Mechanics of Wind Erosion of Soils*

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1 Soil blowing away

When wind velocity exceeds the threshold $U_t$ soil blowing initiates. It is possible to express density of wind energy $E$ necessary for soil blowing with the use of kinetic energy of flow at threshold velocity, $E = 0.5U_t^2$. Variables $U_t$ (m/s), $q$ (kg/(m$^2$.s)), $\tau$ (N/m$^2$), $E$ (J/kg) are sufficient to characterize the process of soil blowing away. It is possible to make of them only two independent dimensionless combinations $B = gU_t/\tau$ and $Z = U_t^2/\tau_q$. $B$ is mass exchange parameter, similar to mass exchange parameter in physical theory of evaporation. It is used to describe gas-dynamic values on the interface between turbulent boundary layer and solid surface [1] and was proved to be $B = c_w - c_\delta$. Here $\tau$ – shear stress, known [9] to be $\tau = \kappa \rho_k U_a^2$, where $K$ – empirical coefficient, $c_w$ – concentration of soil particles in a thin boundary layer, separating soil from atmosphere, $c_\delta$ - concentration of soil particles in air flow far outside the roughness layer. Since concentration of soil particles in a flow above the roughness layer $c_\delta$ tends to zero with height then mass exchange parameter appears to be the concentration (kg/kg) of those soil particles that have lost intermodular coupling due to wind action. These particles are ready to be blown out by wind. According to the $\pi$-theorem [8] processes of soil blowing away can be determined with the use of function $B = f(Z)$, relating dimensionless parameters. At first the kind of this function was based on practical consideration [1,2,3,4,5] and later it was derived theoretically [6]

$B = B(1) e^{-\left(\frac{U_t^2}{\tau_q}\right)}$ (1)

This is a soil blowing away equation in which $\alpha$ is a soil constant and $B(1)$ is a mass-exchange parameter when $U_t = U_c$. The equation of blowing away represents a flux of soil particles directed from soil surface to the atmosphere. To strike a balance of wind erosion it is necessary to estimate a backward flux, directed from atmosphere to soil.

2 Taking off velocity of soil particle

Since movement of soil particles is attributed to vortices then their velocity should be related to this of vortices. Consequently soil particles of different size will take off with equal velocity depending first of all on vortices velocity, which is an unknown function of average flow velocity $U$. This is why we are introducing averaged soil particle, representative of all soil particles, flying due to action of wind of a given velocity. To find the taking off velocity of averaged soil particle $v_0$ we apply the law of mass conservation to soil particles flux through a surface of soil. Resulting [8] is the equation of taking off velocity of an averaged soil particle starting from the surface due to wind, $U_c$

$v_0 = \kappa \ U_c$ (2)

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The \( \kappa \) value depends on aerodynamic roughness of soil surface and in each particular case it should be determined experimentally.

3 Trajectory equation

Equation (1) gives a total flux of soil particles moving from ground to the atmosphere, kg/(m² • s). To predict soil loss it is necessary to divide the flux into two components, one of which is a flux of bouncing particles and another of flying particles. This can be done with the use of equation of trajectory of individual soil particle. Let us assume that outside thin roughness layer, commensurable with the height of roughness elements (vegetative cover, soil aggregates, ridges) wind velocity is constant \( U \). To determine a trajectory we shall consider forces, acting on particle in a flow. The starting soil particle has a vertical velocity \( v_0 \), according to equation (2). In the direction of \( x \) every particle of normalized radius \( r_i \) is subjected to drag

\[ F_d = K_s \pi r_i^2 \rho_s (U - u_i)^2, \]

where \( K_s \) - drag coefficient; \( u_i \) - horizontal component of particle velocity in the direction of \( x \) axis. Due to drag action soil particle gains acceleration in the direction of axis \( x \), therefore according to the second law of Newton

\[ m \frac{dv_i}{dt} = F_d \]

Here \( m \) - mass of soil particle of density \( \rho_s \), equal to \( (4/3) \pi r_i^3 \rho_s \), \( t \) - time.

Lengthwise \( y \) axis soil particle is subjected to Zhukovsky’s lift force \( F_z = K_{s*} \pi r_i^2 \rho_s U^2 \), directed vertically upwards, and Archimedes’ force \( F_a = \frac{2}{3} \pi r_i^3 \rho U^2 \), resulting of which is directed downwards, and Stokes’s force \( F_s = \phi \eta \pi r_i v_i \). Here \( K_{s*} \) - lift coefficient; \( g \) - gravity acceleration; \( \phi \) - shape factor of a particle in the equation of Stokes; \( \eta \) - viscosity of air; \( v_i \) - component of particle velocity along \( y \) axis. The equation of soil particle movement in projection to axis \( y \) is:

\[ m \frac{dv_i}{dt} = F_d - F_z - F_a \]

Substituting variables in (4), dividing both of its parts by \( r_i \) and introducing new variables,

\[ a = 4r_i^2 \rho_s / 3 \phi \eta \quad \text{and} \quad b = r_i (K_{s*} \rho_s U^2 - 4r_i g (\rho_s - \rho_a) / 3 / \phi \eta) \]

we are coming to

\[ a_i \frac{dv_i}{dt} = b_i - v_i \]

When solved \(^6\) this equation gave height of rise of soil particle \( y \) as a function of time

\[ y = b_i t - a_i (b_i - v_{i0}) \left( 1 - e^{-at} \right) \]

Substituting in (6) time \( t \) from equation (3) we are coming to

\[ y = b_i x / U - a_i (b_i - v_{i0}) \left( 1 - e^{-a_i x / U} \right) \]

Equation (7) describes a trajectory of soil particle \( r \), which has initial taking off velocity \( v_{i0} \), gained due to wind \( U \) action. The initial taking off velocity is provided with the equation (2). At the beginning of wind erosion process \( t = 0 \), both horizontal and vertical coordinates are equal to zero \((x = y = 0)\). If wind velocity \( U \) is not high enough to carry soil particle away irrevocably the particle will fall to the ground at some distance \( x_c \), from the starting point. This is why its vertical coordinate will become zero again. Hence, it is possible to find a length of hop of soil particle, \( x_c \), by substituting \( y = 0 \) in the equation (7)

\[ b_i x_c / U = a_i (b_i - v_{i0}) \left( 1 - e^{a_i x_c / U} \right) \]

This transcendental equation can be solved graphically, because the left part of equation (8) is a straight line and the right part is exponential curve. The abscissa of crossing of these two functions corresponds to the length of hop of a particle, \( x_c \).
Increase in flow velocity is accompanied by increase both in length $x$ and height of hop, $y_h$. When flow velocity becomes critical $U_t$, for particles of size $r_i$, then length of their hop becomes infinite and height of their hop tends to a limit $y_h = H$, which is the height of their horizontal flight \(^{(9)}\)

$$H = \frac{4 \kappa n^2 \rho_x U_{t2}^2}{3 \eta \phi}$$

Radius of particles that are flying at a constant height due to constant flow velocity $U_e$, we shall name critical, $r_c$. After expiration sufficient time ($t \to \infty$) vertical component of velocity of particles, being of critical radius, is found from (5) to be

$$v_i = b_i$$

Equating initial expression for $b_i$ to zero we receive critical radius

$$r_i = \frac{3 K_{ssp} \rho_a U_e^2}{4 g (\rho_i - \rho_a)}$$

4 Equation of mass preservation for soil continuum in dusty air flux

To describe a soil-air flux by methods of mechanics it is necessary \(^{(7)}\) to involve a concept of multispeed continuum and interpenetrating movement of components of a dusty airflow. Components of a flux in this case are the air continuum and the set $m$ of soil multispeed continua, all moving in the same volume of space. Air continuum is characterized by average velocity $U$ lengthways the direction of abscissa, and invariable density. Every $i$-th soil continuum constitutes a set of all soil particles of the $i$-th grade, filling up the same volume of space as other soil continua and the air. It means that all the continua are interpenetrating. The first derivative of density of the $i$-th continuum as a function of time and space was deduced in \(^{(6)}\). Due to incompressibility of the air flux it should be equal to zero

$$\frac{\partial \rho_i}{\partial t} + u_i \frac{\partial \rho_i}{\partial x} + v_i \frac{\partial \rho_i}{\partial y} + w_i \frac{\partial \rho_i}{\partial z} = 0$$

The relation between density and soil particles concentration $c_i$ was found and the total differential of a function $d\rho_i(x,y)$ was proved to be equal to zero, $d\rho_i(x,y) = 0$. It means that along any $i$-th trajectory concentration of particles of the $i$-th grade $c_i$ remains constant! This opens up fresh opportunity for detailed study of the flow structure.

5 Structure of a dusty air flux

If $U_t < U_e < U_{t2}$, then particle bounces, if $U_t \geq U_{t2}$, then particle flies. Threshold velocity $U_{t2}$ and appropriate height of horizontal flight $H$ are dependent on particles properties (9, 11).

If $U_t \geq U_{t2}$ and soil in the infinite field consists of uniform particles then all of them will eventually be involved in motion according to equation (4). They will ascend along concave trajectories (7) at any point of which mass concentration $c_i$ remains constant. Since starting points are arbitrary and trajectories do not intercross, this conclusion appears to be true as applied to any point of space enclosing a flow.

If $U_t < U_e < U_{t2}$, then soil particles of the $i$-th grade move along convex trajectories. Since particles are assumed to be identical, then their trajectories are similar. Let's consider a trajectory, beginning in any point $A$ (Fig.1) and terminating in point $B$, the latter being strictly determined by the equation of trajectory (7). Hence, only one trajectory $AB$ can terminate in point $B$. Due to the same reason only one trajectory
BC can start from it. Again, only one trajectory EA can terminate in point A and so on. Hence, both in points A and B concentration of soil particles remains constant, $c_i$, due to equal number of incoming and departing particles. Since point A is arbitrary, then this conclusion is fair for a whole surface.

Each trajectory of saltating particle has upward (AD) and downward (DB) portions. Trajectories of identical particles are supposed to be of similar shape. This is why upward portion of such a trajectory (AD) will be traversed by infinite set of descending branches of similar trajectories, initiating on segment EA. In exactly the same way downward portion of such a trajectory (DB) will be traversed by infinite set of ascending branches of similar trajectories initiating on segment AB. Consequently, in any point of trajectory AB, except for starting and finite, concentration of soil particles of the $i$-th grade is equal to $2c_i$.

Since the trajectory AB is arbitrary, then, this conclusion is true for the whole of the layer of their bouncing.

In case of two grades of bouncing soil particles the structure of two-phase air flow above piece of an infinite field can be represented as consisting of the following layers: 1) soil surface ($c = c_1 + c_2$); 2) layer of bouncing particles both of the first and the second grade ($c = 2(c_1 + c_2)$); 3) layer of bouncing particles of the second grade ($c = 2c_2$); 4) layer of pure air ($c = c_o$). Adding of $m$ grades of flying particles to a soil and consequently to a flow will result in replacement of the layer of pure air with a layer of these particles. Concentration of soil in it will be $c_a = \sum_{i=1}^{m} c_{ia}$. In other three layers concentration of soil phase will also increase by the same value. Considering arbitrary number of grades of bouncing particles, $n$, we shall observe adequate increase in number of layers, but the essence will not change. Establishing continuous function of distribution of grades of soil particles complicates the task strongly, but also does not change the result as a whole, having in mind that our main task is forecasting of soil loss due to wind erosion.

### Table 1 Input data for a model of wind erosion event in Sudan on 11 August 1991

<table>
<thead>
<tr>
<th>$\rho_n$</th>
<th>$\rho_o$</th>
<th>$\varphi$</th>
<th>$\eta$</th>
<th>$B_{sp}$</th>
<th>$\alpha$</th>
<th>$T_0$</th>
<th>$R$</th>
<th>$\Delta$</th>
<th>$\kappa$</th>
<th>$U_e$</th>
<th>$U_{sp}$</th>
</tr>
</thead>
<tbody>
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<td>kg/m$^3$</td>
<td>kg/m$^3$</td>
<td>kg/(m$^3$)</td>
<td>kg/kt</td>
<td>°C</td>
<td>°C/kgt/K</td>
<td>°C/100m</td>
<td>m/s</td>
<td>m/s</td>
<td></td>
<td></td>
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<td>0.0037</td>
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<td>30</td>
<td>287</td>
<td>0.65</td>
<td>0.0042</td>
<td>8</td>
<td>7.6</td>
</tr>
</tbody>
</table>

### 6 Forecasting of soil loss on an infinite field

To solve the problem of forecasting the amount of soil loss from a limited surface $S$ of an infinite field due to wind erosion we shall consider the law of reservation of mass of soil articles flow through this surface. If wind velocity $U_e \geq U_i$, then soil articles flux $q$ a ears, directed from the ground to the atmosphere. If $U_e \geq U_a$ for all of the grades of articles $r_i$ resent, then no backward flow from the atmosphere will occur. In this case all of the articles taken off from the surface will be carried away irrevocably. However, such a situation is improbable. In fact there is only a share (A) of articles $r_i$ in a superficial layer of soil, which can be carried away by wind $U_i$ irrevocably (those, for which $U_i < U_e$) and some other share (B) of articles, which can be forced to bouncing (those, for which $U_e < U_i < U_a$). Then maximal possible loss of soil $Q$ during time $t$ due to wind erosion from arbitrary site of an infinite field within the framework of resent theory can be calculated

$$Q = \frac{A}{A + B} \times S \times t \times \tau \times B(1)U_e^{-1} \times e^{-\frac{(U_i^2 - U_e^2)}{U_e}}$$

### 7 Long range transport and deposition of soil particles

To deduce the equation of trajectory of individual soil particle in case of long range transport we have to solve equation (4) taking into account decline of air density with height accordint to $\rho_{a} = \rho_{a0}(1 - \beta y)$, where $\beta = (100g/RA)^{-1}$, $T_0 = 288$ Kelvin, $\rho_{a0}$ - air density at $y = 0$.?
\( \rho_{a1} \) – air density at \( y_1 \), \( R \) – universal gas constant. The solution is a trajectory equation for soil particle, taking into account air density decline with height

\[
y_b = \frac{1}{\lambda} \left( 1 - \frac{4g_R \rho_a}{3 \rho_{a1} K_u U^2} \right) - A_0 e^{-\frac{b_1}{2}}
\]

(14)

All these equations were used to model a three days wind erosion event in Sudan on the 11-th of August 1991 (Fig. 2). The input data (Table) are based on [2,3,4,6,11,12] and Fig.3. We took into account only particles (\( r_i = 40 \) till \( r_i = 60 \) mkm) that are reported to be blown out of Sakhara \(^{10}\). Results of the modeling (Fig. 4) show qualitative agreement with the picture. The most important result is that soil particles collected by a wind from a huge territory are deposited on a small spot.

Fig.2  Red Sea and Dust pall in Sudan (21° N,38° W)08/11/91:NASA, ID STS043-151-086

Fig.3  Sea level wind velocity for the Red Sea region on 08/11/91. NOAA CDC (http://www.cdc.noaa.gov.HistData)

Fig.4  Calculated trajectories of soil particles in Sudan during dust storm of 08/11/91

8 Conclusion

This study was resulted in a development of physically sound theory of wind erosion of soils which is intended to become a core of experimental work on wind erosion in any location that suffer of wind. The theory itself is rather complicated but the resultant equations are simple enough and are ready for use
because they need variables easy to measure and are readily developed with the use of existing data. The theory describes full-scale wind erosion phenomenon including soil blowing away, soil particles saltation, transportation and deposition. It is applicable for use both in local and global scales in particular for the purposes of dust transportation monitoring.

References