as follows: ϕ_1 - 4 parameters, ϕ_2 - 1, ϕ_3 - 0, ϕ_4 - 1, ϕ_5 - 3, ϕ_6 - 0, or a total of nine parameters.

EVALUATION OF MODEL PARAMETERS

The first step in the construction of any model is to determine the structure of the model and the operating rules for each of the elements of the model. These rules include also the sequencing of mathematical or logical manipulations that are involved in the operation of the model. The sequencing adopted affects the results obtained with any model because computations are carried out for discrete intervals of time, and it makes a difference, for example, if input to a storage element is admitted before or after distributing its contents between a number of possible outputs.

Once a decision has been reached on the structure and operating rules of a model, the next step is to evaluate the various parameters needed for the operation of the model. There are basically two different approaches to the problem of evaluation of model parameters. One approach, related mostly to "physical" models, assigns the values of the parameters according to measurements of some physical entities in the watershed without regard to the goodness of fit between the observed data and the output of the model. The second approach is based on making the output of the model agree as closely as possible with recorded data. This is done with reference to a specified objective function which depends on the values of the parameters and which will attain either a minimum value or a maximum value as a result of choosing the optimal set of parameter values.

The selection of the numerical values of the various parameters is done in the second approach so as to obtain the "best" agreement, in terms of the objective function, with a set of data that is considered to be representative for the watershed investigated and for the purposes for which the model will be used. In many cases the parameters are not allowed to assume any value that will produce good agreement. The values are restricted, in such cases, within limits that are considered to be consistent with the physical concepts involved in the construction of the model. Thus, the threshold storage of an element cannot assume a negative value or exceed some higher limit even if such values may improve the prediction. Similarly, ordinates of a unit hydrograph are restricted to have only positive values.

The optimal set of values for the parameters of a model of given structure and operating rules depends on two important factors. One is the data selected for the comparison of the performance of the model, and the second is the form of the objective function that was adopted for definition of best agreement between observed and synthetic data.

The data used in the comparison are, of course, only a sample of the population of possible data that the prototype watershed is capable of producing given sufficient time. It is, of course, assumed that the prototype is stationary and that its structure does not change with time. Being a sample of a limited number of observations, the characteristics of the sample differ from those of the population to an extent depending on the number of observations. Obviously, the larger the number of observations, the nearer will the sample represent the original population and the better will the model parameters be for generating synthetic data that are presumed to be equivalent to data from the original population. The effect of choice of data is evident in those cases where a set of data is split so that one half can be used for determination of model parameters and the second for testing the adequacy of the model. If the roles of the two halves are interchanged, the optimal parameters derived from the two halves will not be equal and will be different from values obtained with the complete set of data. The effect of the data is also noticeable in cases where the parameters of a model are revised after a few years during which additional data are gathered.

The second factor influencing the values of the optimal parameters of a model is the choice of the objective function that defines what is meant by a good agreement between the synthetic data generated by the model and the historic data available for the watershed concerned. Comparison between observed and synthetic data is done with reference to the deviations between the two types of data at a finite number of discrete points. Even if the output is continuous, the comparisons are usually based on a number of discrete points along the continuous record. The objective function is defined in terms of these deviations, but any definition adopted is only one of the many definitions that can be formulated.

The most common definition for an objective function \mathcal{J} is the sum of the squared deviation between the historic data y_h and the synthetic data y_s produced by the model.

$$\mathcal{J} = \sum (y_h - y_s)^2 \quad (5)$$

Another common definition is in terms of the sum (or the mean) of the absolute magnitude of these deviations

$$\mathcal{J} = \sum |y_h - y_s| \quad (6)$$

Other definitions used or proposed include the maximum absolute deviation, the sum of the absolute values of the ten largest deviations, etc.

Another element of variation of the objective functions is the introduction of weighting factors. Thus, it may be felt that extra weight should be given to deviations in certain parts of the record than to other deviations. The above definition based on a sum of the square deviation would become

$$\mathcal{J} = \sum W(y_h - y_s)^2 \quad (7)$$

where W signifies the weight assigned to the particular values. Examples for such weighting can be found in cases where a better fit is desired for ordinates of the runoff hydrograph near the peak of the curve than in other parts.

A more complicated problem of objective function definition presents itself if the model produces more than one output. In such cases, a decision must be made on the relative weights assigned to the deviations of the various outputs and a method for combining deviations that may be of different dimensions.

None of the definitions of the objective function is, of course, better than the other definitions, at least not from a theoretical point of view. Each definition will produce a distinct set of values which are optimal values of the parameters of the model in terms of the objective function employed. The only guide for a choice of an objective function in any particular case is possibly some considerations of the future use of the model. It is thus quite conceivable that a model of a given structure will have several sets of optimal parameters depending on the type of information it is producing.

PARAMETER EVALUATION TECHNIQUES

The problem of evaluation of model parameters has been formulated above in terms of minimizing (or maximizing) an objective function while the values of the parameters are subjected to a set of constraints determined independently from the physical considerations related to the structure of the model. With the above formulation, many of the techniques available in linear or dynamic programming can be applied to obtain a set of optimal values of the watershed parameters.

One of the simplest techniques available for parameter evaluation is the simultaneous solution of a set of equations obtained when the objective function is differentiated with respect to each of the parameters defined by the model, and the resulting expressions are equated to zero. Thus, if the objective function S is a function of n independent parameters A_1, A_2, \dots, A_n

$$S = f(A_1, A_2, A_3, \dots, A_n) \quad (8)$$

it is possible to obtain n equations of the form

$$\frac{dS}{dA_i} = 0 \quad (i = 1, 2, 3, \dots, n) \quad (9)$$

and to solve for the n values of the parameters that will simultaneously satisfy the set of equations.

The method is applicable mostly to those cases where the functional relationship between the objective function and the n parameters is continuous and can be differentiated, and where the resulting equations are linear with respect to the parameters.

Where these conditions are not met and if the number of parameters is not large, a trial and error or a systematic mapping technique may be found useful. In this technique, the parameters are assigned arbitrary values, and the value of the objective function is calculated. The values of the parameters are then varied either in some systematic manner or randomly, and the value of the objective function is recalculated for each new set of parameter values. The resulting "map" of the values of the objective function is then inspected, and the set of parameters giving the minimal value of the objective function is chosen as the optimal parameters.

The random choice of parameters is used in cases where the range of possible values of the parameters is large or in cases where local minima of the objective function may be present. After finding the minimum by the random procedure, an additional systematic search for the set of optimal parameters is usually advisable in the vicinity of the optimal set found by the random procedure. In this search, the value of each of the parameters is varied systematically above and below the solution found by the random process.

A natural development of the above mapping technique is that of gradient climbing techniques. In these techniques the values of the parameters are changed from one computation to the next in such a way that the value of the objective function is continuously decreasing (or increasing) along the direction of its steepest rate of change. The first step is to compute, for a given set of parameter values, the partial derivatives of the objective function with respect to each of the parameters. The partial derivatives G_i are computed by incrementing each of the parameters A_i in turn by a small amount ΔA_i , noting the resulting change ΔS_i in the objective function while the other parameters are held constant at their original values and computing the partial derivative

$$G_i = \frac{\Delta S_i}{\Delta A_i} \quad (10)$$

If the changes in the parameters are all made of equal relative magnitude, for example by making the increment of each parameter equal to 10% or 5% of the value of the parameter, the resulting values of the change in the value of the objective function will be a measure of the sensitivity of the model to changes in the values of the parameter. Comparing the sensitivity of the model with respect to the various parameters will indicate those parameters that influence the value of the objective function to the largest extent.

After computing the partial derivatives there are a few possibilities that are available for proceeding to find the set of optimal parameters. One is to hold all parameters constant except for the one having the largest partial derivative. This parameter is varied by an arbitrary amount in such a way that the value of the objective function is decreasing. At each stage the value of the objective function and the values of the partial derivatives are recomputed, and the process is repeated until the changes in the value of the objective function or in the values of the parameters are less than some prescribed limits.

An alternative procedure is to change at each stage of the computations the values of a few or all the parameters. The relative change in the value of each parameter is made proportional to the sensitivity of the model with respect to this parameter. After each change, the value of the objective function is evaluated and if it has not reached a minimum value, further changes in the values of the parameters are made, keeping the relative changes as before. After reaching a minimum value for the objective function in the direction adopted for the changes, a new set of partial derivatives or sensitivities are computed, and the above scheme of computations is repeated until the desired optimal set of parameters is obtained.

A number of algorithms for carrying out the computations for optimal parameters are available. They are related to the gradient climbing technique discussed above, but each proceeds to achieve the goal of finding the optimal parameters by its own methods. Methods developed are oriented towards solution by digital computers, taking advantage of the capabilities of such computers. The descriptions of two recent methods are quoted below (with changes in notation to agree with that used herein). The first quote is from a paper by Dawdy and O'Donnel (1965):

Of several optimizing procedures available, one well suited to the catchment model problem is that developed by Rosenbrock (1960). The particular class of problems for which the method was developed is one in which (1) the parameters, A_i , are restricted by physical considerations and must fall within specific limits, and (2) the function, S , dependent on those parameters, and whose value is to be

maximized or minimized, is such that partial derivatives of S with respect to the various A_j cannot be stated analytically in usable forms.

If there are n parameters on which the function S depends optimization consists of a search in an n -dimensional vector space (formed by n orthogonal parameter axes and bounded by limits set on the n parameters) until the optimum value of S is found. Rosenbrock's method is recursive in that it makes this search in a series of repetitive stages. Each stage is terminated by evaluating a new set of n orthogonal directions along which the search during the next stage is conducted. The evaluation of the new directions is based on the movements made along the n directions of the current stage. Only in the first stage are the orthogonal directions coincident with the n parameter axes. In subsequent stages, the first component of the new directions lies along the direction of fastest advance.

During each stage, movement is made along each orthogonal direction in a series of steps. A step of arbitrary length, e , is attempted first. This is treated as successful if the resulting new value of S represents an improvement of, or is equal to, the previous value. If a success, the step is allowed, and e is multiplied by $\alpha > 1$; if a failure, the step is not allowed, and e is multiplied by $-\beta$, in which $0 < \beta < 1$. A new attempt is then made. These attempts are terminated as soon as at least one successful attempt, followed by one failed attempt, has been achieved in each of the n directions. Then the new orthogonal directions used in the next stage are evaluated. An attempt in the end must succeed for each direction, because e becomes so small after repeated failures that it causes no change in S .

The second quote is from a paper by DeCoursey (1968), reporting on work by DeCoursey and Snyder (1969).

After the form of the model is established, and the data to which it is to be fitted are collected, initial estimates of the parameters in the model are made. The data are then processed through the model and the

difference, residual error, between the predicted and observed value of the dependent variable is recorded for each set of observations. Optimum values of the parameters are found by an iterative reduction of this residual error. The method of optimizing the parameters is based on the "Method of Differential Correction" (Nielsen, 1957) with the technique of "Principal Component Analysis" (Kendall, 1957) used to relate the residual error to the parameters (see DeCoursey and Snyder, 1969).

The residual error in any prediction is the cumulative result of errors in each of the parameters. A measure of the size of the error associated with any one parameter is given by the change in the dependent variable caused by an incremental change in the parameter. If the ratios of the changes for each of the parameters are used as weighting factors, then the following equation can be used to relate the total error to corrections for each of the parameters.

$$E_i = h_1 \frac{\Delta \hat{y}_i}{\Delta A_1} + h_2 \frac{\Delta \hat{y}_i}{\Delta A_2} + \dots + h_m \frac{\Delta \hat{y}_i}{\Delta A_m} \quad (11)$$

where

E_i the prediction error for each observation, i ;

h_j a correction to the initial estimate of the numerical parameter, A_j ;

A_j a numerical parameter in the functional relationship between the dependent variable y and n independent variables, X_1 ;

$\Delta \hat{y}_i = \hat{y}_i - \hat{y}_{iA_j}$ the difference between the predicted value of y_i and the predicted value of y_i with A_j incremented a small amount, and

$\Delta A_j = A_j - (A_j + \Delta A_j)$ the increment by which A_j has been changed.

Equation 11 is linear; however, it is highly probable that the "independent" variables will not be independent, thus it would not be advisable to use multiple regression techniques to solve the equation for h_j by minimizing the

sums of squares of E_i . An alternative to the problem is to use components regression, in which the orthogonal, truly independent, components of the system are used to solve for h_j .

Components regression solves equation 11 for a minimum sum of squares of E_i by assigning values to h_j that are orthogonal with respect to each other. This is accomplished by developing the correlation matrix of the $\Delta \hat{y}_i / \Delta A_j$ data. The orthogonal components h_j used to correct the initial estimate of the parameters are transformations of the eigenvectors of the characteristic equation of the correlation matrix. (See DeCoursey and Snyder, 1969).

The correction terms h_j are added to initial estimates of the parameters and the new values are used as estimates for a second pass. This iterative reduction of the residual error is repeated until changes in the parameter values reach a minimum value.

PARAMETER EVALUATION FOR SURFACE RUNOFF MODELS

The parameter evaluation techniques discussed above are applicable to most watershed models. There are, however, some special techniques that were developed for the purpose of evaluation of the parameters of particular types of models. The most notable example of such special methods is found in models representing the surface runoff portion of the watershed response.

Surface runoff models receive as input a portion of the total rainfall designated as rainfall excess (I) and produce as output a portion of the total runoff hydrograph which is considered to be direct surface runoff (Q). The relationship between input and output is assumed to be either linear or quasi-linear.

In the first case, the model representing the relationship between rainfall and runoff has constant valued parameters, but in the quasi-linear case the parameters are assumed to be constant only during the occurrence of each event. Between events, the values of the parameters may change depending on antecedent conditions in the watershed.

The relationship between the rainfall excess input and the direct surface runoff output for the linear or quasi-linear model is given by the convolution integral

$$Q(t) = \int_0^t I(\tau) H(t-\tau) d\tau \quad (12)$$

where H is the instantaneous unit hydrograph or the impulse response function of the system.

The purpose of parameter evaluation techniques for surface runoff models is to find appropriate values of the parameters of the impulse response function H , while the form of the function is predetermined by the structure of the model adopted to represent the relationship. The basis of the various techniques is to estimate from the rainfall and runoff data a number of numerical characteristics that are related to the parameters sought. A set of simultaneous equations is thus obtained with the parameters as unknowns. Usually the number of equations is made equal to the number of unknown parameters so that a unique solution is obtained. If the number of equations is greater than the number of unknowns, a compromise or best fit solution is obtained.

The best example for this type of technique is that based on matching of moments. It can be shown that the moments of the impulse response function of linear (or quasi-linear) systems can be evaluated from the corresponding moments of the input and output functions of the system. On the other hand, once the structure of the model is fixed, it is possible to derive expressions relating the parameters used in the definition of the model to the moments of the impulse response function.

Considering, for example, a model composed of N equal linear reservoirs in series, the first two moments about the origin of the impulse response function is given by

$$M_1 = NK \quad (13)$$

$$M_2 = N(N+1)K^2 \quad (14)$$

where K is the time constant of the reservoirs. The two moments can be evaluated in terms of the moments of the input and output from

$$M_1 = M_{Q1} = M_{I1} \quad (15)$$

$$M_2 = M_{Q2} - M_{I2} - 2M_{I1}M_1 \quad (16)$$

where M_{Q1} , M_{Q2} , M_{I1} , M_{I2} are the first and second moments about the origin of the output and input functions, respectively. Solving for the two parameters of the model, the following expressions are obtained:

$$N = M_1^2 / (M_2 - M_1^2) \quad (17)$$

$$K = (M_2 - M_1^2) / M_1 \quad (18)$$

Higher moments are employed if the model contains more than two parameters.

Instead of moments, other characteristics such as times to peak or values of Laplace transforms may be used for the selection of the values of the parameters.

THE LINEAR CORRELATION MODEL

As an example of some of the ideas discussed above, the operation and evaluation of a simple rainfall-runoff model is given below. The model used for this illustration may be called the linear correlation model. The model is one of the simplest rainfall-runoff models, and it is suitable for describing the relationship between annual runoff volume and annual depth of precipitation. The structure of the model may be described in terms of only three elements or subsystems shown in Figure 2. The operators that define the three subsystems perform the following operations:

Operator ϕ_1 receives as input the annual precipitation P and divides it into two outputs R_1 and L_1 according to the following scheme:

$$\text{if } P \leq C \quad L_1 = P \quad \text{and} \quad R_1 = 0 \quad (19)$$

$$\text{if } P > C \quad L_1 = C \quad \text{and} \quad R_1 = P - C \quad (20)$$

where C is a constant parameter.

Operator ϕ_2 receives as input the output R_1 of the first subsystem and divides it into two outputs R and L_2 according to the following scheme

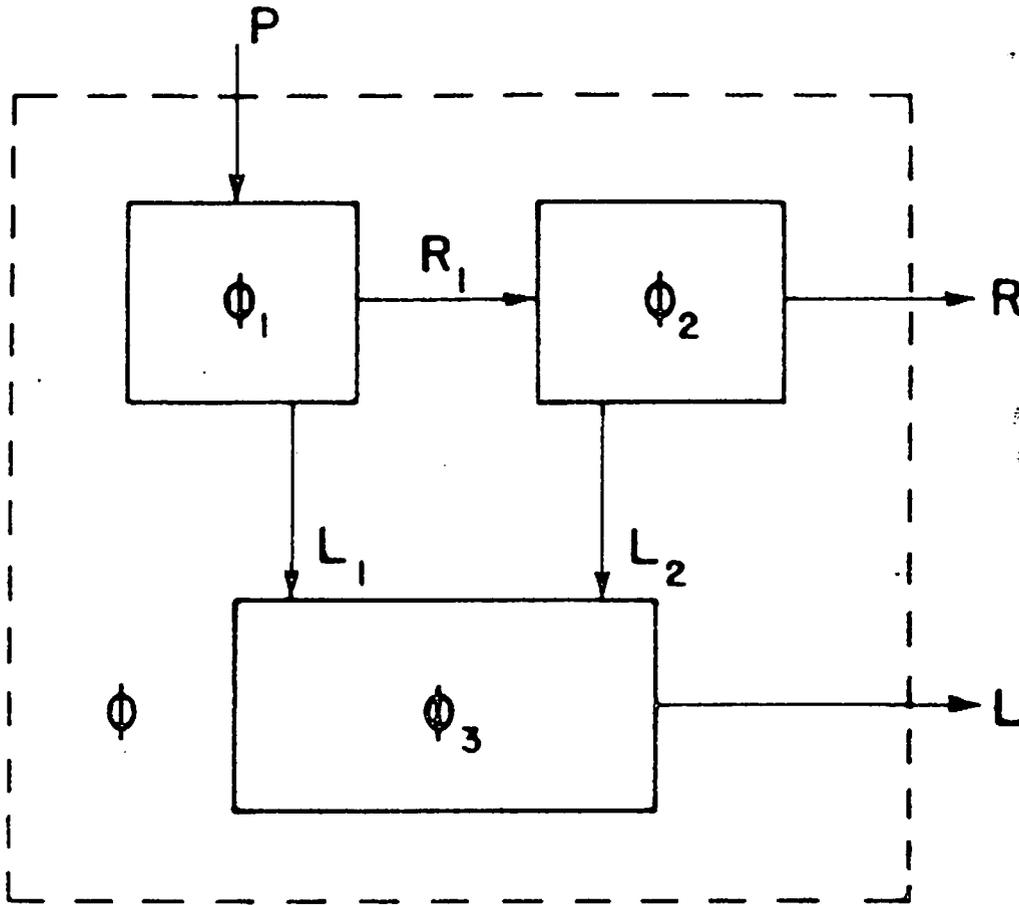


FIGURE 2. LINEAR CORRELATION MODEL

$$R = A \cdot R_1 \quad \text{and} \quad L_2 = (1-A) R_1 \quad (21)$$

where A is a constant parameter.

The third operator ϕ_3 is a summation operator receiving as inputs L_1 and L_2 and producing an output L which is the sum of the two inputs

$$L = L_1 + L_2 \quad (22)$$

The complete system represented by the dashed line box in Figure 1 and by the operator ϕ is thus seen to receive an input R equal to the annual depth of precipitation and to produce two outputs, the annual volume of runoff R and the annual volume of losses L which are mostly losses by evapotranspiration. Combining the operations of the first two subsystems (1 and 2) leads to the following direct relationships between annual volume of runoff and annual depth of precipitation

$$\text{if } P \leq C \quad R = 0 \quad (23)$$

$$\text{if } P > C \quad R = A(P-C) = AP - B \quad (24)$$

where $B = AC$ is a constant parameter. Similarly, the relationship between annual losses and annual precipitation is given by the following expressions:

$$\text{if } P \leq C \quad L = P \quad (25)$$

$$\text{if } P > C \quad L = (1-A)P + B \quad (26)$$

If all or nearly all values of the depth of precipitation are larger than the parameter C, the problem of evaluating the parameters A and B (or A and C) resolves itself into a least squares fitting procedure for the straight line described by Equation 24. The values of the parameters are given in this case by

$$A = \frac{\Sigma (RP) - N \bar{R} \bar{P}}{\Sigma (P^2) - N \bar{P}^2} \quad (27)$$

and

$$B = \bar{R} - A \bar{P} \quad (28)$$

where N is the number of observations, \bar{P} is the mean of the precipitation observations, and \bar{R} is the mean of the runoff observations.

The values of the parameters thus obtained are those minimizing the sum of the squared deviations between measured and predicted runoff volume.

The problem is, however, more complicated if the rainfall records are such that the annual precipitation values for an appreciable number of years of record are below the limiting value for the production of runoff. Such may be the case in arid or semiarid watersheds where the annual rainfall has a high variability and its mean value is not far from the limiting, no-runoff, value. An example of such a set of data is given in Table 1 and is shown graphically in Figure 3. The data have been prepared for the example discussed and do not represent any particular watershed.

If the above equations (Equations 23 and 24) for evaluation of the parameters are applied to all observations as given, the values obtained (line A in Figure 3) will indeed minimize the sum of the squared deviations of the whole set of data, but some of the predicted values of runoff for low values of rainfall will be negative. This is, of course, an impossible result, and it also contradicts the structure of the model as described above (line BC in Figure 3). The defect can be corrected by specifying that the predicted value of runoff is set to zero, whenever its value is negative, but doing so will increase the sum of squared deviation and will not correct the slope of the line and its location. The objective of minimizing the sum of the squared deviations between predicted and measured runoff values will thus not be achieved.

The optimal set of parameters can, of course, be obtained by a graphical procedure of passing a straight line (by eye) as in Fig. 3, but if a subjective method is desired, it is possible to employ a mapping technique. Values of the parameters A, B are assumed and the sum of squared deviations \mathcal{S} is computed for each such set of values. The resulting map for the data given in Table 1 is given in Figure 4, indicating that an optimal value of the parameters may be chosen from the points bounded by the line $\mathcal{S} = 5.0$, giving the following ranges for the parameters:

$$0.63 < A < 0.75 \quad (29)$$

$$-5.2 < B < -3.9 \quad (30)$$

TABLE 1.--Annual precipitation, runoff and losses data.

Ovservation No.	Annual precipitation	Annual runoff	Annual losses
	(in)	(in)	(in)
1	2.79	0.00	2.79
2	3.61	0.01	3.60
3	4.14	0.08	4.06
4	4.63	0.00	4.63
5	5.11	0.20	4.91
6	5.20	0.00	5.20
7	5.58	0.00	5.58
8	6.29	0.02	6.27
9	6.45	0.42	6.03
10	6.87	0.25	6.62
11	7.09	0.16	6.93
12	7.12	0.59	6.53
13	7.54	0.40	7.14
14	7.93	1.03	6.90
15	8.41	0.22	8.19
16	8.60	1.82	6.78
17	9.20	1.80	7.40
18	9.55	2.91	6.64
19	10.19	2.63	7.56
20	10.98	2.83	8.15
21	11.60	3.65	7.95
22	12.60	2.99	9.61
23	12.63	4.36	8.27
24	13.21	4.20	9.01
25	13.78	5.61	8.17
26	14.51	5.30	9.21
27	15.99	6.64	9.35
Mean	8.578	1.782	6.796

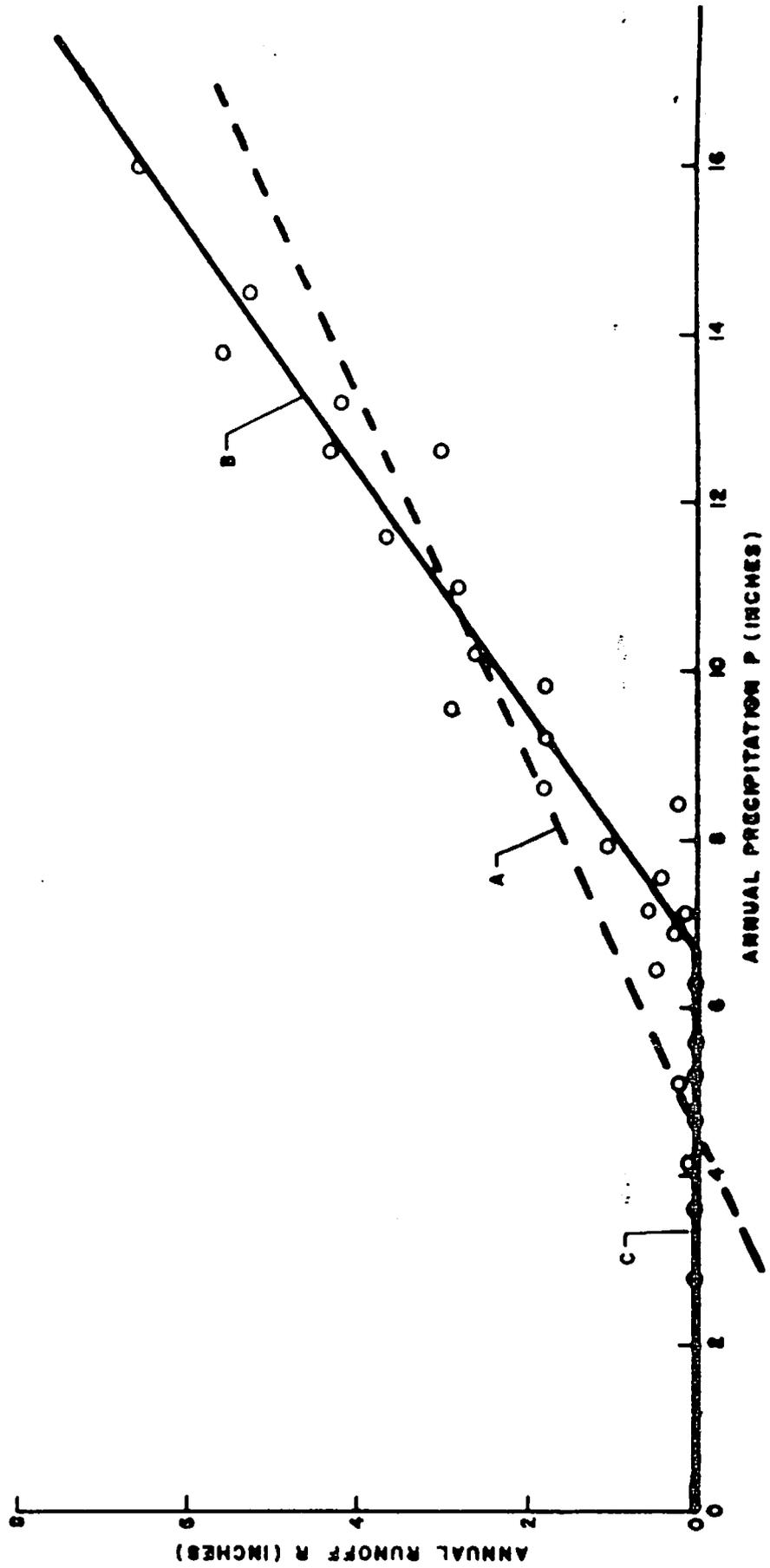


FIGURE 3. ANNUAL RUNOFF—RAINFALL RELATIONSHIP FOR LINEAR CORRELATION MODEL.

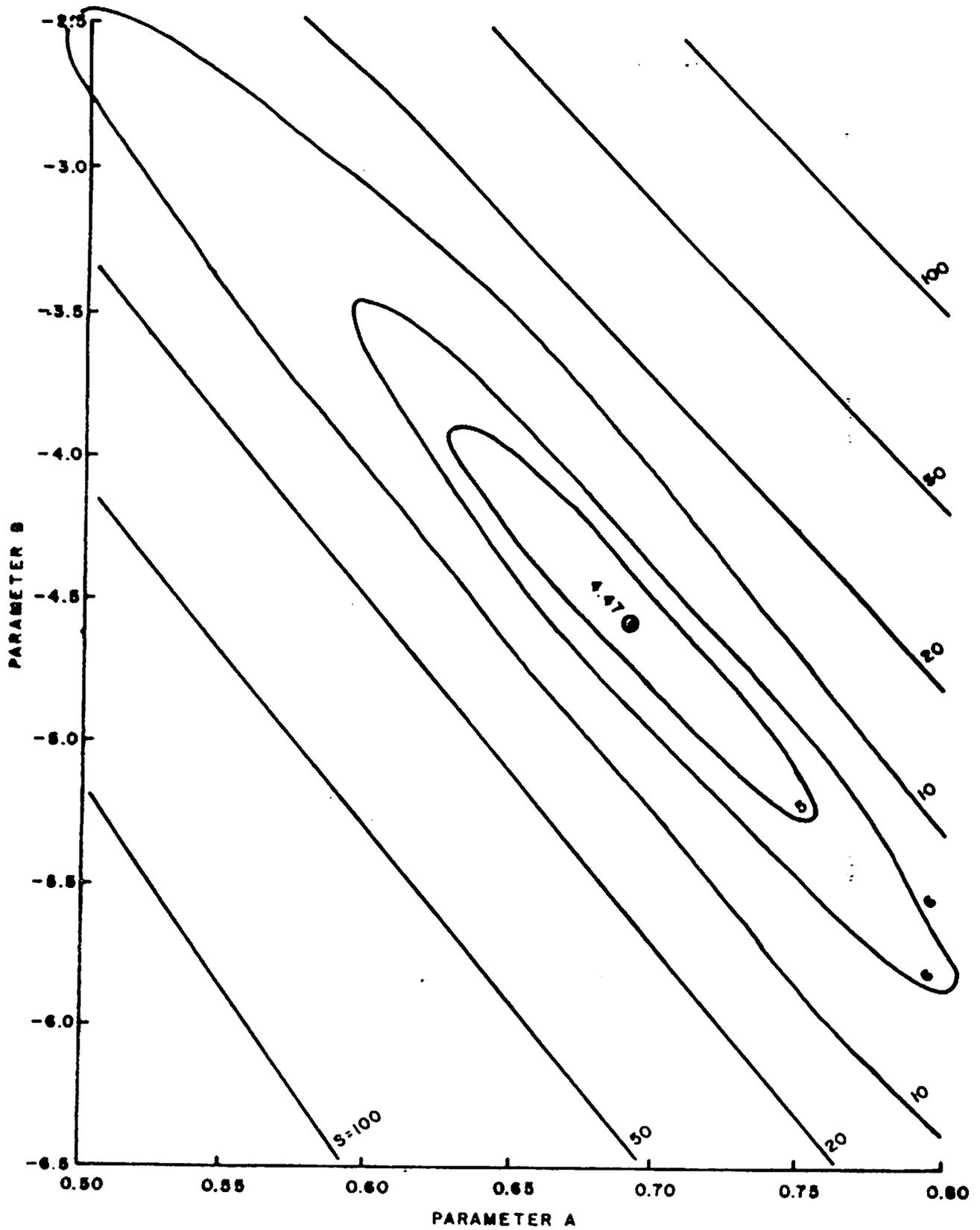


FIGURE 4. SUM OF SQUARED DEVIATIONS FOR LINEAR REGRESSION MODEL $S=f(A,B)$

As an alternative to the mapping, a gradient climbing technique may be employed. Starting with any assumed values for the parameters, the solution will follow the steepest gradient on the surface shown in Figure 4 until the minimal point, or a point near the minimum, is reached.

A special procedure suitable for this particular model is as follows. The data are arranged in increasing values of the observed precipitation P , as in Table 1, the observed data are then divided into two groups by choosing one of the values of P as a division point P_0 and the parameter evaluating equations (Equations 23 and 24) are applied only to observations in the group for which the precipitation is higher than the division point, ($P > P_0$). The straight line thus obtained will be the best fitting line for the observations within the group so defined. The sum S of the squared deviation of the observed values of runoff and the values predicted by the line

$$R = AP + B \quad \text{for } P > P_0 \quad (31)$$

or by the line

$$R = 0 \quad \text{for } P \leq P_0 \quad (32)$$

will thus be a function of the division point P_0 .

$$S = f(P_0) \quad (33)$$

if the point P_0 is varied systematically, it is possible to plot the functional relationship between S and P_0 and to determine the value of P_0 that will minimize the function S . The values of the parameters computed for this division point will be the optimal in the sense that the objective function S has obtained its minimal value consistent with the adopted structure of the model. The curve obtained for the data given in Table 1 is shown in Figure 5. The optimal point obtained by this method is shown also on Figure 1. The solution provided by this method appears to be identical with that provided by the mapping technique.

CONCLUSIONS

A number of hydrologic models for representing the relationship between the total rainfall on a watershed and the total runoff at the outlet of the watershed are available. Some of these models are used to further our understanding of the behavior of the watersheds but some are used only as tools for generation of synthetic data. Methods and techniques are available for the evaluation of the optimal values of the parameters used

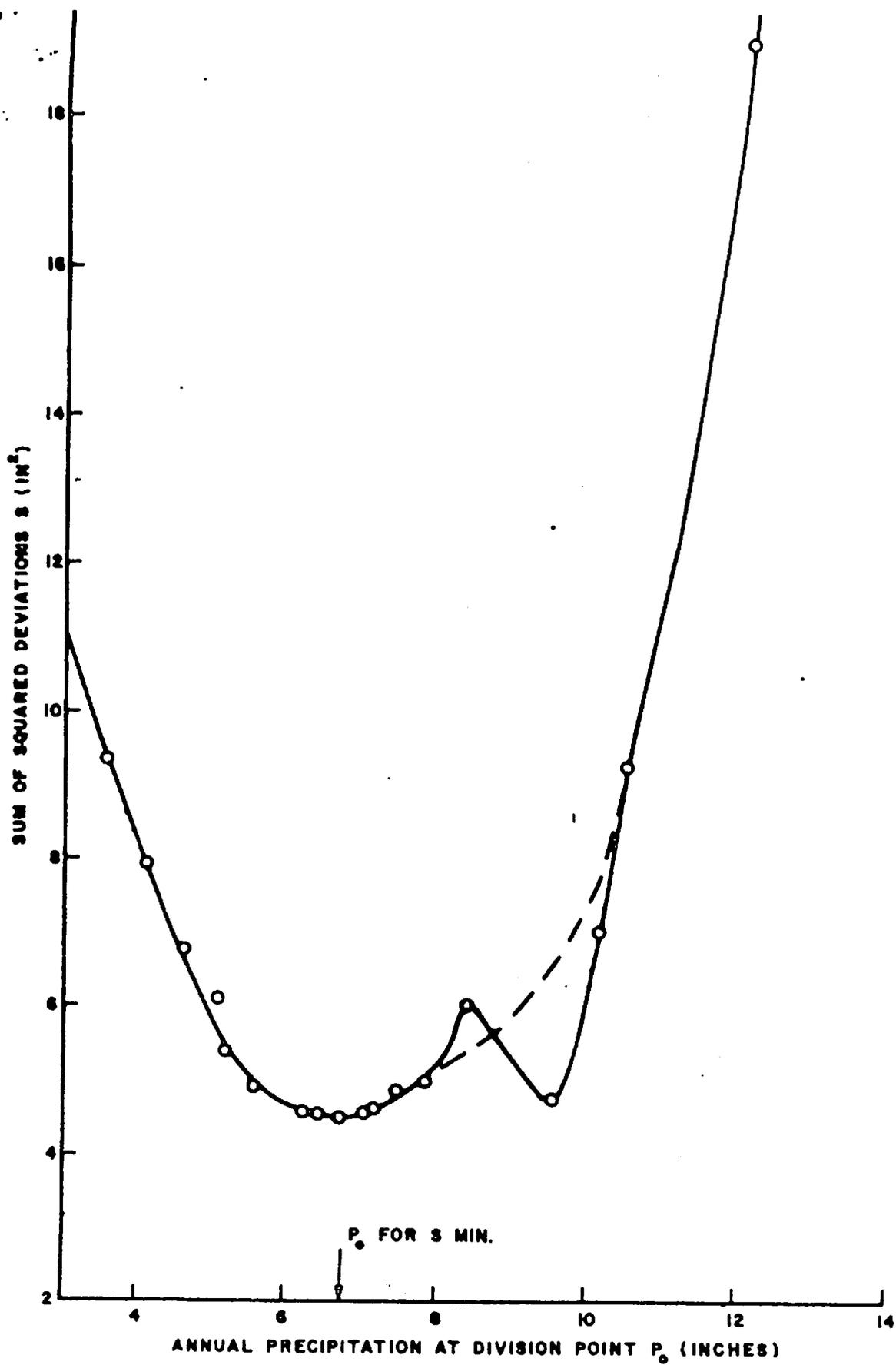


FIGURE 5. OBJECTIVE FUNCTION FOR LINEAR REGRESSION MODEL.

to define the structure and the operation of the model. The optimal set of values depends on the definition of the objective function used to specify the goodness of fit between the synthetic data and the historic data. The values depend also on the quality of the data used and the number of observations available.

Further research is needed to determine the effects of the above factors and the effects of the optimization scheme employed on the optimal values of the parameters of the model. The interrelationship between the structure of the model and the sensitivity and stability of the parameters also deserves some research effort.

A problem which will probably receive attention in the future is that of producing efficient special purpose models. These models may be defined as having the simplest structure consistent with the objectives for which the synthetic data are needed, while retaining some of the physical concepts used to construct the more complicated models designed to reproduce all processes that take place in the watershed.

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