Hydraulic Modeling of Irrigation-Induced Furrow Erosion

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ABSTRACT

In the experimental Version 4.xx series, erosion science is introduced into the surface-irrigation simulation model, SRFR. The hydraulics of water flow in furrows for individual irrigation events is predicted by numerical solution of the unsteady equations of mass and momentum conservation coupled to generally applicable empirical equations describing infiltration and soil roughness and to a known furrow configuration and inflow hydrograph. Selection of appropriate field values for the infiltration and roughness coefficients yields infiltration distributions and surface flows (including runoff) in reasonable agreement with measurements. The erosion component consists in applying the simulated hydraulic flow characteristics to site-specific empirical determinations of soil erodibility, to general empirical sediment-transport relations, and to general physically based deposition theory to provide estimates of soil erosion, flux, and deposition at various points along the furrow as functions of time. Total soil loss off the field and ultimate net erosion and deposition along the furrow follow. At this initial stage of the investigations, a single representative aggregate size is assumed adequate for the analysis. Results are compared to measurements of sediment concentrations in the furrow quarter points and in the tailwater. For a given representative aggregate size, the results are heavily dependent on the choice of transport formula. The Laursen (1958), Yang (1973), and Yalin (1963) formulas are programmed for investigation, as are a variety of computational options. Preliminary comparisons suggest the superiority of the Laursen formulation, with the Yang and Yalin formulas significantly over-predicting transport.

INTRODUCTION

Since its release in 1995, users have noted problems in applying WEPP (Water Erosion Prediction Project) to irrigated furrows, citing major discrepancies between simulation and field-measurement of both infiltration and erosion / deposition profiles (e.g., personal communications from T.L. Spofford, Natural Resources Conservation Service (NRCS); Bjorneberg et al, 1999; Bjorneberg and Trout, 2001). A review of the supporting documentation and literature revealed a number of unnecessary and possibly flawed assumptions within the hydraulic components of the model. Not long after, in a parallel development, the U.S. Water Conservation Laboratory (USWCL, USDA/ARS) released SRFR, Version 3, a comprehensive surface irrigation-simulation program rooted in hydraulic theory.

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Model Attributes

The USWCL model for surface irrigation-induced furrow erosion yields as output -- sediment-transport hydrographs at user-selected points along the furrow (grams per second), and in concentrations by weight and volume; post-irrigation cumulative erosion and deposition depth profiles (local kg/m²), separately and combined (net); and soil-loss figures (tons/hectare) from the successive reaches of the furrow. For experimentation, one of the three aforementioned transport-capacity formulas can be selected by a user. In addition, several other simulation assumptions, described below, can be incorporated, to test their effect.

The erosion detachment/transport/deposition process

The rate of detachment of soil granules or aggregates from the soil bed for entrainment into the flow is assumed (as in WEPP, 1995 and Fernandez, 1997) to depend on calculated stream erosivity and an empirical, site-specific soil erodibility. A key factor in stream erosivity is average shear stress \( \tau \) on the bed,

\[
\tau = \rho g S_f R
\]

Here, \( \rho \) is the mass density of the water, \( g \) is the ratio of weight to mass, \( S_f \) is the friction slope, and \( R \) is the hydraulic radius (cross sectional area divided by wetted perimeter); all are known on the SRFR time-space grid. Usually, in furrows steep enough for erosion to be a problem, the assumption, \( S_f = S_o \), can be made, implying a bottom slope \( S_o \) sufficiently large that gravitational and drag forces are essentially in equilibrium.

The potential soil detachment rate (i.e., by a stream of clear water) is assumed in the form (Foster, 1982),

\[
D_p = K_r (\tau - \tau_c)
\]

in which \( D_p \) is the detachment rate per unit wetted width and length (Kg per second per meter width, per meter length). The soil erodibility, \( K_r \), and critical shear stress, \( \tau_c \), are determined by measurements of sediment transport rate under various flow rates of clear water, at the site. According to this theory, unless the threshold, \( \tau_c \), is exceeded by the local shear, there no detachment. Plots of \( D_p(\tau) \), such as those of Fernandez (1997), yield a slope, \( K_r \), and an intercept, \( \tau_c \). Fernandez reports a consistent, small decrease of \( K_r \) with time, which he quantifies by a curve-fit equation (for \( t > 1 \)),

\[
K_r = a + b \ln(t)
\]

in which \( a \) and \( b \) are constants, with \( b \) negative. Equation 3 is programmed into SRFR (as is Equation 2 and the others presented here), although to the present time, numerical entries for \( b \) have not been obtained for US soils, and \( K_r \) is simply entered as a constant.

Foster (1982) suggests that the shear stress in Equation 2 be restricted to the resistance of the soil grains only, and not the drag of bed forms (e.g., dunes) or plant parts. In principle, in the empirical determination of \( K_r \), the fraction of the total shear stress used for the abscissa values in the plots must be identified, so that the same convention can be used in calculating \( D_p \) in a subsequent simulation. Actually, in practice, experimental values of \( K_r \) are usually determined in furrow segments free of dunes, debris, or plant parts, so that the total shear constitutes also the shear due to particle drag. But then, in subsequent simulation, the total shear is partitioned into a fraction due to particle drag and that estimated for dunes or other factors. In the present version of SRFR, with but one representative grain size, the default option is to use total shear in the simulation. However, in the erosion calculations, SRFR does allow substitution, instead, of a shear value based on grain size,

\[
\tau_s = \left( \frac{\rho V^2}{58.2} \right) \left( \frac{D_s}{R} \right)^{1/3}
\]

derivable from Strickler’s relation between Manning n and equivalent sand-roughness size (see, e.g., p. 98, Henderson, 1966). Here, \( V \) is water velocity, i.e., local flow rate divided by local cross-sectional area (standard hydraulic solution variables in SRFR), and \( D_s \) is the representative grain size. As another alternative for experimentation, the Darcy-Weisbach relationship,

\[
\tau_{Dw} = \frac{f \rho V^2}{8}
\]

can be substituted, with the Darcy-Weisbach \( f \) given as in WEPP (1995) as a constant,

\[
f = 1.11
\]

despite an uncertain theoretical basis. Flows in irrigation furrows are relatively deep, compared to the very shallow overland flows on which Equation 6 is based (see, e.g. Henderson, 1966, p. 97, for factors influencing \( f \), as well as typical numerical values in open channels). Equations 4 and 5 are relatively more severe treatments of grain shear than partitioning, in which sand-grain shear is always a fraction, less than or equal to one, of total shear.

The influence of sediment already entrained upon actual detachment at some location down a furrow is characterized as in (Foster and Meyer, 1972),

\[
D_f = D_p \left( 1 - \frac{G_s}{T_C} \right)
\]

in which \( D_f \) is the actual detachment rate per unit width and length, and \( G_s \) is the local sediment load (Kg/sec) carried by the stream at that point; \( T_C \) is the local transport capacity of the flow (Kg/sec). When load equals transport capacity, detachment stops. The erosion rate \( E \) per unit length, then, is simply

\[
E = D_f W
\]

in which \( W \) is the wetted width or perimeter of the stream, and \( E \) is in units of Kg/sec per meter length.

In general, then, the sediment load increases with distance down the furrow. If at some point the sediment load matches the transport capacity of the flow, one of two scenarios can take place. If the hydraulic variables (and transport capacity) were to remain constant with distance downstream, the sediment load, too, would remain constant from station to station, until field end, where it would discharge into the receiving drain. But if transport capacity decreases with distance, e.g., if flow rate decreases, the stream begins to drop some of its entrained load. Because of
finite fall velocities, loads typically exceed transport capacity for a distance -- as required to deposit the excess to the bed. If it is assumed that the sediment is well mixed in the flow over the depth, $y$, and any excess load over transport capacity falls at the rate $w_s$, then, in the time, $\delta t / V$, it takes for the sediment vertical profile to be advected over a small distance, $\delta x$, the fraction of the excess that reaches the bed is $(w_s \delta x / V) / y$. Thus, the distance-rate of deposition, $D$, (Kg/sec per unit distance) is

$$D = \frac{(G_s - T_c) w_s}{V y}$$

(9)

Following Fernandez (1997) and WEPP (1995), which view the overall sediment-transport phenomenon as a sequence of steady states, the sediment load at any point in the flow $G_s(x,t)$ is given by the solution of the ordinary differential equation,

$$\frac{dG_s}{dx} = E - D$$

(10)

Fig. 1, drawn from a frame of the animation displayed by SRFR during the simulation, illustrates typical behavior of the transport-capacity function and resultant sediment loads at one instant of time (60 minutes into the irrigation). Note the region behind the stream front in which the transport capacity and detachment are zero. The flow rate there is so small that the boundary shear lies below the threshold for entrainment (discharge continually decreases with distance down the field, because of infiltration). Far upstream, the sediment load grows the fastest at the clear-water inflow, where the transport capacity is a maximum and the existing sediment load zero. With distance downstream, the transport capacity decreases due to infiltration, and the sediment load increases due to upstream entrainment; both factors lead to reductions in further growth in the load. Eventually, though, transport capacity is exceeded, and some of the load starts to deposit back onto the bed. In accord with Equation 9, some excess of load over transport capacity persists over a short distance.

In view of the significant role played by transport capacity in the entire detachment/transport/deposition process, the transport-capacity formulas accessible in SRFR are reviewed next.

**Transport-capacity formulas --**

Laursen, 1958 --

Total-load transport capacity was given by Laursen (1958) for a single, representative sediment size $D_s$ and assumed specific gravity of 2.65 as

$$T_{s,Laursen} = 0.01 \rho Q \left( \frac{D_s}{R} \right)^{7/2} \left( \frac{\tau_s}{D_s} - 1 \right) F_r(\tau_s)$$

(11)

In this expression, $Q$ is the local flow rate, $\tau_s$ is the boundary shear attributable to the grain roughness (Equation 4, specifically), and $\tau_{s,c}$ is the critical shear given by the Shields diagram (see, e.g., Henderson, 1966) as a function of particle Reynolds number. In the event $\tau_s < \tau_{s,c}$, transport capacity is zero. The factor, $F_r$, is a function of the ratio, $r_s$, between (total) shear velocity and sediment fall velocity $w_s$, i.e.,

$$r_s = \frac{\sqrt{\tau / \rho}}{w_s}$$

(12)

and is given by an empirically determined curve in Laursen (1958) reflecting his particular choice of data sets of measured total loads. In SRFR, both the Shields diagram and Laursen’s curve are fitted with analytic expressions to facilitate computation. Interestingly, Laursen, in his correlations between Equation 11 and measured total load, did not utilize the Reynolds-number argument of the Shields critical stress. Instead, he postulated several constant values of dimensionless critical shear, $\tau_{s,c}$, depending on the data source. With two of his data sets, he assumed $\tau_{s,c} = 0.08$ and 0.16, respectively, while the remainder used $\tau_{s,c} = 0.04$, near the lowest point of the Shields diagram.

The fall velocity, $w_s$, if not given as input data, is calculated in SRFR by the formula of Rubey (Simons and Senturk, 1992, cited by Fernandez, 1997),

$$w_s = \left( \frac{2}{\sqrt{3}} + f_{c,v} - \sqrt{f_{c,v}} \right) \sqrt{g D_s \left( \frac{\rho_s}{\rho} - 1 \right)}$$

(13)

in which

$$f_{c,v} = \frac{36 \nu^2}{g D_s \left( \frac{\rho_s}{\rho} - 1 \right)}$$

(14)

with $\nu$ the kinematic viscosity of the water and $\rho_s$ the mass density of the sediment.
Yang, 1973 –

Yang developed a formula for total-load transport capacity with an approach based on the excess of stream power -- rather than bed shear -- over a critical value. Specifically, Yang’s transport capacity depends on the excess of a relative stream power,

$$R_p = \frac{V_s}{w_s}$$

over a critical value, given by one of two formulas, depending on the particle Reynolds number (ratio of grain size to thickness of the laminar sublayer):

$$R_{PCR} = \left( \frac{25}{\log_{10} \left( \frac{\tau / \rho D_s}{\nu} \right)} - 0.06 \right)^{0.06} S_0$$

for $\frac{\tau / \rho D_s}{\nu} < 70$ and

$$R_{PCR} = 2.05 S_0$$

for $\frac{\tau / \rho D_s}{\nu} > 70$ (16)

Then, for $R_p > R_{PCR}$,

$$T_{cY} = 0.000001 \rho Q 10^4$$

$$\lambda = 5.435 - 0.286 \log_{10} \left( \frac{w_s D_s}{\nu} \right) - 0.457 \log_{10} \left( \frac{\tau / \rho}{w_s} \right) +$$

$$0.409 \log_{10} \left( \frac{w_s D_s}{\nu} \right) - 0.314 \log_{10} \left( \frac{\tau / \rho}{w_s} \right) \log_{10} \left( R_p - R_{PCR} \right)$$

If $R_p < R_{PCR}$, the transport capacity is zero. The constants in Equation 19 were selected for the best fit to data (primarily sands), while that in Equation 18 reflects the fact that Yang’s concentration formula is expressed in parts per million by weight. Worthy of note, the right side of Equation 16 is undefined for sufficiently small grain sizes, and the equation for transport capacity fails (Fernandez, 1997) at a particle Reynolds number of about 1.15. In his book (Yang, 1996), Yang puts a lower limit on $\sqrt{\tau / \rho D_s / \nu}$ of 1.2, in Equation 16.

Yalin, 1963–

The transport capacity of a flow is given by Yalin as

$$T_{cY} = 0.635 \rho \lambda \left[ \frac{\tau}{\rho \gamma_s} \right] \left[ \frac{\tau}{\tau_s} \right] \left[ 1 - \frac{1}{\sigma} \right]$$

in which

$$\sigma = 2.45 \left( \frac{\rho}{\rho_s} \right)^0.4 \sqrt{\frac{\tau_s}{\rho_s \gamma_s}} \left( \frac{\tau}{\tau_s} \right) - 1$$

(21)

In these expressions, Yalin utilized the average total shear stress given by Equation 1, with, furthermore, $S_0 = S_{90}$. Alternate formulations suggest that $\tau$ should be restricted to that portion of the total drag for which the sediment grains themselves are responsible. Consequently, the choices available in SRFR for experimentation are full shear (Equation 1), Strickler shear (Equation 4), or the WEPP (1995) assumption (Equation 6).

It may be worthwhile to note the typo in Foster, 1982, p.341, in which the term $\rho / \rho_s$ of Equation 27 is inverted.

The Erosion Component of SRFR

SRFR (Strelkoff et al, 1998) is a computer program for simulating surface-irrigation. It solves the equations of mass and momentum conservation of general physics, coupled to empirical formulas for time-dependent infiltration and the hydraulic drag of bed roughness and submerged plant parts upon the surface stream. The formulas are complemented with site-specific coefficients, input to SRFR as data, along with system geometry and inflow. The equations are solved in a series of time steps over the length of the surface stream, found as part of the solution (and leading to the advance and recession functions of time). Thus, at every computational time level, the flow depths and velocities are known at a sequence of points within the surface stream. In SRFR 4.xx, these provide the local bed shear and other flow factors entering into the erosion/transport/deposition equations as described above, and allow numerical solution of Equation 10 at each successive time level, $t_i$, to yield the sediment fluxes $G_s(x,t_i)$ at each point. These, in turn, lead to the other erosion-related outputs described under Model Attributes, above.

As noted, the user of the erosion component in SRFR 4.06 can select one of the three above transport-capacity formulas, along with pertinent assumptions. For orientation purposes, Fig. 2 illustrates how transport capacity varies in one particular example furrow with the different approaches. The furrow is of trapezoidal cross section (base width of 150 mm and side slopes 1:1) and is set on a 1.3% bottom slope. Manning n is 0.04, while the representative size of the soil grains is 0.05 mm. Bed shear, and hence transport capacity, increase with increasing flow rate in the furrow.

Figure 2. Comparison of sediment-transport-capacity formulas. Representative particle size $D_c=0.05$ mm, furrow base width = 150 mm and side slopes = 1:1.
Sample applications

Some preliminary comparisons of simulated and field-measured sediment transport were undertaken as a first test of our characterization of the actual mix of particle and aggregate sizes present in the furrow bed by a single representative particle size and density. All experiments were run in Portneuf silt loam soil at the ARS Northwest Irrigation and Soils Research Laboratory, Kimberly, Idaho, in 1994 (see, e.g., Trout, 1996) and in 1998 (see also Bjorneberg and Trout, 2001, for complete soil description).

For each comparison, the infiltration and roughness parameters entered into SRFR were selected to yield as much agreement as possible between simulated and measured advance times and runoff volumes. For all runs, the representative particle size was selected as $D_s=0.05$ mm, somewhere in the midrange of actual particle sizes, along with a specific gravity of 2.65. The site-specific critical shear in Equation 2 was selected for all runs as $\tau_c=1.2$ N/m$^2$.

The erodibility, $K_r$, also intended to be common to all runs, was initially selected to equal 0.0003 s/m, a value suggested by experiments with WEPP; however, the very first SRFR run, with Trout’s, 1996, beans data, showed the inadequacy of this $K_r$ value. In view of the many uncertainties regarding the operation of WEPP on its data, the idea of using WEPP results to yield a suitable $K_r$ were abandoned, and a value was sought that would merely satisfy, more-or-less, all of the SRFR runs.

Fig. 3 compares calculated sediment-load hydrographs with average values at the quarter points in the furrow, gleaned from averages of the 1994 Trout beans data of July 1 (in the Kostiakov [1932] cumulative-infiltration formula, $d=kt^n$, $k=40$ mm/hr$^{0.6}$, while Manning $n=0.04$). The value of $K_r=0.001$ s/m input for the simulation was calibrated from the comparison between measured and calculated hydrographs at the first quarter point, before transport capacity plays much of a role in limiting sediment loads.

These limitations are clearly evident at subsequent quarter points in both measured and simulated data, the latter obtained with the Laursen transport-capacity formula. The importance of the transport-capacity formula in the computations is underscored by Fig. 4, which shows the same comparison, but with the Yang and Yalin formulas in force. The same calibrated $K_r$ value (again based on the first quarter point), $K_r=0.001$ s/m, was found for this case as well.

The increased growth in calculated sediment loads at the half and further-downstream points reflects the differences in transport capacities evident in Fig. 2.
Figure 6. Comparison of simulated sediment transport hydrographs at furrow quarter points with averages from measured (Bjorneberg and Trout, 2001) data of Aug 5, 1998. Furrow 8. Site-specific $K_r=0.001$ s/m, $\tau_c=1.2$ Pa. Laursen (1958) transport-capacity formula in effect. Transport overpredicted.

Figure 7. Comparison of simulated sediment transport hydrographs at furrow quarter points with averages from measured (Bjorneberg and Trout, 2001) data of Aug 5, 1998. Furrow 3. Site-specific $K_r=0.001$ s/m, $\tau_c=1.2$ Pa. Laursen (1958) transport-capacity formula in effect. Transport greatly overpredicted.

Fig. 5 is drawn for 1994 Trout corn-data averages for June 2/3, characterized by quite different infiltration parameters ($k=80$ mm/hr$^{0.7}$, $n=0.036$), but the same erosion characteristics ($D_s$, $K_r$, $\tau_c$) postulated as for the beans data. Qualitative agreement is again evident, although the computed transport capacity and sediment flux peters out before the field values did. The simulations in Fig. 5, like those in Fig. 6 and 7 were also obtained with the Laursen transport-capacity formula. Fig. 6, for Furrow 8, a run from August 5, 1998 ($k=142$ mm/hr$^{0.57}$, $n=0.036$), shows a three-fold increase in computed values of transport over measured ones -- still well within the scatter evident in the data selected as bases for the transport-capacity formulas by their respective authors. Perhaps significantly, the tests of August 5 were all run at such low flows that erosion was generally quite small. In Fig. 7, for Furrow 3 ($k=230$ mm/hr$^{0.58}$, $n=0.036$), the discrepancy increases to a factor of 10, suggesting the need for caution, in spite of the promising results of Fig. 3, 5, and 6.

**SUMMARY AND CONCLUSIONS**

With the Yang and Yalin formulas applied to the Trout data, transport capacity and erosion based on a representative aggregate size in the mid range of measured sizes proved greatly over-predicted, and deposition in the lower furrow sections under-predicted. The Yalin formula provided the poorer predictions, corroborating the WEPP experiences of Bjorneberg and Trout (2001). It is noteworthy that the Laursen and Yang transport-capacity formulas were recognized by Alonso et al. (1981) as superior to the Yalin formula in predicting transport capacity in long channels, both in flumes and in the field. But the Yalin formula was selected for WEPP, because it best predicted erosion in the very shallow rain-fed overland flow on concave hillsides (see, e.g., Foster, 1982).

Yang's approach of stream-power excess over a critical value of stream power is conceptually quite different from shear-stress excess over a critical value, at the core of many other formulas. Yang's approach, likewise, looks at the problem of total-load transport capacity in a considerably broader way than the single-phenomenon, particle saltation approach of Yalin. Laursen’s formula, on the other hand, is a classical exercise in dimensional analysis, with appropriate contributions from physical reasoning and even a little intuition – with the final results both confirmed and defined empirically.

With so great a dependence on empirical formulas in simulating soil entrainment and transport, furrow erosion by upstream inflow may be sufficiently different from hillside erosion by rain-fed overland flow that acceptable predictive approaches and formulas for the latter problem may not be satisfactory for the former. For example, pre-wetting phenomena have been shown to have a significant effect on detachment, but virtually all of the WEPP erosion database is for pre-wetted (rained-on) soils, which do not exhibit the violent fine-scale commotion observed at the front of a wave of irrigation water in a dry, powdery bed.

Further experimentation with SRFR is needed to evaluate the sensitivity of results to the input data. The transport formulas are sufficiently complex to defy sensitivity analysis without numerical experiments. The numerous thresholds in the various formulas suggest the need for some description of the *mix* of particles to avoid step responses. Allocation of portions of the total transport capacity to the fractions in the mix, along the lines of WEPP, would be an initial step.

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**REFERENCES**


